Accepted Manuscript

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Please cite this article as: Peter Wanke, Abul Kalam Azad, Ali Emrouznejad , Efficiency in
BRICS banking under data vagueness: A two-stage fuzzy approach, (2017), doi: 10.1016/
j.gfj.2017.05.001

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Efficiency in BRICS Banking under Data Vagueness: A Two-Stage Fuzzy Approach

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ABSTRACT

This study analyzes the efficiency levels of the banking industry in the BRICS countries (Brazil, Russia, India, China, and South Africa) from 2010 to 2014, using an integrated two-stage fuzzy approach. Very often the reliability of data collected from BRICS is questionable. In this research, we first use fuzzy TOPSIS to capture vagueness in the relative efficiency of BRICS banking over time. In the second stage, we adopt fuzzy regressions based on different rule-based systems to enhance the power of significant socioeconomic, regulatory, and demographic variables to predict banking efficiency. These variables are previously identified by using bootstrapped truncated regressions with conditional α-levels, as proposed by Wanke, Barros, and Emrouznejad (2015a). The results reveal that efficiency in the banking industry is positively associated with country gross savings and the GINI index ratio, but negatively associated with relatively high inflation ratios. Fuzzy regressions proved far more accurate than bootstrapped truncated regressions with conditional α-levels. We derive policy implications.

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1. Introduction

Studies using alternative models to measure banking performance have been increasing in number over the years (Behzadian, Khanmohammadi Otaghsara, Yazdani, & Ignatius, 2012; Berger & Humphrey, 1997; Kahraman, Onar, & Oztaysi, 2015; Liu, Lu, Lu, & Lin, 2013a, 2013b; Ou, Hung, Yen, & Liu, 2009; Sampaio, 2013). These models can be classified as parametric or nonparametric (Berger & Humphrey, 1997; Fethi & Pasiouras, 2010; Porcelli, 2009). Broadly speaking, parametric models allow various types of inferences to be drawn directly from performance estimates (Kumbhakar, Parmeter, & Tsionas, 2013). Nonparametric models fall short because they need statistical properties for a robust examination of the roots of inefficiency in light of contextual variables. Thus far, bootstrapping—i.e., performance error resampling—is the only statistical tool available to remedy this situation (Bogetoft & Otto, 2010).

While SFA (Stochastic Frontier Analysis) remains the most popular of the parametric models (Sampaio, 2013), DEA (Data Envelopment Analysis) is the most popular among the nonparametric ones (Amsler, Lee, & Schmidt, 2009; Liu et al., 2013a, 2013b; Paradi & Zhu, 2013; Zhou, Ang, & Poh, 2008) and is widely applied in the banking industry (Paradi & Zhu, 2013). The majority of the banking papers using DEA have focused on developed countries, although there are some recent studies on developing economies (Liu et al., 2013b; Paradi & Zhu, 2013; Porcelli, 2009; Wanke, Azad, & Barros, 2016a; Wanke, Azad, Barros, & Hadi-Vencheh, 2016b).

In a traditional DEA model, performance is calculated using historical data on inputs and outputs (Berger & Humphrey, 1997; Charnes, Cooper, & Rhodes, 1978). Battese and Rao (2002) showed that this method discriminates more finely—i.e., efficiency scores are less biased towards one—if the data encompass several years of observation, as is also true for multicriteria decision-making models (MCDM). MCDM are also nonparametric because there are no

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underlying statistical properties whatsoever. Until now, MCDM such as TOPSIS (Technique for

Order Preference by Similarity to Ideal Solution) have not been used to assess performance in

banking at the level of economic blocks (Behzadian et al., 2012), although a number of

alternative MCDM approaches have been applied to assess performance in banking at a country

level (Doumpos & Zopounidis, 2002; Hwang & Masud, 2012; Hwang & Yoon, 2012; Kahraman

et al., 2015).

TOPSIS is based on the concept that the positive ideal alternative has the best level for all

criteria considered or for the input/output set, while the negative ideal has the worst values for

the input/output set (Wanke, Azad, Barros, & Hassan, 2016c). Although it generally resembles

DEA, in which outputs may be maximized and/or inputs minimized, TOPSIS exogenously

defines the relative weights of criteria (namely, benefits and costs, or simply outputs and inputs,

respectively), whereas in DEA these weights are calculated endogenously (Behzadian et al.,

2012). TOPSIS is also computationally simpler because there are virtually no constraints on the

number of companies and criteria that can be assessed (Wanke et al., 2016c).

Although nonparametric methods may be sufficient to determine performance levels, they

do not afford details on the effects of context. To remedy this, several studies have proposed

two-stage approaches for measuring and explaining bank performance, using either DEA or any

MCDM to compute performance levels (e.g., Wanke et al., 2016a; Wanke, Barros, & Faria,

2015b) and regression models to explain their drivers (Wanke, Pestana Barros, & Chen, 2015c).

Importantly, the underlying uncertainty of performance levels—and, therefore, of the

input/output set and their relationships with contextual variables—encompasses both randomness

and fuzziness. While randomness is related to statistical fluctuation resulting from data

collection, fuzziness is related to the underlying vagueness of the data collected (Wanke, Barros,

& Emrouznejad, 2015a).

Banking input and output variables may be both fuzzy and random. On the input side, for instance, credit granting is a variable surrounded by fuzziness because the assessment of loan risk is intrinsically vague (Demirgüç-Kunt & Huizinga, 1999). In contrast, the value of banking production is random by nature because it changes according to the country's economic conditions. In BRICS countries even banking production may be subject to vagueness because of poor data (Agrawal & Kumar, 2012;[1] Jerven, 2013; Koch-Weser, 2013[2]).

The techniques used in the two-stage approach adopted here take advantage of new developments in scientific computing technologies, such as the software R, which can be used to support systematic theory testing and development (James, Witten, Hastie, & Tibshirani, 2013). We use the fuzzy version of the TOPSIS technique in the first stage, while in the second stage we use bootstrapped truncated regressions, as Wanke and colleagues (2015a) proposed, to evaluate the sign and significance of the effect of contextual variables on banking performance. We also use fuzzy regressions to boost the predictive power of these significant variables under varying modelling assumptions.

The contributions of this paper are fourfold. First, we evaluate the evolution of efficiency in BRICS banking, adding to the scarce literature on banking performance at the level of economic blocks (Cheng, Gutierrez, Mahajan, Shachmurove, & Shahrokhi, 2007; Choudhury & Ashraf, 2016; Zhang, Jiang, Qu, & Wang, 2013). Second, we use a two-stage fuzzy approach to assess vagueness and randomness in banking inputs and outputs. Third, we apply different types of fuzzy regressions to complement statistical bootstrapped methods (see Arunraj, Mandal, & Maiti, 2013). Fourth, as we analyze the years after the world financial crisis of 2007–2008, this study sheds some light on the BRICS banking industry's effectiveness in handling financial distress.

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[1] http://www.eastasiaforum.org/2012/04/06/nagaland-s-demographic-somersault-how-reliable-are-india-s-official-statistics/

[2] http://www.uscc.gov/sites/default/files/Research/TheReliabilityofChina'sEconomicData.pdf

The remaining parts of this study are organized as follows. Section 2 depicts the setting, while Section 3 reviews the literature. Section 4 describes our methods, including fuzzy TOPSIS and fuzzy regression. Section 5 discusses the empirical results and their implications for policy-making, and Section 6 draws conclusions.

