Performance Analysis of Packet Layer FEC Codes and Interleaving in FSO Channels

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Abstract

The combination of forward-error-correction (FEC) and interleaving can be used to improve free-space optical (FSO) communication systems. Recent research has optimized the codeword length and interleaving depth under the assumption of a fixed buffering size, however, how the buffering size influences the system performance remains unsolved. This paper models the system performance as a function of buffering size and FEC recovery threshold, which allows system designers to determine optimum parameters in consideration of the overhead. The modelling is based on statistics of temporal features of correct data reception and burst error length through the measurement of the channel good time and outage time. The experimental results show good coherence with the theoretical values. This method can also be applied in other channels if a Continuous-Time-Markov-Chain (CTMC) model of the channel can be derived.

Keywords: FEC codeword length, interleaving depth, FSO communication, CTMC model

1. Introduction

Free-space optical communication, which benefits from its high rate, flexible installation and licence-free spectrum, is now a hot topic again as the devices required have been developed rapidly over the last several years. The review [1] systematically introduces all aspects of FSO communication. It's widely known that FSO communication systems are vulnerable to weather effects and atmospheric turbulence as well as pointing errors. As a result, adaptation methods and transmission protocols are being actively studied.

FSO transmission provides high-speed connections for disaster recovery and temporal networks. However, for many scenarios such as real-time transmission and broadcasting, retransmission is either expensive or impossible to implement, so using forward error correction (FEC) is promising and has been widely studied [2][3][4]. Recent reviews [5] and [6] give an overview of coding methods for optical transmission. An important aspect of FEC coding is the selection of the code rate and codeword length, and it is introduced in [7]. Interleaving is also necessary because in high speed FSO transmission, an outage caused by atmospheric turbulence, which often lasts for several milliseconds, will cause a burst of packet erasures. The long burst error means that the interleaved FEC buffering size will be very large and optimization between the FEC codeword length and interleaving depth is necessary. Optimization between the codeword length and interleaving depth was first discussed with consideration of overhead [8], with the assumption of constant buffering size, which means the product of codeword length and interleaving depth is fixed. And based on that, later work [9] proposed a near-perfect interleaving depth to define an upper bound of the effective interleaving depth and used the interleaving-first (IF) algorithm to achieve better performance with lower complexity of coding and decoding. Recent work [10] focuses on reducing FEC processing delay by adopting a short block size and controlling the interleaving depth flexibly.

These studies discuss the optimization of codeword length and interleaving depth all based on the assumption that the total buffering size is fixed. However, the buffering size is determined by designers and its optimization also needs to be considered. Current studies are usually based on average features of errors such as the probability of bit or packet error and the correlation factor, instead of temporal features such as the error length or outage time distribution function, which is essential in buffering size and FEC recovery threshold determination.

It's widely known that the performance of FSO transmission systems is affected by various factors. In [11] and [12], Markov chains are used to model FSO channels but do not give the required statistical features of errors. However, a definition of Time Share of Fade (TSF) has been proposed [7] to describe the probability of the fading time being larger than T_f. Similarly, there have been measurements of the statistics of channel good time [13]. Using these results, we can conclude the statistical features of channel good time and outage time, and based on that, the distributions of burst error length and correct data length can be calculated with a given transmission time of the packet, according to which we can design channel-aware adaptation methods.

This paper proposes a model of system performance as a function of buffering size and FEC code rate, based on a CTMC model of an FSO channel. This approach allows us to optimize the FEC and interleaving strategy without fixing the product of FEC codeword length and interleaving depth. The model is verified through comparison with data from Monte-Carlo simulations.

The paper is organised as follows: Section 2 introduces the channel model we use. Section 3 proposes the system performance model and analyses the packet layer FEC codes and interleaving without fixed buffer size. Section 4 simulates the system and analyses the principle of the simulations we implement. Section 5 discusses the simulation results and the comparison of the calculated and simulated values of system performance. Finally, conclusions are drawn in Section 6.

2. FSO Channel Model

2.1 Discrete-Time-Markov-Chain (DTMC) Model

We assume that FEC is used at the packet level so that the channel can be considered as a binary erasure channel (BEC), which means the packet is either correctly received or discarded. Therefore, a general Gilbert-Elliot (GE) model [14], which is widely used when considering FEC coding and interleaving, is shown as Fig. 2.1. In the model, the channel condition is summarized by two states: good (or the channel is ON) and bad (or the channel is OFF). The channel good state is defined as: data transmitted in this state will all be received correctly. Similarly, the channel bad state is defined as: data transmitted in this state will be erased or discarded. Let p and q denote the transition probabilities from state G to state B and from state B to state G, respectively. The transition probability matrix is given by

