Mach Edges: A critical test of the nonlinear 3rd derivative model for edge detection

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1. Introduction to edge detection

Edges are key points of information in visual scenes. But, how are they extracted from the eye’s neural output? It is widely accepted that the retinal image is filtered by even- and odd-symmetric spatial operators of various scales, early in the visual pathway [1].

One view of these operators is that their role is to compute the spatial gradients and higher derivatives of the scales, early in the visual pathway [1].

2. Rationale and stimuli

As a critical test of the model, we add a linear luminance ramp to the blurred triangle waves used previously. This ramp has no effect on the second or higher derivatives, but the nonlinear 3rd derivative model predicts: (1) For ramp gradients of less than ±1 (below, 2 left plots) the gradient (1st derivative) profile has both positive and negative parts - 2 edges are predicted. (2) For steeper added ramp gradients (below, 3 right plots) the gradient (1st derivative) does not change sign. This causes one 3rd derivative peak to be blocked by a rectifier in the model - one edge predicted.

However, our previous feature-marking experiments found that peaks in the (inverted) 3rd derivative can signify edges where there are no 1st derivative peaks nor 2nd derivative zero-crossings [4]. These results on “Mach edges” the edges of Mach bands) were nicely predicted by a new non-linear model based on 3rd derivative filtering (Georgeson et al., 2007, in press). The model uses 2 stages of rectification to move spurious edges (the troughs shown here).

3. Experiment 1: Yes-No paradigm

Procedure
- Images were presented once for 0.3s.
- Task: indicate whether 1 or 2 edges were seen at the centre of the image.
- Interval between presentations was at least 1s.
- 1 session had 10 repetitions of the 18 conditions at one blur width, in random order, and took about 5 minutes.
- 4 subjects each performed 6 sessions for blur width 8 pixels followed by 6 sessions for blur width 2 pixels.

Results
- Data broadly similar for all 4 subjects & both phases: averages shown here.
- Error bars calculated from the standard deviation in binomial sampling:
  \[ \sqrt{-\frac{p}{n}} \]
  where \( p \) is the probability of response and \( n \) is the number of trials (460 here).
- Results were almost identical for the two blur widths.
- Two edges were seen reliably with added gradients up to ±0.5. This matched model predictions closely (solid line).
- With steeper added ramps, reports of two edges fell away as the model predicts, but more gradually than predicted.

4. Experiment 2: Feature-marking paradigm

Procedure
- Images were flashed repeatedly (0.3s on, 0.6s off).
- Task: mark the position and polarity of all edges and bars seen
- The marker comprised two black dots, each 1 x 3 pixels, vertically arranged, each 32 pixels (0.6 deg) from image midline.
- Subjects were instructed to fixate midway between the dots.
- 1 blurr width (8 pixels) = 9 ramp gradients = 18 conditions.
- 1 session consisted of 6 repetitions of each of the 18 conditions, in randomised order, and took about 30 minutes.
- 4 subjects each performed 4 sessions.

Main Results
- Data were broadly similar for all 4 subjects; averages are shown here.
- The task was reliable. Error bars are too small and are shaded behind symbols.
- All subjects marked a central bar flanked by 2 edges for gradients up to ±0.5.
- For gradients beyond ±0.5, bars and one edge were marked less frequently - smaller symbols here.
- Position and polarity of marked edges was well predicted by the 3rd derivative model. Absolute position was better for phase 180 than phase 0.

5. Conclusions and model refinement

Conclusion
- Two Mach Edges were reliably seen in a triangle-wave, but one disappeared when a steep linear ramp was added. This pattern of results was nicely predicted by the nonlinear 3rd derivative model.
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Model refinement - Add a retina
An additional pre-filter was added to the model. It has a Difference-of-Gaussians receptive field profile, based on P cells in central vision [8]. RF shape and spatial frequency response are shown above.

- Its effect is to accentuate edges as shown here (left). The filter modifies this waveform so that it now has a small central negative gradient, which allows both 3rd derivative edges to pass through the rectifiers in the model.
- The revised predictions, which also assume a noisy decision process, match the data well (right).