2. Setting

Since O'Neill (2001) enlightened the world about the patterning of the BRICS, an emerging literature has examined various aspects of their potential, such as market opportunities (Cheng et al., 2007; Mobarek & Fiorante, 2014; O'Neill, 2011), rising powers (Jacobs & Van Rossem, 2014; Laïdi, 2012), competitive input of factors (Radulescu, Panait, & Voica, 2014), and strong foreign currency reserves (Radulescu et al., 2014). Researchers such as Jacobs and Van Rossem (2014) and O'Neill (2011) have argued that the minimal effect of the 2008 global crisis on these countries, combined with their growth potential in the last decades compared to the G7 or G20 (cf. Luna, 2016), has shifted world focus from developed economies to emerging economies, especially cross-regional integrations such as BRICS. Additionally, the high levels of foreign currency reserves and high investment rates among these countries have fueled the integration of BRICS vis-à-vis developed countries (Radulescu et al., 2014).
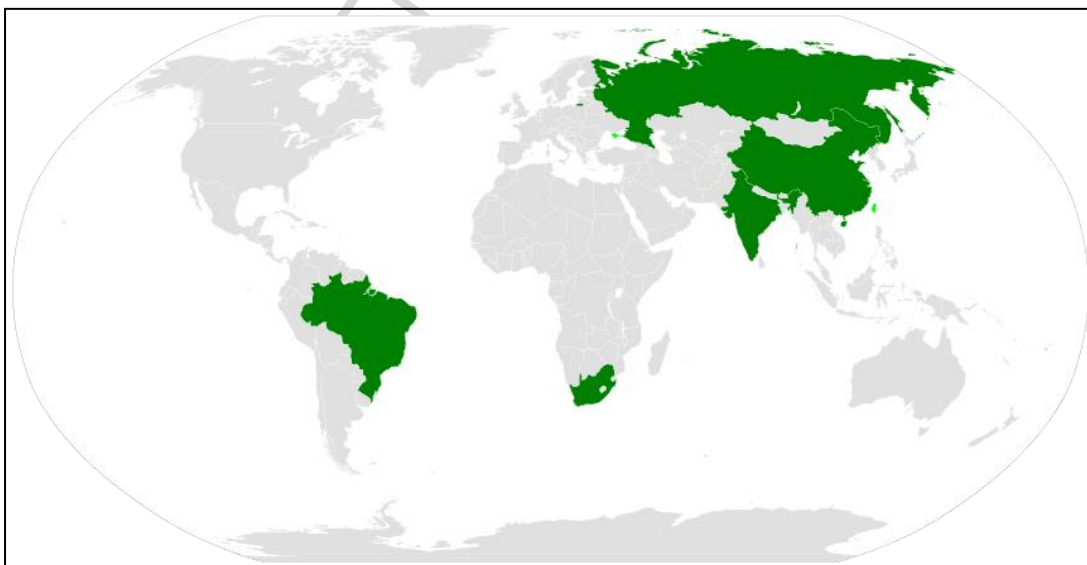
Fig. 1. Geographical location of BRICS countries (highlighted in green).

Source: public domain, organized by Felipe Menegaz.

According to World Bank data for 2015, the BRICS countries have more than 3 billion people, a number that is more than the combined population of the USA and Canada and accounts for 42% of the world's population. In the long term, such a huge untapped market may be the best hedge for globalization among these emerging economies. All of these countries are G20 members, and their combined nominal GDP of USD 16 trillion represents one-fifth of total gross world product. However, in aggregate value, BRICS is outperformed by the G7, and individual BRICS countries differ significantly in economic indicators and other indexes.

In particular, the patterns and volumes of banking indicators show significant variation. For instance, capacity for risk (capital adequacy ratio in Fig. 2a) has been increasing for India and China, but declining in the remaining countries. Fig. 2b shows that BRICS countries differ in asset quality (nonperforming loans). Bank branches for all countries have been increasing over the period (Fig. 2c), as have bank loans to the domestic private sector, except in China (Fig. 2d). Moreover, in the current decade individual BRICS countries have not performed uniformly. Brazil, which was already experiencing economic recession in earlier years, saw worse depression than in the 1930s. Because of the recent sharp decrease in the price of fuels and the weight of sanctions, Russia, too, is in recession. Thus, the economic dynamics of the BRICS region call for further research, especially on banking.
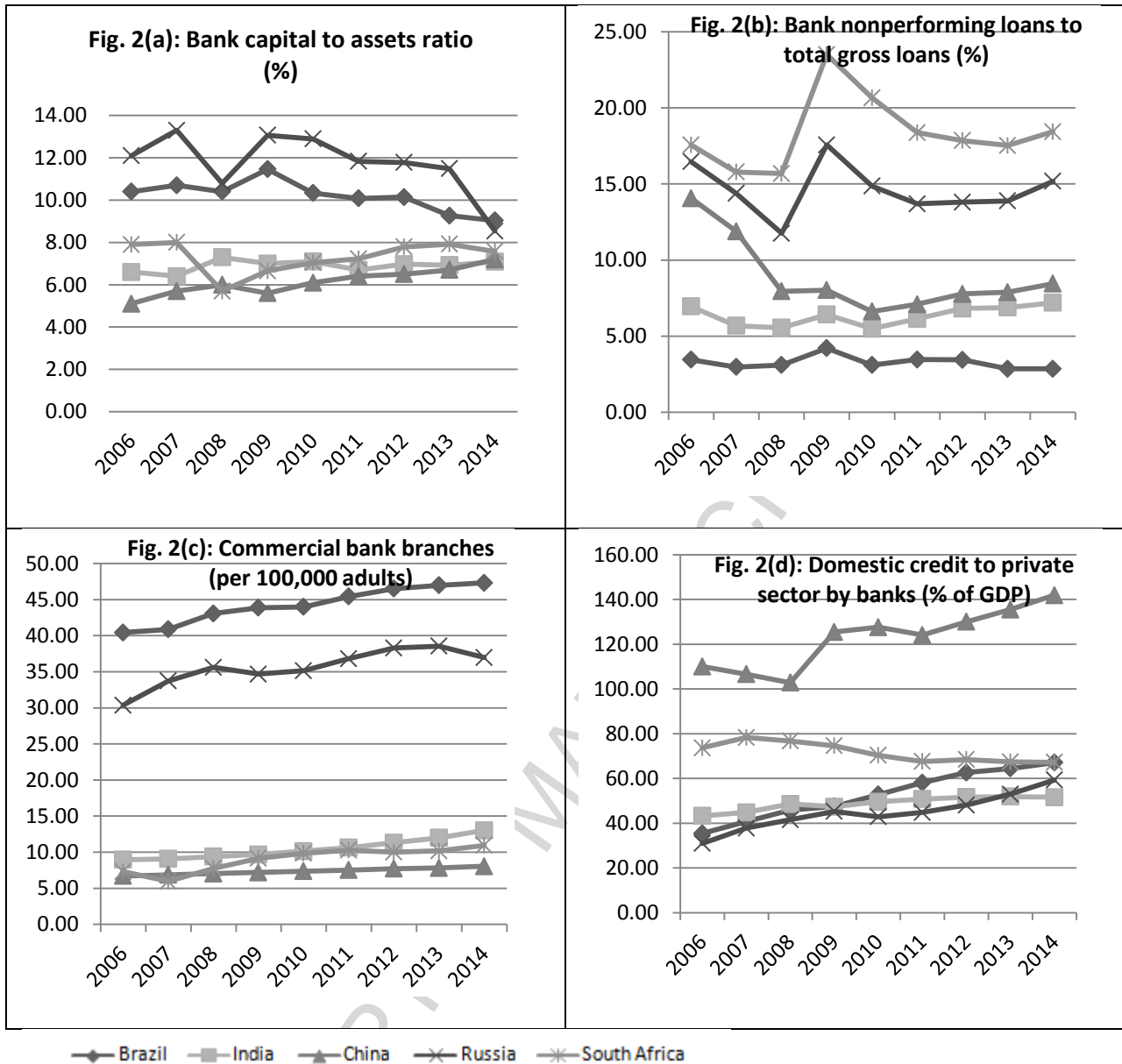
Fig. 2. Bank data for BRICS countries.

Source: World Bank.

3. Literature review

In an earlier survey of 130 international financial efficiency studies, Berger and Humphrey (1997) found that most used one of five approaches. Major nonparametric approaches included DEA and its return-to-scale and convexity constraint variants such as the free disposal hull (FDH). Among the parametric approaches, SFA stands out as the most frequently used,

followed by the thick and the distribution-free frontier approaches (TFA and DFA, respectively).

A more recent survey on DEA (Liu et al., 2013b) and SFA (Amsler et al., 2009) revealed that

these efficiency models are most frequently applied to banks and other financial institutions. Of

course, each model has its advantages and limitations (Berger & Humphrey, 1997).