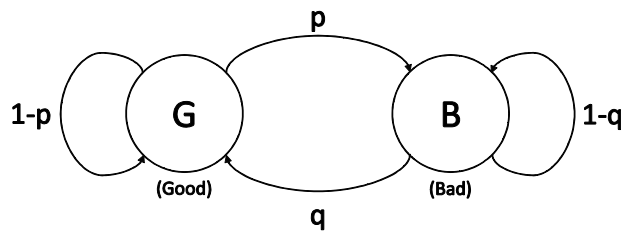$$T = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}.$$

Fig. 2.1 DTMC model of the FSO channel

Suppose $P^k = \begin{bmatrix} \pi_G^k & \pi_B^k \end{bmatrix}$ is the probability matrix of kth transmission state, where π_G^k and π_B^k denote the probability of being in the good state and bad state, respectively, then we have

$P^{k+1} = P^k T$. The final steady state $P^\infty = \begin{bmatrix} \pi_G^\infty = \dfrac{q}{p+q} & \pi_B^\infty = \dfrac{p}{p+q} \end{bmatrix}$ can be derived

through solving $P^\infty = P^\infty T$ and $\pi_G^\infty + \pi_B^\infty = 1$, where π_B^∞ also indicates the overall packet loss ratio (PLR) of the basic channel, i.e. before adaptation methods are implemented. A correlation coefficient ρ, defined as $\rho = 1 - p - q$, describes the probability of remaining in the current state [8]. When interleaving is added with an interleaving depth of d, the

distance between two formerly adjacent packets in an FEC frame will be d, so the new parameters in the DTMC model become

$$p' = \frac{p(1 - \rho^d)}{p + q} \text{ and } q' = \frac{q(1 - \rho^d)}{p + q}, \tag{1}$$

and the new correlation coefficient will be $\rho' = 1 - p' - q'$ [9]. From the formulae above, we know that when $p + q$ is close to 1, the channel correlation is very small and hence the channel is almost memoryless, so adding interleaving will not give a large improvement. But when $p + q$ is small, the channel is more likely to remain in the current state, which means the length of burst errors would be larger, so adding an interleaving strategy with a large d will decrease ρ', and therefore decrease the average length of burst error, so that FEC decoding would be much easier.

2.2 Continuous-Time-Markov-Chain Model

To design the adaptation system, the number of erasures that an FEC codeword can recover should be determined by the probability of recovering every burst error longer than a certain length, which in turn is determined by the system requirements. This value varies as a result of different distributions of burst error lengths even if the average error ratio is fixed, but the DTMC model gives only a fixed distribution. Measurements have been made of the temporal distribution of the fade time [7][15], and experimental data in [13] gives the distributions of channel good time. These results allow us to make a CTMC model of the channel so that we can derive the statistical features of the burst error lengths with a given packet transmission time. Then we can make a CTMC model of an FSO channel as shown in Fig. 2.2. However, in practice the ON/OFF statistics should be systematically measured before a strategy of FEC coding and interleaving is adopted.
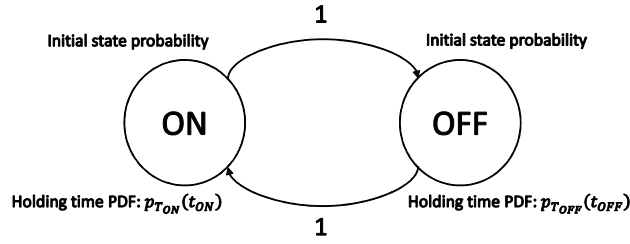
Fig. 2.2 CTMC model of FSO channel

The correlation coefficient is important to the adaptation strategy, so we now calculate ρ according to the CTMC model. Let t_P be the packet transmission time, μ_{ON} be the mean ON state holding time (ON time), and μ_{OFF} be the mean OFF state holding time (OFF time). In ON states, the average number of packets transmitted is $\dfrac{\mu_{ON}}{t_P}$. One of these packets is followed by an erasure and the other $\dfrac{\mu_{ON}}{t_P} - 1$ packets are followed by correctly received packets. Therefore, knowing the parameter p in the DTMC model denotes the probability of transition from state G to B, we now have $p = \dfrac{t_P}{\mu_{ON}}$ and similarly $q = \dfrac{t_P}{\mu_{OFF}}$. Thus, we have

$$\rho = 1 - \frac{t_P}{\mu_{ON}} - \frac{t_P}{\mu_{OFF}} \tag{2}$$

Furthermore, we can conclude that the system is highly correlated when the mean ON or OFF time is much greater than the packet transmission time.

