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Optimal Load Shedding in Electricity Grids with Renewable Sources via Message Passing

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Abstract

The increased penetration of volatile and intermittent renewable energy sources challenges existing power-distribution methods as current dispatch methods were not designed to consider high levels of volatility. We suggest a principled algorithm called message passing, which complements existing techniques. It is based on statistical physics methodology and passes probabilistic messages locally to find the approximate global optimal solution for a given objective function. The computational complexity of the algorithm increases linearly with the system size, allowing one to solve large-scale problems. We show how message passing considers fluctuations effectively and prioritise consumers in the event of insufficient resource. We demonstrate the efficacy of the algorithm in managing load-shedding and power-distribution on synthetic benchmark IEEE data and discuss the role of weights in the trade-off between minimising load-shedding and transmission costs.

Keywords: Electricity distribution; load shedding; message passing; networks; optimisation; power flow; renewable energy; uncertainty

1. Introduction

Recent EU legislations aims for 20% of all of generated power to be from renewable sources [1] by 2020 in order to limit the use of destructive and unsustainable power sources such as fossil fuels and nuclear plants. While renewable sources such as wind and solar offer a clean, mature and tested technology they are fluctuating and are

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not fully under human control. This, in addition to the fluctuations in demand, results in higher levels of uncertainty in planning power generation and distribution at the economic dispatch stage over the 15-60 min time window. The current distribution method was not designed to consider such high levels of uncertainty and a new method is sought. An effective distribution algorithm must also consider scenarios where generation does not meet demand, especially with the inevitable increased fluctuations in power systems. This paper suggests representing uncertainties as probability distributions and using message passing, which inherently considers probabilistic scenarios to find a global solution at the economic dispatch stage of power distribution.

Power distribution is currently modelled using optimal power flow (OPF) [2]. Assuming initial voltages, the first step to adjust voltages uses Newton Raphson or a fast-decoupled method to satisfy Kirchoff's law. The second step seeks a better solution using steepest descent or similar techniques and the algorithm is repeated until an optimal is found. OPF has been used for a long time with little change, it makes small adjustments and uses linear techniques to consider network fluctuations, but is less effective when considering the large scale fluctuations introduced by renewable sources. Existing alternative methods of optimal power distribution include: chance constrained OPF [3, 4], which builds on previous work [5], suggests that adding probabilities to the hard constraints of OPF is sufficient for considering renewable fluctuations; and the interior point method [6], which uses matrices to solve network constraints and a predictor-corrector technique to improve the solution. The majority of existing methods are heuristic and consider fluctuations superficially.

Message passing (MP) [7, 8] is a principled technique which passes conditional probabilities locally to find an optimal solution with modest computational complexity. This enables the algorithm to inherently consider the probabilistic nature of renewable sources and address very large scale problems. Message passing algorithms have been developed to consider bandwidth [9] and fluctuations [10].

This paper extends previous work on fluctuations to include load shedding when demand exceeds power generation. One aspect of the extension with respect to previous work is the consideration of load shedding in the presence of weighted nodes, where weights indicate the importance of nodes, section 2. We demonstrate the effects of the suggested algorithm on synthetic benchmark IEEE networks and randomly generated random-regular graphs in section 3. Section 4 discusses the advantages of message passing and possible future directions.

2. Methodology

Consider a network of N nodes (consumers/generators/ substations), connected to c others (degree connectivity can be heterogeneous but we keep them uniform within derivations for simplicity), each node j has a capacity Λ_j, which indicates if the node is a consumer (negative) or a generator (positive). In power distribution, generator i sends power y_{ij} to consumer j via power lines (edges) such that $y_{ij} = -y_{ji}$, in order to satisfy the constraint that all nodes need to be satisfied, i.e., have non-negative resource. As mentioned, in some cases there is insufficient power to satisfy all consumers and the algorithm distributes what is available. To model this one adds a *load-shedding* variable ζ_j to the constraint, which should be minimised. Mathematically the constraint can be written as in Eq. (1):

$$\sum_{(ij)} \mathcal{A}_{ij} y_{ij} + \Lambda_j + \zeta_j \geq 0 \tag{1}$$

for each node j, where (ij) describes any pair of nodes, $\mathcal{A}_{ij} = 1$, if nodes i and j are connected by an edge, and 0 otherwise; this is one of many constraints a power flow model must consider (others include bandwidth over edges and keeping generated power between the minimum and maximum capacities). A power distribution algorithm must also minimise functions such as: power loss, load shedding and generation costs; this paper describes the first two. To consider minimising transportation costs we use the equation $\phi = \frac{y_{ij}^2}{2}$ (loosely based on the power loss equation, but other loss functions and models can be considered). To minimise load shedding we minimise the deficit of each node multiplied by some predetermined weight indicating its importance: $\psi = \frac{\alpha_j \zeta_j^2}{2}$, where α_j is the importance weight of the node j. The general energy equation of the network can be written as:

$$\mathrm{E} = \sum_{(ij)} \mathcal{A}_{ij} \phi(y_{ij}) + \sum_j \psi(\zeta_j) \tag{2}$$