More recently, Kahraman, Onar, and Oztaysi (2015) examined the application of different

MCDM methods of efficiency estimation to real-world decision making and revealed that the

earlier frontier approaches had oversimplified a complex, ill-structured problem (Carlsson &

Fullér, 1996). Moreover, the bank-level data that are often used in examining efficiency are

vague and incomplete. For such an environment, fuzzy MCDM is the best alternative analytical

method (Dubois, 1980; Kahraman et al., 2015). The latest developments and major applications

of fuzzy MCDM may be found in Abdullah (2013) and Kahraman and colleagues (2015). The

major subdivisions of MCDM are multiattribute decision making (MADM) for discrete problems

and multiobjective decision making (MODM) for continuous MCDM problems. Kahraman and

colleagues (2015) reported on 20 popular fuzzy MADM methods and 3 fuzzy MODM methods.

Further discussions of these methods can be found in Hwang and Masud (2012). Some major

applications of fuzzy MCDM include fuzzy analytical hierarchy process (f-AHP) (Mandic,

Delibasic, Knezevic, & Benkovic, 2014; Wanke et al., 2016b), fuzzy TOPSIS (Lima-Junior &

Carpinetti, 2016; Mandic et al., 2014; Tansel İç, 2012; Wanke et al., 2015c), fuzzy f-ELECTRE

(Doumpos & Zopounidis, 2002), and fuzzy VIKOR (Gul, Celik, Aydin, Gumus, & Guneri,

2016).

A number of studies of banking efficiency have taken individual BRICS countries into

consideration: Wanke, Barros, and Faria (2015b) examined the recent efficiency of Brazilian

banks, Shi and Zou (2016) examined Chinese banks, and so on. However, bank performance in

the BRICS as a whole has yet to be examined. Moreover, using fuzzy TOPSIS to examine bank

performance and then using fuzzy regression to estimate sources of efficiency would contribute to the existing literature on this region.

4. Method

This section explains the major computational steps performed in this study. Section 4.1 presents the contextual (socioeconomic, regulatory, and demographic) variables and the input/output set used in this study, and then explains the two stages of the fuzzy approach. Section 4.2 describes the fuzzy TOPSIS method used in the first stage, and Section 4.3 reviews the fundamentals of truncated regression with bootstrapping at each conditional α-level, the technique used in the second stage (see Wanke et al., 2015a). Lastly, section 4.4 addresses a number of different possible rule-based systems embedded within the environment of fuzzy regressions as discussed by Riza, Bergmeir, Herrera, and Benítez Sánchez (2015).

4.1. The data

The data on BRICS banking were obtained from different datasets, such as the Bankscope and World Bank databases, and encompassed the period from 2010 to 2014. As far as these data allowed, we used the same negative and positive criteria used by previous researchers. For the fuzzy TOPSIS model described in Section 4.2., the negative criteria (input variables) included lower reserves for impaired loans/NPLs, total capital ratio, Tier 1 ratio, loan loss reserves/gross loans, loan loss provision/net interest revenues, fixed assets, nonearning assets, equity, total liabilities and equity, loan loss reserves, liquid assets, overheads, loan loss provisions, and tax. The positive criteria (output variables) included ratios of equity to assets, equity to net loans, equity to short term funding, equity to liabilities, cost to income, and net loans to total assets; net interest margin; return on average assets (ROAA) and on average equity (ROAE); recurring earning power; interbank ratio; loans; total earning assets; total assets; deposits and short term

funding; other (noninterest bearing); net interest revenue; other operating income; profit before tax; and net income. Table 1 presents their descriptive statistics.

In addition, Table 1 includes GDP per capita growth (annual %), gross savings (% of GDP), and inflation (our socioeconomic variables); the GINI index estimated by the World Bank (our demographic variable); and bank capital to assets ratio (%) (our regulatory variable). A considerable body of research has examined the effect of socioeconomic variables on bank performance (Andries, 2011; Demirgüç-Kunt & Huizinga, 1999; Grigorian & Manole, 2002; Johnson & Kuosmanen, 2012; Sufian & Habibullah, 2010). Recently, socioeconomic variables have been tested in most of the two-stage efficiency studies (Grigorian & Manole, 2002; Hoff, 2007), which have revealed fairly similar results: GDP per capita growth, gross savings (% of GDP), inflation, and GINI index increase the efficiency of the banking industry, while inflation decreases it.

Table 1

Descriptive statistics.

Variables		Min	Max	Mean	SD	CV
	Lower Reserves for Impaired Loans / NPLs	0.000	15.425	13.212	0.360	0.027
	Total Capital Ratio	0.000	4.426	3.907	0.254	0.065
	Tier 1 Ratio	0.000	4.323	3.765	0.171	0.045
	Loan Loss Res / Gross Loans	0.000	3.598	1.668	0.638	0.382
	Loan Loss Prov / Net Int Rev	0.000	6.136	5.822	0.148	0.025
	Fixed Assets	0.000	15.227	4.202	1.940	0.462
	Non-Earning Assets	0.000	15.750	13.365	0.414	0.031
	Equity	0.000	14.993	6.916	1.606	0.232
	Total Liabilities & Equity	0.000	17.702	9.094	1.885	0.207
	Loan Loss Reserves (Memo)	0.000	12.917	11.339	0.323	0.028
	Liquid Assets (Memo)	0.000	16.858	11.711	0.586	0.050
Negative criteria	Overheads	0.000	12.904	4.942	1.852	0.375
	Loan Loss Provisions	0.000	10.629	10.375	0.245	0.024
	Tax	0.000	11.854	7.992	0.492	0.062

	Equity / Total Assets	0.000	4.801	3.553	0.287	0.081
	Equity / Net Loans	0.000	7.565	6.883	0.177	0.026
	Equity / Cost & Short Term Funding	0.000	6.561	4.132	0.426	0.103
	Equity / Liabilities	0.000	6.703	3.523	0.450	0.128
	Net Interest Margin	0.000	4.705	3.731	0.143	0.038
	Return on Avg Assets (ROAA)	0.000	4.188	3.557	0.127	0.036
	Return on Avg Equity (ROAE)	0.000	7.038	5.985	0.154	0.026
	Cost to Income Ratio	0.000	6.432	3.983	0.516	0.130
	Recurring Earning Power	0.000	4.426	3.391	0.153	0.045
	Interbank Ratio	0.000	6.937	5.197	0.777	0.149
	Net Loans / Total Assets	0.000	4.929	3.956	0.462	0.117
	Loans	0.000	16.737	9.465	1.332	0.141
	Total Earning Assets	0.000	17.555	10.958	1.016	0.093
	Total Assets	0.000	17.702	9.094	1.885	0.207
Positive criteria	Deposits & Short-term Funding	0.000	17.596	11.301	0.939	0.083
	Other (Non-Interest-bearing)	0.000	14.025	6.312	1.495	0.237
	Net Interest Revenue	0.000	13.883	6.033	1.559	0.258
	Other Operating Income	0.000	13.123	9.242	0.404	0.044
	Profit before Tax	0.000	13.535	10.000	0.450	0.045
	Net Income	0.000	13.130	9.986	0.405	0.041
Socio-economic and demographic variables	Bank Capital to Assets Ratio (%)	6.100	12.896	8.425	1.999	0.237
	GDP Per Capita Growth (Annual %)	-1.139	10.099	3.900	3.218	0.825
	Gross Savings (% of GDP)	14.351	50.603	28.452	12.587	0.442
	Inflation, GDP Deflator (Annual %)	0.853	15.914	6.869	3.218	0.468
	GINI Index (World Bank Estimate)	33.850	63.380	46.300	10.013	0.216

4.2. *Fuzzy TOPSIS*

The original TOPSIS method is designed to order multidimensional objects in a linear way (Dymova, Sevastjanov, & Tikhonenko, 2013; Hwang & Yoon, 2012). Broadly speaking, this task involves ordering objects from best to worst according to a latent measure that is not subject to direct observation or measurement (Baykasoğlu & Gölcük, 2015; Jefmański & Dudek, 2015). TOPSIS typically takes into consideration how far an evaluated object is from its

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negative-ideal and positive-ideal solutions (Roszkowska & Kacprazak, 2016; Tavana, Zandi, & Katehakis, 2013). Barros and Wanke (2015) and Wanke, Pestana Barros, and Chen (2015c) apply the TOPSIS method to measuring performance.