3. Analysis of Packet Layer FEC Codes and Interleaving without Fixed Buffer Size

When an adaptation strategy is evaluated, the performance of the system and the overhead are often considered. In this section, we first introduce the restrictions of FEC codeword length n and interleaving depth d and then propose the model of system performance as a function of FEC tolerable error ratio and the buffering size, and finally estimate the overhead. Based on these analyses, the system designer can compare the improvement of performance and the extra overhead to decide which pair of parameters are selected, according to the specific scenario.

3.1 Restrictions of Interleaving Depth

Extending the codeword length of FEC and increasing the depth of interleaving are commonly known as effective ways to address burst errors. It is known experimentally that the performance of the transmission system is related to the buffering size $S = n \times d$ [8]. When S is fixed, the final PLR of the system is essentially fixed. Based on this assumption, the parameters n and d are optimized to reduce the overhead under the condition that the system performance is similar [9].

In highly correlated channels, although the FEC tolerant error ratio is higher than the average channel error ratio, packet FEC still fails because erasures often appear in bursts. And highly correlated error bursts require an extremely large FEC codeword length which will cause an unacceptably large coding and decoding overhead. Consequently, the most efficient solution is interleaving because it scatters the burst errors into every codeword so that the number of erasures in each codeword is averaged and thus tolerable. Also, because the complexity of FEC coding and decoding increases with the codeword length n, increasing the interleaving depth d is preferable. From (1) we know that ρ' approaches 0 as d increases. To describe this, we used experimental data from [7] and [13] to plot ρ' as a function of interleaving depth, as shown in Fig. 3.1. Experimental work [7] measures the holding time when the receiving

power is above and below the power threshold that the receiver can receive the data correctly, and gives the statistics of the OFF time. The power threshold is chosen based on the design decision [7] that the OFF probability is 0.3. The statistics of the ON time was measured in a different experiment [13], but this is not consistent with the average OFF probability in [7]. To ensure the consistency, we use the shape of the ON time distribution and adjust the mean and standard deviation proportionally to make the overall OFF probability 0.3. Assuming the information rate is 10Gbps and the code rate is 0.65, the total transmission rate is 15.385Gbps. Using 1046B as the packet size [7], we can derive the packet transmission time is 0.544μs, and $\rho = 0.999935$. The channel is a typical highly correlated FSO transmission channel.
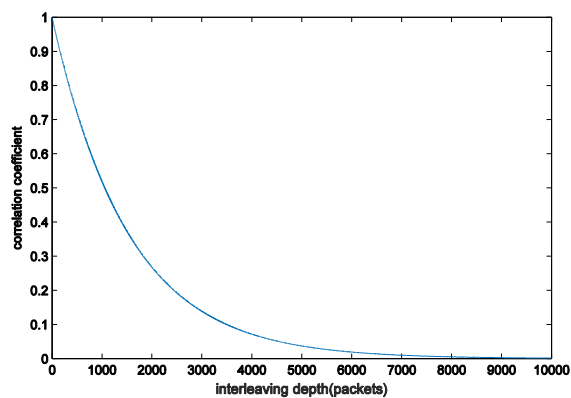

Fig.3.1 Correlation coefficient as a function of the interleaving depth

From Fig 3.1 we can see that the correlation coefficient asymptotically approaches zero for large d, thus increasing d becomes an increasingly inefficient way to reduce the channel correlation. To find a proper d, a threshold θ is defined in [9]: when $p' + q' \geq \theta$, the channel is considered to be nearly uncorrelated and using a larger d will have little impact on system performance. The parameter θ is determined by the application scenario, and when θ approaches 1, the system performance approaches that of a perfectly interleaved system. The corresponding minimum interleaving depth is $d(\theta) = \left\lceil \dfrac{\log(1 - \theta)}{\log(1 - p - q)} \right\rceil$, which is referred

to as the Near-Perfect (NP) interleaving depth (d_{NP}). According to (2), the near-perfect interleaving depth, or the maximally efficient interleaving depth is derived as

$$d_{NP}\left(\theta\right) = \left\lceil \frac{\log\left(1 - \theta\right)}{\log\left(1 - \dfrac{t_P}{\mu_{ON}} - \dfrac{t_P}{\mu_{OFF}}\right)} \right\rceil. \tag{3}$$

3.2 Restriction of FEC Codeword Length

However, even if the interleaving depth is smaller than d_{NP} , the codeword length n can't be too small because it leads to coding inefficiency [7]. For maximum distance separable (MDS) codes, the whole n-packet codeword that includes k information packets can be recovered if at least k packets are received. But for non-MDS codes, the receiver has to receive more than k packets to recover the erased packets. This effect is called coding inefficiency, and the ratio of the extra part needed is called the inefficiency factor Ψ. Ψ is typically negatively correlated with n when the type of FEC code is determined, denoted as $\psi\left(n\right)$. As a result, increasing n appropriately will reduce the overhead of redundancy, and therefore increase the effective throughput when the total throughput is fixed. Some common examples of LDPC codes are shown in Table 3.1.