The fuzzy TOPSIS method was first developed by Chen (2000), with subsequent applications by Chang and Tseng (2008), Uyun and Riadi (2013), Yayla, Yildiz, and Ozbek (2012), and Kia, Danaei, and Oroei (2014). The major difference between the fuzzy TOPSIS method and the original one is that the former uses fuzzy numbers in computing firms' performance scores and rankings. In this research, triangular fuzzy numbers (TFN) are used to capture vagueness in banking inputs and outputs. As Fig. 3 shows, a TFN may be represented by (l, m, u), where l, m, and u denote, respectively, the minimal, the mean, and the maximal value of a given variable. A TFN may be symmetrical around the mean or not. TFNs are the most common and intuitive way to represent vagueness, since they allow inputs and outputs to be measured simultaneously in terms of these three values (Wanke et al., 2016b).

To capture the vagueness in data collection, we made several assumptions about the nature of output and input data in BRICS banks over time. First, we assumed that variations in inputs and outputs were linear. Second, we represented all inputs and outputs by TFNs. Third, we defined the minimal (maximal, mean) value of the TFN as the minimum (maximum, mean) input or output between 2010 and 2014.
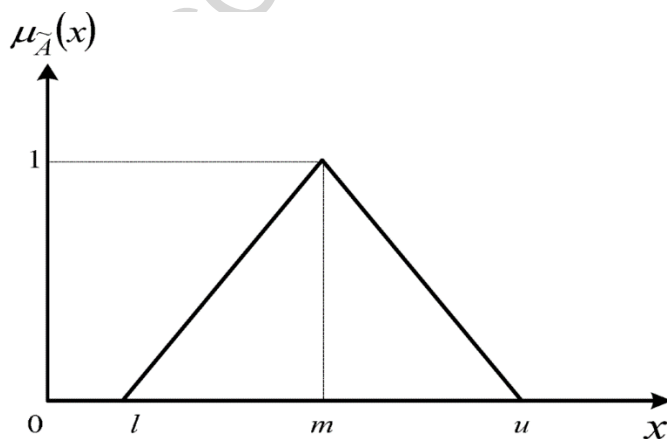

Fig. 3. Example of a triangular fuzzy number.

Let us assume that a certain set of alternatives $A = (Ai \mid i = 1,..., n)$ and a set of criteria $C = (Cj \mid j = 1,...,m)$, where $\tilde{X} = \{\tilde{x}_{ij} \mid i = 1, ..., n, j = 1, ..., m\}$ stands for an evaluation criteria fuzzy set and $\tilde{W} = \{\tilde{w}_j \mid j = 1, ..., m\}$ for a fuzzy weights set. The linear ordering of objects, using the above assumptions, requires the following steps (Chen, 2000):

Step 1. Normalization of the fuzzy criteria. Normalization means transforming criteria to range between zero and one. In this study, the criteria are the inputs and outputs chosen for performance analysis, while the alternatives are the samples in the dataset. For each one of the five years of the sample, we collected samples from 87 banks in Brazil, 87 banks in Russia, 52 banks in India, 125 banks in China, and 14 banks in South Africa, creating a total of 1825 samples.

$$\tilde{Z}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{x}_{ij}^2}} \ , \qquad i = 1, ..., n; \ j = 1, ..., m. \tag{1}$$

Step 2. Weighting of the normalized fuzzy criteria. In this step, the exogenously defined weights, also defined as TFN, are incorporated into the computations. Very often weights are set as equal in exploratory studies and/or in the absence of different opinions from foreign experts.

$$\tilde{v}_{ij} = \tilde{w}_j \tilde{z}_{ij} \tag{2}$$

Step 3. Computation of positive-ideal and negative-ideal solutions A^+ and A^-:

$$\tilde{A}^+ = \{\tilde{v}_1^+, \tilde{v}_2^+, ..., \tilde{v}_m^+\} = ((\max_i \tilde{v}_{ij} \mid j \in J_1), (\min_i \tilde{v}_{ij} \mid j \in J_2) \mid i = 1, ..., n) \tag{3}$$

$$\tilde{A}^- = \{\tilde{v}_1^-, \tilde{v}_2^-, ..., \tilde{v}_m^-\} = \left((\min_i \tilde{v}_{ij} \mid j \in J_1), (\max_i \tilde{v}_{ij} \mid j \in J_2) \mid i = 1, ..., n\right), \tag{4}$$

where J_1 and J_2 are respectively the positive criterion set and the negative criterion set.

The spider graph in Fig. 4 illustrates how the positive ideal and negative ideal solutions are spread out through the different fuzzy TOPSIS criteria. The ideal points (positive ideal

solutions) are the ones where all the positive criteria are maximal and the negative criteria are minimal. The anti-ideal points (negative ideal solutions) are the ones where all the positive criteria are minimal and the negative criteria are maximal.
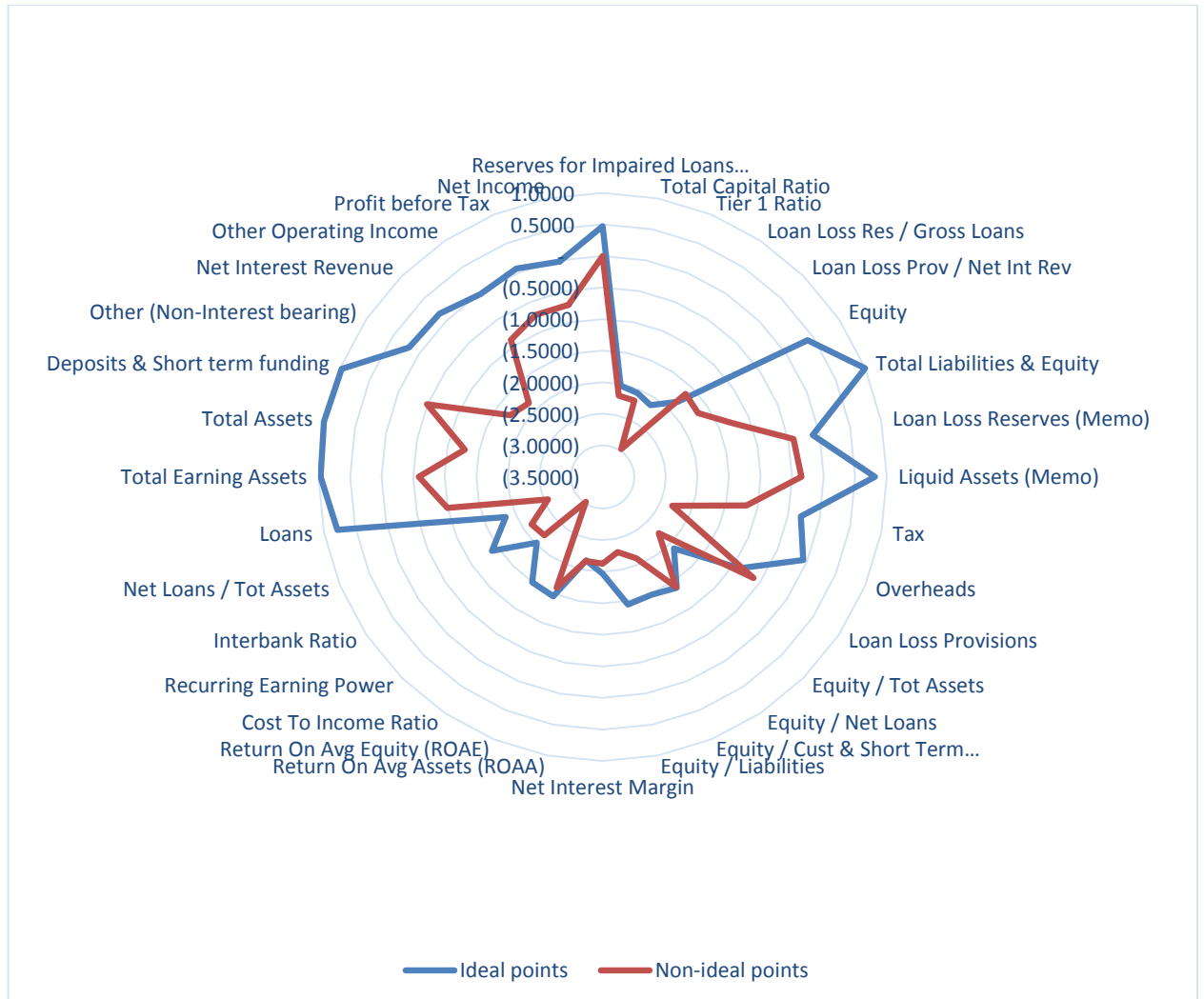

Fig. 4. Fuzzy TOPSIS positive ideal and negative ideal solutions based on rescaled variables.

Step 4. Distance measurement for each object from positive-ideal and negative-ideal solutions d_i^+ and d_i^-. This distance is computed simply by observing the Euclidean norm for the distance between two distinct points in the space.

Step 5. Computation of a synthetic measure:

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$$CC_i^+ = \frac{d_i^-}{d_i^+ + d_i^-} \quad , \qquad i = (1, \ldots, n). \tag{5}$$

Results from eq. (5) are rescaled using a unit interval. The smaller the distance of an object from a positive-ideal solution and the larger from a negative-ideal solution, the closer the synthetic measure is to one. The other way around, the closer the synthetic value is to zero.

Step 6. Ranking the objects: the best object has the largest synthetic measure.

In summary, fuzzy TOPSIS first normalizes fuzzy numbers according to the formula of linear scale transformation, and second models the weights of particular criteria as TFN. In this study, for the sake of simplicity, the same system of weights has been assumed for all variables; therefore, the parameters' values of fuzzy numbers representing weights are the same, i.e., 1. We used the R codes provided by Jefmański and Dudek (2015) to compute the fuzzy TOPSIS scores.

4.3. Bootstrapped truncated regression with conditional α-levels

Fuzzy α-level analysis (also known as α-cut analysis) is widely used in assessing uncertainty or vagueness in the measurement of a variable. Uncertain variables can be treated as fuzzy numbers, such as the triangular fuzzy number depicted in Fig 3. They can be manipulated by specially designed operators—in our case, the different levels of α—by assigning a given value ranging between 0 and 1 (say, 0, 0.1,…). The alpha-level is the degree of sensitivity of a given variable to vagueness: when α is equal to one, there is no fuzziness (vagueness) and the data are fully reliable (apart, of course, from random intrinsic effects). At some point, as the information value diminishes (with lower values of α implying higher values of fuzziness or vagueness), one no longer wants to be "bothered" by the data. In many systems, because the observation mechanisms are inherently limited, the information becomes suspect below a certain level of reliability.

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Wanke and colleagues (2015a) departed from the model of Simar and Wilson (2007) and proposed a truncated regression with conditional bootstrapping at each α-level to regress the respective crisp efficiency scores computed from fuzzy efficiency methods (where scores range between 0 and 1) onto a set of contextual variables:

$$\theta_j \mid \alpha = k + Z_j \delta + F_j \gamma + \varepsilon_j, j = 1,....,n .$$ (6)

In eq. (6), α is a value ranging from zero to one and represents the level of the membership function for the efficiency score; k is the constant term; ε_j is statistical noise; F_j is a vector of dummy variables that represent the fixed effects for the type of the fuzzy models used whenever different models are used; and Z_j is a vector of the contextual variables for observation j that is expected to be related to the observation's efficiency score, θ_j, taken as a crisp value.

Noting that the distribution of ε_j is restricted by the condition $\varepsilon_j \geq 1 - k - Z_j \delta - F_j \gamma$ (since both sides of (7) are bounded by unit), Wanke and colleagues (2015a) followed the steps proposed in Simar and Wilson (2007) and assumed that this distribution is truncated normally, with zero mean (before truncation), unknown variance, and (left) truncation point determined by this same condition. If we replace the true but unobserved regress and, in (6), replace θ_j by the fuzzy efficiency estimate $\bar{\theta}_j$, the conditional econometric model formally becomes

$$\bar{\theta}_j \mid \alpha \approx k + Z_j \delta + F_j \gamma + \varepsilon_j, j = 1,....,n,$$ (8)

where

$$\varepsilon_j \sim N(0, \sigma_\varepsilon^2), \text{ so that } \varepsilon_j \geq 1 - k - Z_j \delta - F_j \gamma, j = 1,...,n,$$ (9)

which is evaluated by maximal likelihood estimation as regards $(\delta, \sigma_\varepsilon^2)$ obtained from the data. It should be noted that Wanke and colleagues used only one type of fuzzy TOPSIS model,

implying the discard of vector F_j, and that their computations used R codes; for further details

see Wanke and colleagues (2015a) and references therein.

In summary, the approach used here starts off with fuzzy TOPSIS models where positive

and negative criteria are treated as TFN with minimal and maximal bounds determined by the

dataset and culminates with the proposed conditional bootstrapped truncated regression. They are

performed each time for a given α-level (say 0; 0.1; 0.2; ...; 1). Readers should be aware that the

α-level values within this set are primarily used in the fuzzy TOPSIS so as to determine crisp

values for the input and the output bounds, thus enabling computation of their respective

efficiency levels.

4.4. Fuzzy rule-based systems (FRBS) *in fuzzy regression*

A theory of fuzzy sets was originally derived by Zadeh (1965) in a seminal work

expanding classical set theory towards sets with different degrees of membership, or α-levels. In

traditional sets an object either is or is not a member; in a fuzzy set, membership is measured on

a wide range of possibilities between zero and one. While an α-level of 1 means that an object

belongs to a set, an α-level of zero means the opposite. An α-level somewhere in between 0 and

1 shows partial membership (Riza, Bergmeir, Herrera, & Benítez Sánchez, 2015; Pedrycz &

Gomide, 1998; Klir & Yuan, 1995).

Fuzzy rule-based systems (FRBS) extend classical rule-based systems, frequently

expressed in the form "IF A, THEN B." In FRBS, A and B are operationalized as fuzzy sets, so

that "if GDP growth rate is higher, then efficiency levels are higher" or even "if inflation rates

are above a certain level, then efficiency levels are lower than a given threshold." Broadly

speaking, to model an FRBS, one must take two important steps: structure identification and

parameter estimation (Riza et al., 2015). Nowadays, both steps are covered by several algorithms

that generate fuzzy IF-THEN rules automatically from numerical data. In the present study, the

numerical data include not only the efficiency estimates for different α-levels, but also the

underlying contextual variables. The algorithms use various approaches: neuro-fuzzy techniques, heuristic procedures, clustering methods, squares methods, genetic algorithms, etc. As regards rule structure, two classical models prevail: the Mamdani and the Takagi-Sugeno-Kang (TSK) (González, Perez, & Verdegay, 1994).

The task of building an FRBS implies defining all its components, whether manually or automatically, particularly the database partition and the underlying rule bases. There are two approaches to composing an FRBS (Wang, 1994). The first relies on information gathered from human experts. The second extracts information from alternative learning methods and eventually makes them compete against each other so that useful conclusions regarding their predictive ability can be drawn (Jang, 1993; Pedrycz, 2012; Sugeno & Yasukawa, 1993). Following an emerging trend in the field (Antonelli, Ducange, Marcelloni, & Segatori, 2016; Rodríguez-Fdez et al., 2016), the present study adopts the second approach, comparing learning methods that are usually classified into different groups (Riza et al., 2015): space partition, clustering, and neural networks. An FRBS can be used just like other regression models and their corresponding packages in R. The principles of the technique are described in Table 2.

Table 2

Parameters for the FRBS.

Methods	Parameters
ANFIS	num.labels = 5, max.iter = 300, step.size = 0.01, type.mf = 3
HYFIS	num.labels = 5, max.iter = 200, step.size = 0.01
SBC	r.a = 0.3, eps.high = 0.5, eps.low = 0.15
DENFIS	Dthr = 0.15, max.iter = 5000, step.size = 0.01, d = 2 "
FIR.DM	num.labels = 5, max.iter = 1000, step.size = 0.01
FS.HGD	num.labels = 5, max.iter = 100, step.size = 0.01, alpha.heuristic = 1
GFS.FR.MOGUL	persen_cross = 0.9, max.iter = 300, max.gen = 200, max.tune = 500, persen_mutant = 0.3, epsilon = 0.95
WM	num.labels = 15, type.mf = 3, type.defuz = 1, type.tnorm = 1, type.snorm = 1

5. Analysis and discussion of results

Fig. 5 presents the distributions of the efficiency scores for the BRICS banking industry from 2010 to 2014, computed using fuzzy TOPSIS on our set of positive and negative criteria

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generated based on TFN and the α-level approach. Broadly speaking, the fuzzy estimates fluctuated from to 0.4 to 0.9 and appear to be almost stagnant over these years. Efficiency levels tend to be quite similar between countries. However, readers should note that, although efficiency in Chinese banks is biased upwards, Brazilian banks tend to present more dispersed scores than in other countries. The underlying causes of these results may be related not only to bank ownership within each country, but also to industry fragmentation and the intensity of competition (number of competitors). Regulations restricting foreign banks and recent governmental restructuring in each country, whether focusing solely on the health of financial statements or also on network redesign, may also exert some influence.