Table 3.1 Examples of LDPC inefficiency and other parameters

Code	Codeword Length	Information bits	Redundancy bits	Code rate	Overall efficiency	Inefficiency
Long LDPC	16200	14400	1800	88.9%	88.6%	0.003
Medium LDPC	5940	5040	900	84.8%	84.2%	0.006
Short LDPC	1120	840	280	75%	71.4%	0.036

3.3 System Performance Model

Following our discussion of the consequence of varying n and d and their restrictions, we now discuss how the system performance is related to S and FEC code rate. Note that $P\{X\}$ represents the probability of event X happens, the system performance $\omega = 1 - PLR$ can be derived according to the probability as:

$$\omega = P\{r < \theta_e\} \times 1 + \left(1 - P\{r < \theta_e\}\right) \times \frac{q}{p + q}, \tag{4}$$

where θ_e is the largest tolerable ratio of erasures in an FEC codeword, and $\dfrac{q}{p + q}$ implies the ratio of successfully received packet before adaptation methods.

From [8] we know that the performance of different systems is very close if the buffering size is the same. Assume for now that the buffering size contains only one codeword, so that the system performance can be estimated by the probability of the error ratio in the buffering size being smaller than the tolerable error ratio θ_e. Because the burst error length is proportional to the OFF time and the correct data length is proportional to the ON time, with the same factor t_P, we can use the ratio of OFF time over the total time to describe the error ratio. During the transitions between ON- and OFF- states in the CTMC channel model, determining the buffering size is like determining the sampling size to observe the OFF time ratio in the random process. Provided that the expected ratio of OFF time over the total time is r_0, for a certain sampling size, the observed r from experiments should be close to r_0, with a certain distribution called the sampling distribution, and its deviation should be smaller when the sampling size gets larger. An example is shown in Fig. 3.2, where the area of the shadow part of the figure denotes the probability that the erasures would not be recovered. If the sampling size is large enough, according to the central limit theorem, the distribution will be close to Gaussian.
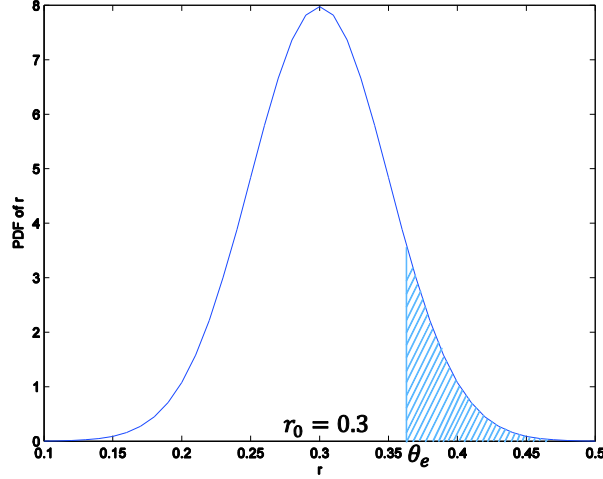
Fig. 3.2 The PDF of the sampling distribution of the OFF ratio

For the current problem, the sampling size equals the transmission time of S packets. If we have $r_0 = \theta_e = 0.3$, there's 0.5 probability that the whole codeword can be recovered and another 0.5 probability that erasures are all discarded. As a result, the ratio of correct packets when $S \to \infty$ would converge at $0.5 \times 1 + 0.5 \times 7 = 0.85$ according to (4).

For a general CTMC model of the channel, let $p_{T_{ON}}(t_{ON})$ be the PDF of the ON time and $p_{T_{OFF}}(t_{OFF})$ be the PDF of the OFF time. If we observe N pairs of ON and OFF transitions, the PDF of r is denoted by $f(r; N)$, and its CDF is denoted by $F(r; N)$. Given θ_e, we can derive the system performance by

$$\omega = F(r; N) \times 1 + (1 - F(r; N)) \times \frac{q}{p + q} = \frac{p \cdot F(\theta_e; N) + q}{p + q}. \qquad (5)$$