Wanke and colleagues (2015b) examined Brazilian bank efficiency using a dynamic slack based model, and their findings suggest that small public and national banks (in which corporatism is strong) are less efficient than large and foreign ones. That is, efficiency varies a good deal between different types of Brazilian banks. Pressure to improve performance in Brazilian banks is low for several reasons: not only does the country's banking industry operate like a cartel, having been heavily concentrated by a series of mergers and acquisitions between 1990 and 2010, but also the huge interest rates offered by the Brazilian government to keep inflation rates under control dampen interest in gaining market share and extinguish desire to ameliorate operational performance. In general, banks in Brazil operate like an extension of the treasury secretary, trading in government bonds.

On the other hand, banks in China have long enjoyed a market safe from competition, increasing their efficiency bias (see Shi and Zou, 2016). China's banking industry is still very fragmented compared to those of the other BRICS countries: there are around 130 banks in operation, and most of them still have to meet operational and financial targets imposed by the several planning committees of the Chinese government. Also, government-driven bank restructuring in China focused on systematically eliminating excess employees and branches,

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while in other BRICS countries this tends to happen case by case with mergers and acquisitions, so that it has a synergistic effect.

As regards the other BRICS countries, in South Africa and India the banking industry is highly concentrated, in just 14 and 52 banks, respectively. In India, there is evidence of differences in performance between foreign and local banks (Tzeremes, 2015), while in South Africa domestic banks have been reduced in number over the past years by a series of mergers and acquisitions (SARB, 2015). The remaining banks in South Africa operate as if under an oligopoly. For Russia, Karas, Schoors, and Weill (2010) found that domestic private banks are not more efficient than domestic public ones, possibly because of low competition levels and higher costs for customers to change banks, besides cultural factors, although efficiency is higher in foreign banks. Banking performance in these three countries, however, is not so widely dispersed as it is in Brazil.

These differences might suggest that socioeconomic, demographic, and regulatory variables are affecting bank efficiency within each country. The literature examining the effects of contextual variables on bank efficiency is vast. Most of the studies have found that contextual variables have either a positive or a negative impact, but the nature and significance of that impact remain highly country dependent (Shi & Zou, 2016; Sufian & Habibullah, 2010; Wanke et al., 2015b, 2016a, c, and d).

It is interesting to note that, in a way quite analogous to what happens with bootstrapped estimates in frontier methods, fuzzy TOPSIS efficiencies are higher when there is no fuzziness at all (α-level = 1, represented by the solid bold line in Fig. 5 on the left). These efficiencies systematically decay with the value of the α-level (that is, as fuzziness increases), reaching their minimal values when fuzziness is maximal (α-level = 0, represented by the dashed fine line in Fig. 5 on the left). Under bootstrapping, the newer efficiency estimates computed statistically tend to be lower than the original ones.
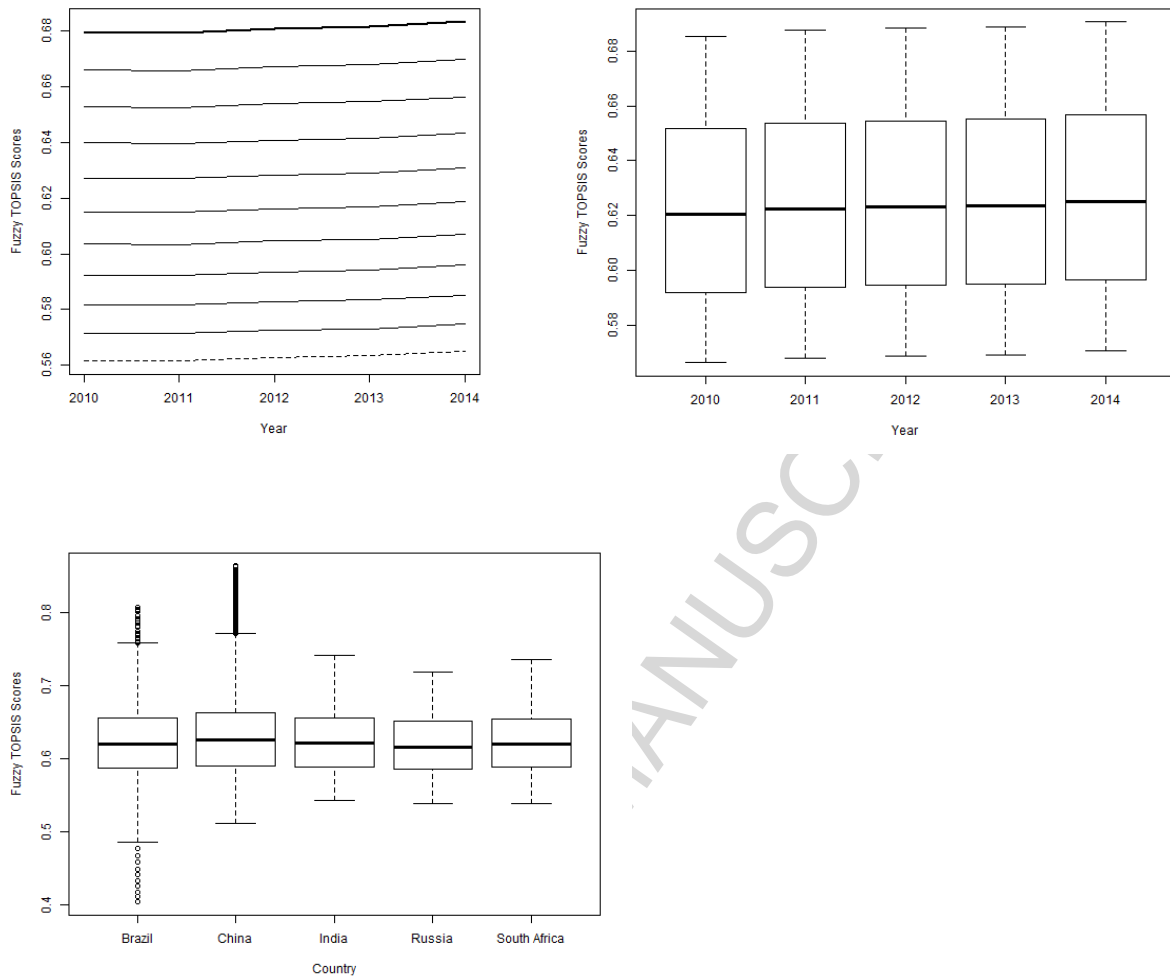
Fig. 5. Fuzzy efficiency levels for BRICS banking.

The results of truncated regression with conditional bootstrapping performed at different α-levels, shown in Fig. 6, reveal the impact of socioeconomic, regulatory, and demographic variables on banking efficiency in BRICS countries under different levels of fuzziness. A number of different conclusions can be inferred with respect to the significance and the sign of the contextual variables. A solid line marking zero indicates whether or not a given variable is significant, regardless of the alpha level.