And using the definition of CDF, we can transform $F(r; N)$ into

$$F(\theta_e; N) = P\{r \leq \theta_e; N\}, \text{ or} \qquad (6)$$

$$F(\theta_e; N) = P\left\{\frac{T_F}{T_N + T_F} \leq \theta_e; N\right\}, \qquad (7)$$

where T_N and T_F denote the sum of the observed ON and OFF time within sampling size N, respectively. Because T_N and T_F are positive numbers, the formula (7) can be transformed into:

$$F(\theta_e; N) = P\{(1 - \theta_e)T_F \leq \theta_e T_N; N\} \tag{8}$$

Then knowing T_N and T_F are mutually independent, we can derive:

$$F(\theta_e; N) = \iint_{(1-\theta_e)t_F \leq \theta_e t_N} f_{T_N}(t_N; N) f_{T_F}(t_F; N) dt_N dt_F , \tag{9}$$

where $f_{T_N}(t_N; N)$ and $f_{T_F}(t_F; N)$ are the PDFs of t_F and t_F, respectively. In order to calculate these distributions, $f_{T_N}(t_N; N)$ can be derived as the PDF of the sampling distribution of T_{ON}, with a sampling size N, whereas $f_{T_F}(t_F; N)$ is matched similarly with T_{OFF}. Particularly, if N is greater than 30, according to the central limit theorem, $f_{T_N}(t_N; N)$ (or $f_{T_F}(t_F; N)$) is close to Gaussian, with a mean value of $N\mu_{ON}$ (or $N\mu_{OFF}$) and a deviation of $\sqrt{N}\sigma_{ON}$ (or $\sqrt{N}\sigma_{OFF}$), where σ_{ON} (or σ_{OFF}) denotes the deviation of T_{ON} (or T_{OFF}). And when N is large enough, we can estimate the sampling size by $N = \dfrac{\mu_{ON} + \mu_{OFF}}{S \times t_P}$. Thus the system performance becomes:

$$\omega = \dfrac{p \cdot \displaystyle\iint_{(1-\theta_e)t_F \leq \theta_e t_N} f_{T_N}\left(t_N; \dfrac{\mu_{ON} + \mu_{OFF}}{S \times t_P}\right) \cdot f_{T_F}\left(t_F; \dfrac{\mu_{ON} + \mu_{OFF}}{S \times t_P}\right) dt_N dt_F + q}{p + q} . \tag{10}$$

Thus, given the required system performance ω_0 according to the particular deployment scenario, we can verify if θ_e and S can satisfy the requirement.

3.4 Bandwidth Efficiency and Overhead

Let B be the total bandwidth of the channel, so that the effective throughput can be calculated by $B \times (1 - R) \times \omega_0$. The minimum FEC redundancy ratio should be $R = \theta_e + \psi(n)$. In order to ensure the effective throughput under the limitation of a certain bandwidth, we define the bandwidth efficiency by

$$\eta = (1 - \theta_e - \psi(n)) \times \omega_0. \qquad (11)$$

Another factor the designer must consider is overhead. The overhead C can include buffering time $C_{buffer}(n \times d)$, FEC coding and decoding time $C_{FEC}(n)$, and the associated costs $C_{\exp ense}(n)$ including coding and decoding power and the codec's price. Designers can weigh them according to the specific scenario with factors α, β and γ:

$$C = \alpha C_{buffer} + \beta C_{FEC} + \gamma C_{\exp ense}, \qquad (12)$$

or add some other overhead they are concerned about.

We can now adjust n and d and then compare the corresponding η and C to decide which pair of coding/interleaving parameters should be adopted. If the system tolerable PLR and the FEC recovery threshold are given, we can derive the needed buffering size, and then adopt IF [9] or other algorithms to optimize the FEC codeword length and interleaving depth. Besides, we can also fix the interleaving depth to a near-perfect value and list several FEC recovery thresholds, so that we can derive both system performance and overhead as functions of the FEC codeword length, after which the optimization can be done.

For example, we can first fix d to $d_{NP}(0.9)$, so that η and C can be calculated as a function of n, and then we select the n that results in the optimum η and C. Specifically, suppose the system tolerable PLR is 0.03 and $\psi(n)$ is fixed, we plotted three performance curves as a function of buffering size with $\theta_e = 0.33, 0.328, 0.325$ in Fig. 3.3. The needed buffering