Only two contextual variables proved to be nonsignificant: GDP per capita growth (socioeconomic) and bank capital/asset ratio (regulatory). These results suggest that bank efficiency levels in the BRICS countries are not geared to growth in the average income level nor

constrained by local regulations that attempt to guarantee financial health. Instead, as the following results show, banking efficiency seems to be geared towards capital accumulation at the country level and how its value is preserved over time.
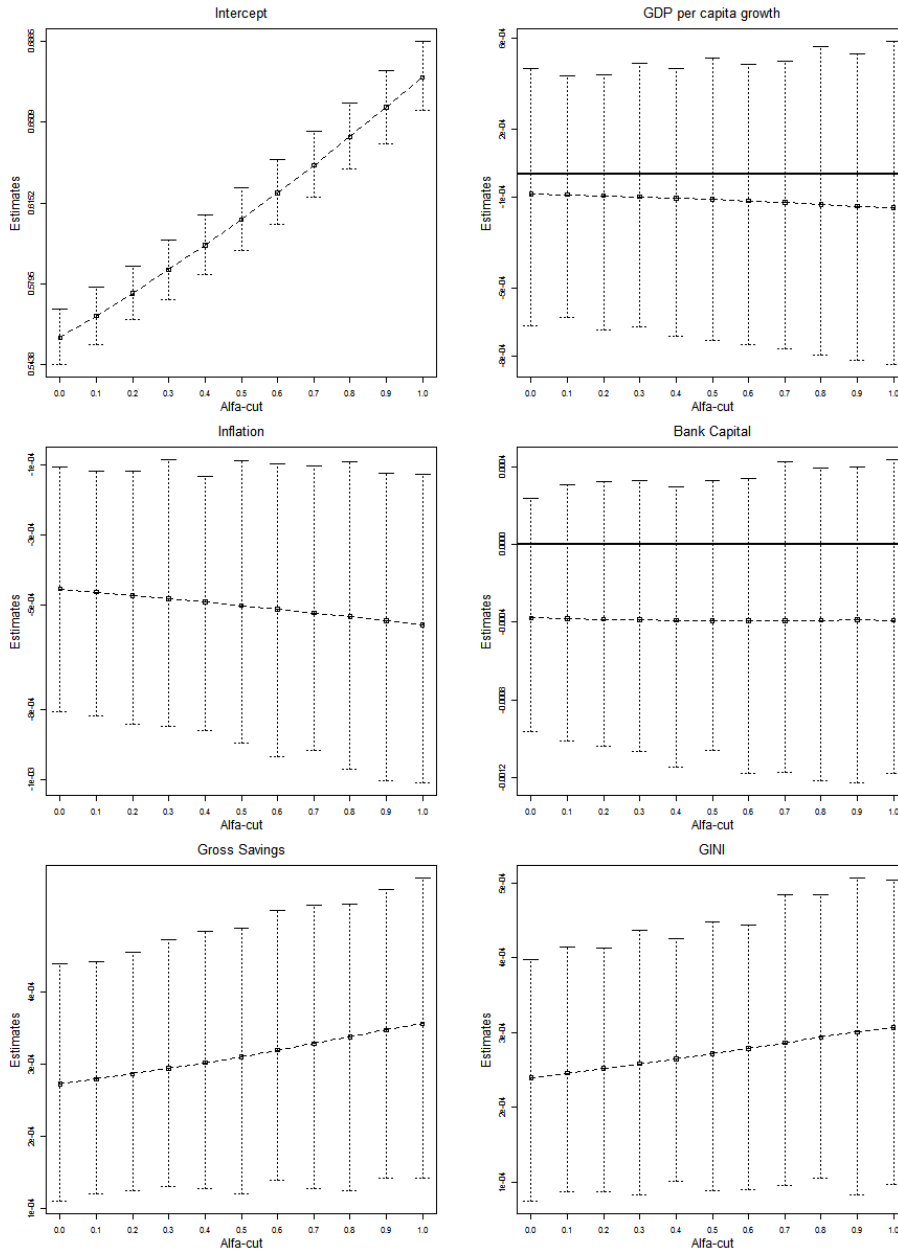

Fig. 6. Estimates for the contextual variables.

All the other contextual variables—country gross savings, inflation ratio, and GINI index—were significant regardless of the α-values, suggesting that, although gross savings may be a prerequisite for higher productivity in banking systems, the impact of the inflation ratio on the loss of productivity cannot be neglected. There is an interesting counterbalance of social

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welfare and capital accumulation in banking productivity in BRICS. Efficiency tends to increase with higher GINI levels, which reflect income inequality, and, to some extent, with capital accumulation. These results suggest that banking efficiency is driven not only by capital accumulation at the country level, but also by its concentration in some population segments. Generally speaking, this would imply a higher value per capita of transactions such as deposits, loans, and investments, the grounds of higher banking productivity.

Readers should note that the signs of the significant variables did not depend on the fuzziness level. One may infer that randomness and uncertainty do not interact at the input/output level. This lack of ambiguity, which is encountered infrequently in fuzzy systems applied to efficiency measurement (Wanke et al., 2015a, 2016d), represents a topic for further research on the predictive power of the significant sources of efficiency under different rule-based systems using fuzzy regressions. As we note above in section 4, fuzzy regressions do not allow us to compute the sign and significance for each contextual variable; still, fuzzy regressions tend to present higher levels of predictability than traditional statistical models.

Fig. 7 presents the bootstrapped results for the log-likelihood estimates for each alpha level. Likelihood is greatest when $\alpha = 0$, i.e, when fuzziness is greatest. Since the confidence intervals of these estimates under different values of alpha do overlap, it is not possible to claim a statistical difference among them. A similar behavior was found by Wanke and colleagues (2015) and by Wanke, Barros, and Nwaogbe (2016d) when they applied this bootstrapped regression to different decision-making contexts. This also suggests that different rule-based systems are needed to assess the problem of predicting efficiency levels in BRICS banking.
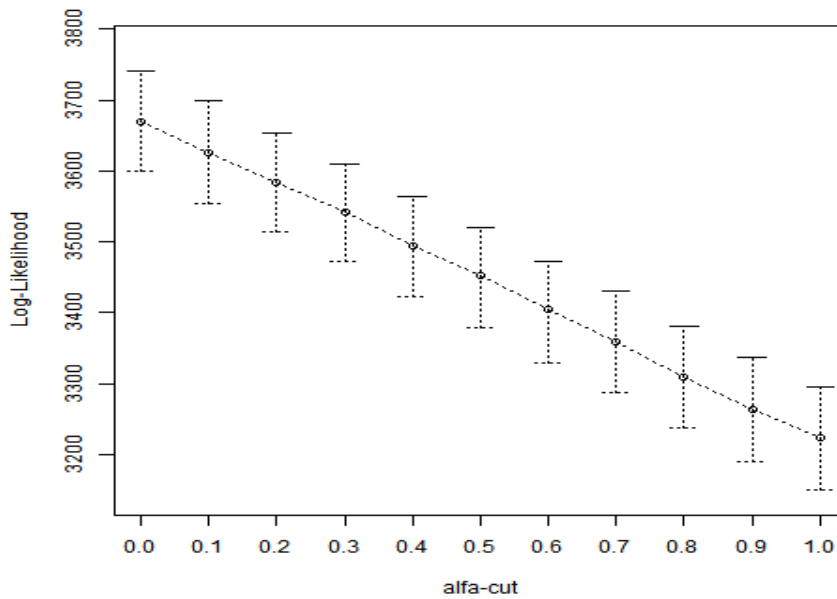
Fig. 7. Log-likelihood estimates and their confidence intervals.

Below, we compare the results of the rule-based systems methods presented in Table 2, computing average percent error (APE) and considering only significant contextual variables. Figure 8 organizes the results by alpha-levels; Figure 9, by methods. All eight FRBS methods tested outperformed bootstrapped truncated regression in terms of MAPE. The average errors were substantially smaller under HYFIS, SBC, FS.HGD, DENFIS, GFS.FR.MOGUL, and WM models, and under ANFIS, and FIR.DM the errors were not comparable even in central tendency and distributional characteristics to the worse results obtained through bootstrap. This suggests that better APE and MAPE can be achieved in several FRBS models, whether the underlying learning method is based on neural networks, space partition, or clustering. The absence of interaction between fuzziness and randomness, as detected in the bootstrapped regression, together with the fact that the best likelihood model is the one with highest input/output fuzziness, may help explain why several FRBS methods showed superior predictive ability. Further research, however, is necessary to confirm this conjecture under circumstances where randomness and fuzziness interact, and to show how these interactions may jeopardize predictive ability in fuzzy regressions and/or variable significance in bootstrapped regressions. In the

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present project, this particular result may be related to the poor quality of the data collected on