sizes are marked in the figure. If it is not essential to reduce the bandwidth efficiency to such a small value in the current scenario, we will certainly choose the parameters with $\theta_e = 0.33$ because it saves 1/3 of the buffering size in comparison to $\theta_e = 0.325$. If $d_{NP}(0.9)$ is so large that $\psi(n)$ is too large, we can use $d_{NP}(0.8)$ or less and compare the corresponding η and C to make a decision.
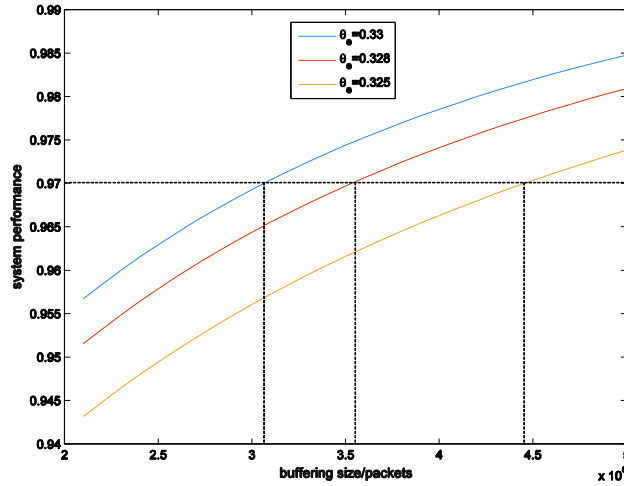

Fig. 3.3 Given the system performance requirement, different S is needed according to different θ_e

4. System Simulation

4.1 Parameter Selection

Just as section 3.1 introduced, in our experiments, the probability of ON-state is fixed by $\frac{q}{p+q} = 0.7$ according to [7]. For the OFF time PDF, we use statistical features of the channel data taken directly from [7]. For the ON time PDF, we use data from [13] with its distribution shape, and adjust the mean and standard deviation parameters slightly and

proportionally in order to achieve the condition $\dfrac{q}{p+q} = 0.7$. The packet transmission time

is set to 0.544μs, and the correlation factor before adaptation is $\rho = 0.999935$.

4.2 Implement

Our model is implemented in MATLAB and the theoretical methods are verified using Monte-Carlo simulations. The process of the simulation is designed and implemented as follows.

Firstly, the channel condition is simulated according to the CTMC model, by a 0-1 sequence that is able to be operated for packet erasures. To achieve this, a sequence of ON and OFF times in the CTMC model $[\ t_1^{ON}\ t_1^{OFF}\ t_2^{ON}\ t_2^{OFF}\ t_3^{ON}\ t_3^{OFF}\ \dots]$ is generated according to the parameters as discussed in section 4.1. This sequence of transition times then allows us to determine if a particular packet is fully received, erased or occurs over a transition boundary. In the latter case where only part of a packet is erased, we assume that inside the packet, an inner FEC can recover errors if more than 70% of the packet is received. The channel sequence is thus derived. The whole process is shown in Fig. 4.1 and the results confirmed that the overall ON time ratio is around 0.7.
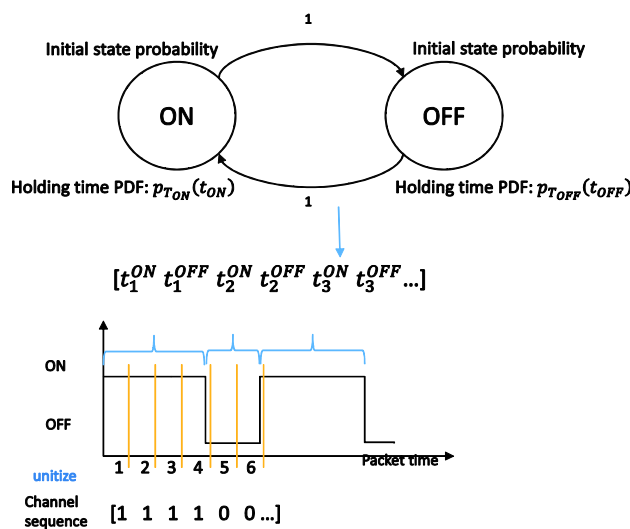
On the basis of the derived channel sequence, the next process in the simulation is coding, transmission and decoding. Before the source data is sent, packet level FEC is applied and interleaving is performed. Then the coded data sequence is modified according to the current channel condition to simulate the successful transmission, or packet erasure. Finally, the received packets are de-interleaved and decoded, so that the system PLR can be calculated. This process is illustrated as Fig. 4.2.
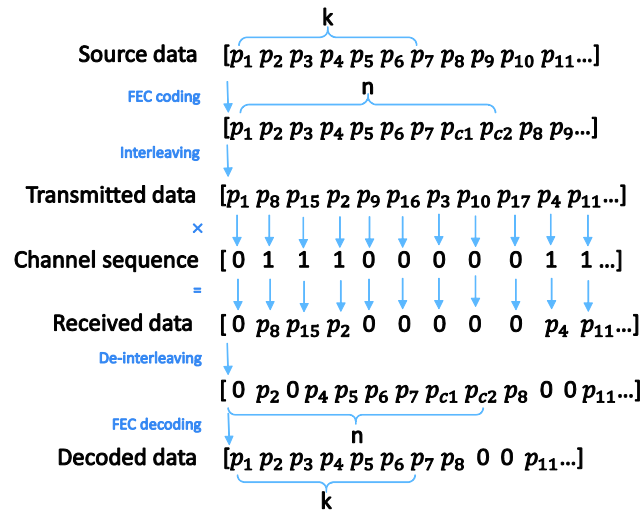