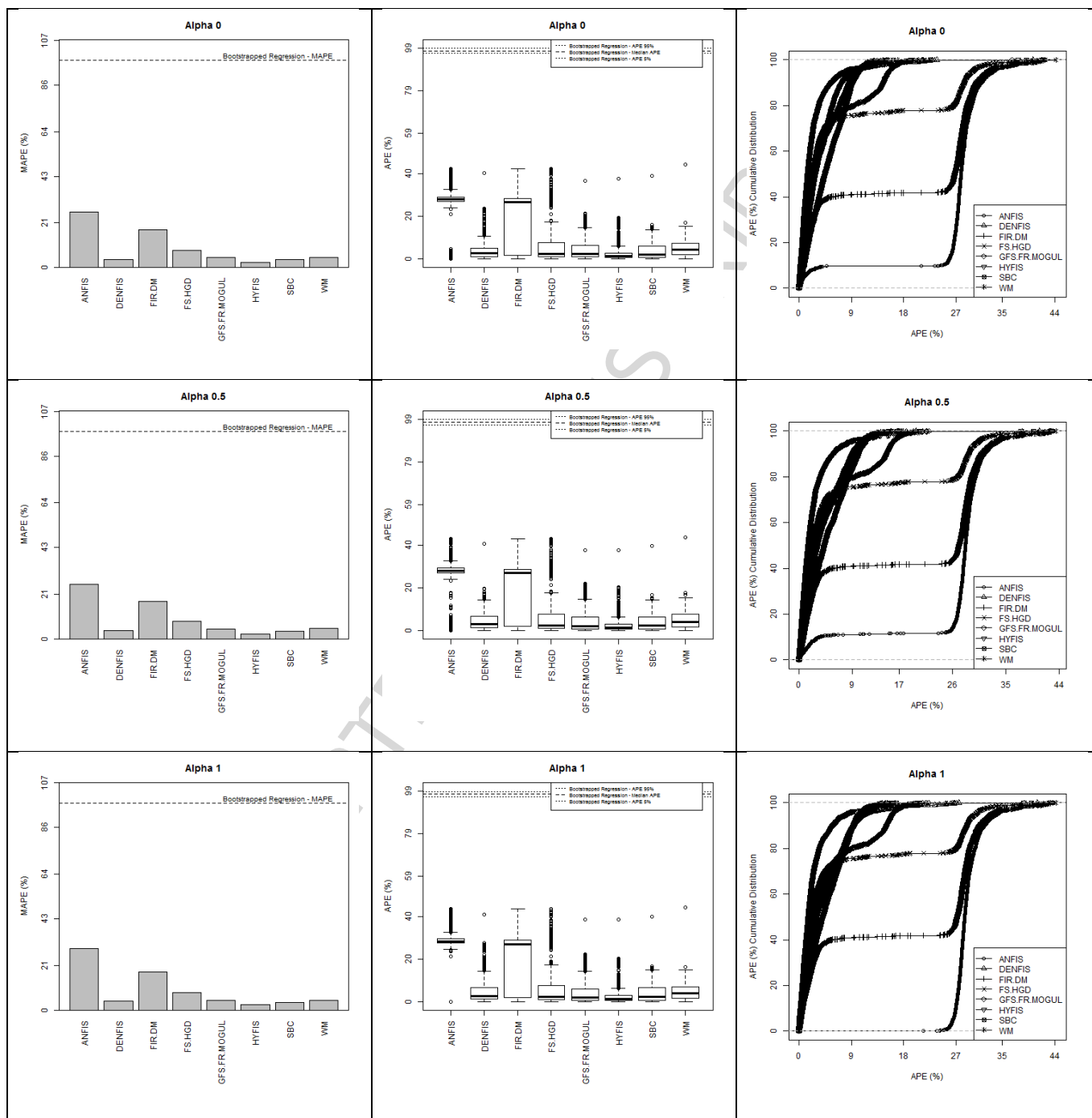BRICS banking and to the vagueness surrounding the data collection.

Fig. 8. FRBS regression results grouped by selected α-levels.

Fig. 9 shows how the degree of fuzziness affects errors under the best FRBS model

(HYFIS). The different alpha-levels have little impact on the distribution of the APE and its

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central tendency (median), although average errors (MAPE) do tend to increase slightly with increasing fuzziness (alpha-level = 0). Under the remaining FRBS models, APE either increases or remains stagnant with lower fuzziness levels. This suggests that FRBS generally work better in fuzzier environments, as we expected. The major implications of these findings are related to the use of neural networks in fuzzy regressions in the HYFIS model. Neural networks showed good predictive power in connecting linguistic terms defined by fuzzy rules with socioeconomic variables. This means that linguistic variables work better than clustering or space partition to describe banking efficiency in BRICS in order to compare the banks of these countries. In practical terms, linguistic variables allow us to derive broader or more general conclusions, such as "higher inflation rates imply lower banking efficiency." The possibility of deriving a discourse on how things happen rather than fitting parameters for space partition or cluster membership helps not only to consolidate theory, but also to establish a common basis of comparison for unreliable quantitative data obtained from different sources, although qualitatively comparable in meaning. Further research should attempt to explain the interactions between sociodemographic variables and financial indicators in the banking industry. Our results suggest that variations in the GINI or inflation indexes do not have uniform effects on the various financial indicators that may help explain banking performance.
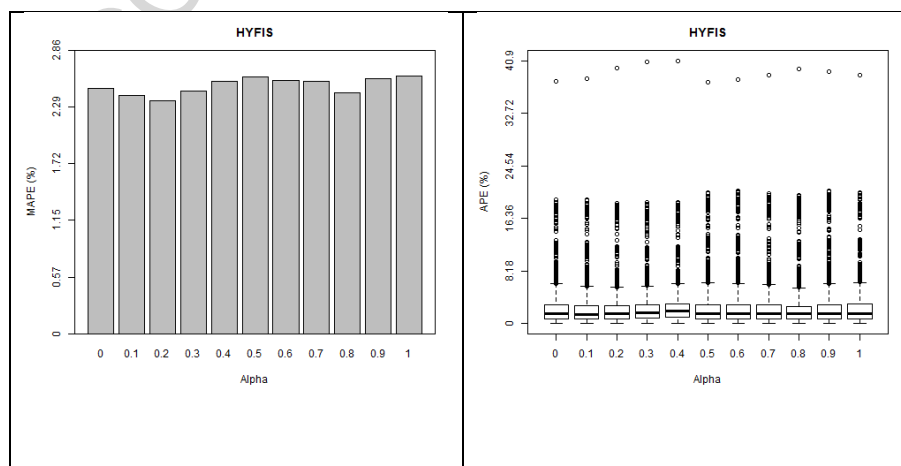
Fig. 9. FRBS regression results for the HYFIS model.

The implications of this study of BRICS banks for decision-makers are related to the fuzziness and randomness of the problem under analysis. Given that interactions between sociodemographic and financial variables may exist and can be detected in sign reversal for different alpha levels, and also that we found no interaction between randomness and fuzziness in our data, we believe that decision makers should consider using bootstrapped conditional regressions together with FRBS regressions such as HYFIS—the former first, to detect significant contextual variables and their signs, and the latter to forecast using these variables.

6. Conclusion

This study analyzed BRICS banking efficiency using fuzzy TOPSIS and fuzzy/bootstrapped regression approaches. Fuzzy TOPSIS enables us to handle various sources of uncertainty and vagueness while computing the efficiency scores. Building upon the fuzzy analysis performed in the second stage, we can identify potential socioeconomic, demographic, and regulatory causes of inefficiency, subsequently boosting their predictive power by means of fuzzy based rules. The complementary use of statistical and fuzzy regression tools constitutes an alternative direction for future research in the field of two-dimensional fuzzy Monte Carlo analysis.

Banking efficiency in the BRICS countries appears to be explained by the countervailing forces of capital accumulation and social welfare, building upon the trade-off between inflation ratio and country gross savings. From the managerial perspective, this paper sheds light on bank efficiency because it uses both business approaches and contextual variables. It also serves as a ground-breaking benchmarking tool to explain the diverse aspects of banking business and unveil their interaction with sociodemographic variables. This paper suggests that to attain

optimal bank efficiency, managers should focus not only on their peer banks, but also on their bank's contextual variables before benchmarking.

For policymakers, our results suggest that they should take into account sociodemographic variables when setting targets and parameters for banking regulation. When inflation and the GINI index are high—that is, when banking efficiency levels are structurally low and social inequality high—banking competition should be nurtured as much as possible. Mergers and acquisitions should be discouraged and the entrance of foreign competitors should be welcomed. In this case the banking industry should focus on financial products for lower income customers and small businesses. On the other hand, when gross savings are high and efficiency levels are structurally high, government should regulate special funding for long-term projects such as infrastructure or social welfare investments with subsidized interest rates.

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Highlights

- This paper uses fuzzy TOPSIS to assess the efficiency of BRICS banking.
- Fuzzy TOPSIS is used with fuzzy regression to predict performance.
- Socio-economic and regulatory variables affect efficiency.