Fig.4.2 Simulation outline of data transmission and FEC coding and decoding

5. Results and Analysis

5.1 FEC Codeword Length and Interleaving Depth

In our numerical simulations, we first simulate the urban environment discussed in [8]. The parameters used are $\frac{q}{p+q} = 0.6$, $\rho = 0.99975$, and $R = \theta_e = 0.5$. Second we use the parameters introduced in section 4.1 and $R = \theta_e = 0.65$. The results of these two

simulations are shown in Fig. 5.1. The result corresponding to the higher value of ρ is shifted to higher values of interleaving depth as expected from our previous discussions.
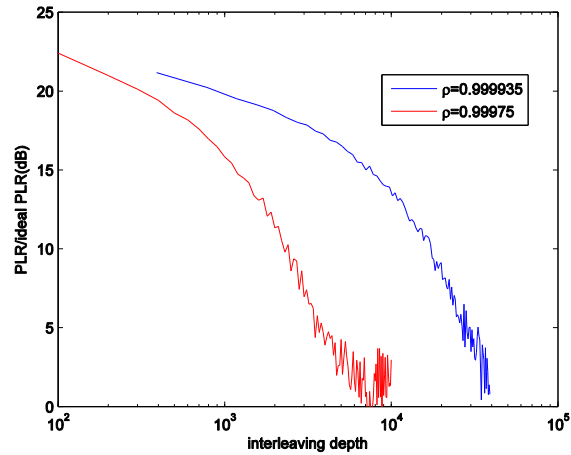

Fig. 5.1 Interleaving causes different performance when ρ is different

We now consider the effect of changing the FEC codeword length n using values of 128, 10000, 20000, 50000 and gradually increase the interleaving depth d, the system performance obtained is shown in Fig. 5.2. We can conclude from the figure that the system performance is uniquely characterised by the buffering size $S = n \times d$, as predicted in [8].
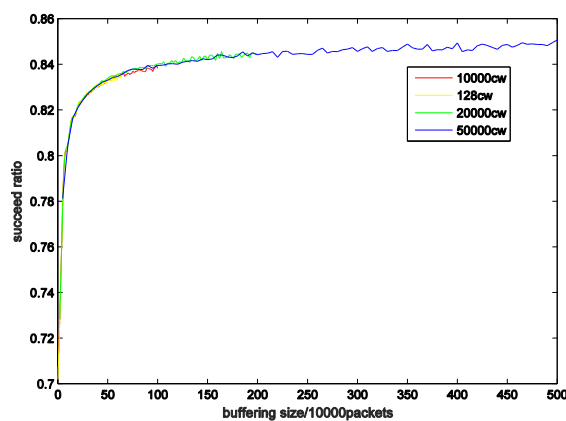

Fig. 5.2 System performance depends on buffering size.

To evaluate the system performance at infinite buffering size in the simulation, we use curve fitting tools to derive the value numerically. We fit the system performance with the three

parameter function $f(x) = a \times x^b + c$. An example of quality of the fitting is shown in Fig. 5.3, the excellent agreement gives us confidence in the derived asymptotic values.
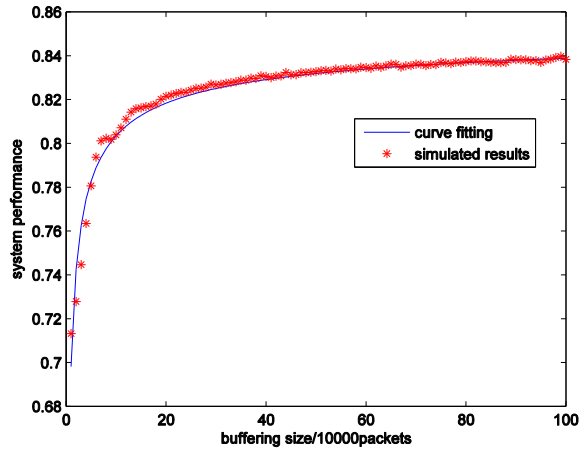

Fig. 5.3 Curve fitting is used to indicate the performance at infinite buffering size in simulation

This procedure is then applied in several groups of experiments in which $R = \theta_e = \dfrac{p}{p + q}$

and the results are shown in Table 5.1. In the function $f(x) = a \times x^b + c$ where b is negative, when $x \rightarrow \infty$, $f(x)$ will converge to c. From Table 5.1, we can see that when

$\dfrac{q}{p + q} = 0.7$, c varies slightly around the theoretical value 0.85, and when $\dfrac{q}{p + q} = 0.8$,

the theoretical value according to equation (4) is $0.5 \times 1 + 0.5 \times 0.8 = 0.9$, and c is close to 0.9, as expected.

Table 5.1 Results of curve fitting

$\dfrac{q}{p + q}$	**Codeword Length (n)**	**a**	**b**	**c**
0.7	50000	-0.07236	-0.5339	0.8545
0.7	20000	-0.1236	-0.6135	0.8514
0.7	10000	-0.1586	-0.473	0.8568
0.8	50000	-0.06187	-0.7018	0.8973

0.8	20000	-0.09936	-0.513	0.9049

5.2 Comparison between Theoretical and Simulated Performance

To verify the strategy, we compare the calculated system performance with the simulated one. The system performance can be calculated based on our theoretical considerations according to (5). In the function $F(\theta_e; N)$, we assume N is large enough and use a Gaussian distribution to predict $f_{T_F}(t_F; N)$. Also because N is large enough, we have the following approximate relation between S with N, $S = N \times (\mu_{ON} + \mu_{OFF})/t_P$, and then plot the calculated system performance in the same figure with the simulated one, as shown in Fig. 5.4. The agreement between the theoretical value and the simulation is worse at small S because the relation $S = N \times (\mu_{ON} + \mu_{OFF})/t_P$ is invalid here and our OFF time distribution is not Gaussian. Specifically, the corresponding S should be greater than about 2,200,000 packets, if N is greater than 30, to satisfy the basis of using the central limit theorem, and as can see from Fig. 5.4, that the agreement is much better in this region of large buffer size.
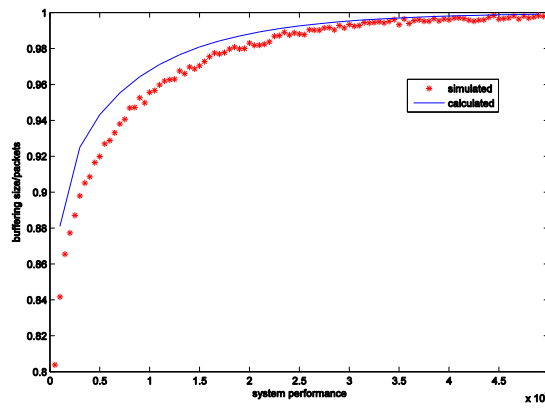

Fig. 5.4 Comparison of calculated and simulated value of system performance

When we change θ_e to 0.33 and 0.314, a group of similar results are derived, as shown in Fig. 5.5, from which we can see our mathematical prediction shows agreement with the simulated values. However, they are also restricted by the requirement of a large N, or a large S, which

is not a problem since the needed buffering size is around 3GB, which is not difficult to achieve now and such large block FEC is necessary and already widely used in highly correlated FSO transmission system.
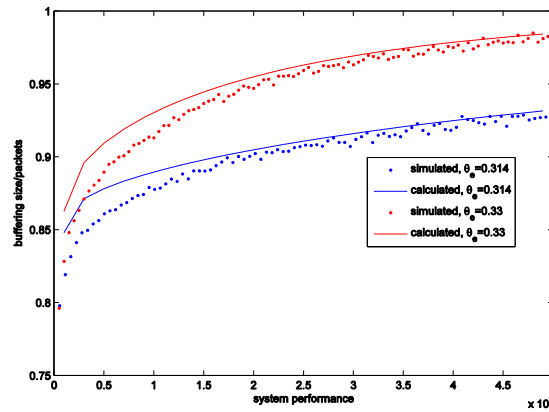

Fig. 5.5 Different groups of comparison when parameters are changed

6. Conclusion

In this paper, we have proposed a theoretical model of system performance as a function of FEC buffering size and recovery threshold, based on a CTMC model of an FSO channel. It allows the designer to estimate the needed buffering size in a specific scenario or to determine the optimum parameters without fixing the buffering size first. We have designed a Monte-Carlo simulation of FSO packet transmission including adaptation of the channel. The model is in very good agreement with the results of the numerical simulations. In the scenario where the raw channel good probability is 0.7, the overall system performance we have achieved is 0.97 using a buffer of 3.1×10^6 packets and FEC recovery ratio of 0.33. This method is applicable in situations such as long-range or mobile transmissions where the packet error rate before adaptation is significant. The CTMC model can be applied to any FSO channel provided the ON and OFF state statistical distribution functions are known. Our

analysis of the interleaved FEC can also be applied in other (non-FSO) channels if a CTMC model of the channel can be derived.

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