The message passing algorithm is based on a statistical physics methodology that assigns a cost to each node according to its state. The free energy F relates to the probability of being in a certain state, considering the corresponding cost (energy) and its multiplicity (entropy); minimising it (at zero temperature) achieves the *ground state* of the system. The free energy is defined as $-T\ln Z$, with Z the normalisation constant:

$$Z = \left(\prod_{(ij)} \int dy_{ij}\right) \prod_j \Theta\left(\sum_i \mathcal{A}_{ij} y_{ij} + \Lambda_j + \zeta_j\right) \quad \times e^{-\frac{1}{T}\left(\sum_{ij} \mathcal{A}_{ij} \phi(y_{ij}) + \sum_j \psi(\zeta_j)\right)} \tag{3}$$

where T is the temperature which represents how strictly one optimises the state and $\Theta(y) = 0$ if $y < 0$ and 1 otherwise. The link between the macroscopic state of a system and the microscopic variables has been highlighted in the literature [11, 12]. This and many other hard computational problems have taken advantage of this relation to gain insight and devise highly successful new algorithms [13-15].

Minimising the free energy is difficult as the computational complexity increases exponentially as the system size grows, alternatively [8] suggests using the principled Bethe approximation. This approximation takes advantage of the sparsity of the network ($N \gg c$) by saying that as the probability of loops is small, the network can be presumed to be locally tree-like and long range correlations can be ignored. The cavity method says if a node j were to be removed, all of its neighbours would be weakly correlated and can be deemed statistically independent, allowing for just one node and its neighbours to be considered at any given time, which reduces the computational complexity. To employ the algorithm we iteratively choose a node j and one of its neighbours i, termed an ancestor; the remaining neighbours k are termed descendants. Now one can rewrite the free energy as a conditional probability $F(y_{ij}|\boldsymbol{T}_j)$ depending on the free energy of its descendants, where T_j represents the tree of node j:

$$F(y_{ij}|\mathbf{T}_j) = -T\ln\left\{\prod_{k=1}^{c-1}\left(\int dy_{jk}\right) \Theta\left(\sum_{k=1}^{c-1} y_{jk} - y_{ij} + \Lambda_j + \zeta_j\right)\exp\left[-\frac{1}{T}\sum_{k=1}^{c-1}\left(F(y_{jk}|\mathbf{T}_k) + \phi(y_{jk})\right) + \frac{\alpha_j \zeta_j^2}{2}\right]\right\} \tag{4}$$

In economic dispatch, a distribution algorithm is used to decide the generation levels of controllable power stations for the next 15-60 min. Fluctuations and uncertainties in renewable generators are problematic because the dispatch model must make decisions on uncertain data. To combat this [10] suggests redefining capacities as Gaussian probability distributions with expected capacity $\overline{\Lambda}$, and variance σ. The *expected* free energy can be found by $\langle F\rangle = -T\langle \ln Z\rangle$ where $\langle * \rangle$ represents the average with respect to Λ. As the infinite tails of the normal distribution hard constraints cannot be satisfied, we change it to a soft constraint which is satisfiable with probability $(1-p)$.

$$\frac{1}{2}\operatorname{erfc}\left(\frac{-\sum_{k\neq i} \mathcal{A}_{jk}(y_{jk} + \varepsilon_{jk}) + y_{ij} - \Lambda_j}{\sqrt{2\sigma_j^2}}\right) > 1 - p \tag{5}$$

Considering the network to be tree-like the free energy can be written as $F = N_{\boldsymbol{T}} F_{av} + F_V(y|\boldsymbol{T})$ where $F_V(y|\boldsymbol{T})$ is the vertex free energy (VFE), $N_{\boldsymbol{T}}$ is the number of nodes in its descendants tree $\boldsymbol{T}$ and F_{av} is the average free energy per node in the tree. The VFE of a node j is the *added free energy contribution of node j* and the *expected* VFE can be written as:

$$\begin{aligned}&\langle F_V(y_{ij}|\mathbf{T}_j)\rangle = \\ &-T\left\langle \ln\left\{\prod_{k=1}^{c-1}\left(\int dy_{jk}\right)\Theta\left(\sum_{k=1}^{c-1} y_{jk} - y_{ij} + \Lambda_j + \sqrt{2\sigma_j^2}\operatorname{erf}^{-1}(2p-1) + \zeta_j\right) \times \exp\left[-\frac{1}{T}\sum_{k=1}^{c-1}\left(F_V(y_{jk}|\mathbf{T}_k) + \right.\right.\right.\right. \\ &\left.\left.\left.\left.\phi(y_{jk})\right) + \frac{\alpha_j\zeta_j^2}{2}\right]\right\}\right\rangle - F_{av}\end{aligned} \tag{6}$$

Setting $T = 0$ for strict optimisation this becomes:

$$\langle F_V(y_{ij}|\boldsymbol{T}_j)\rangle = \min\left[\left\langle \sum_{k=1}^{c-1}\left(F_V(y_{jk}|\boldsymbol{T}_k) + \phi(y_{jk})\right) + \frac{\alpha_j\zeta_j^2}{2}\right\rangle\right] - F_{av} \tag{7}$$

$$\text{s.t.} \qquad \sum_{k=1}^{c-1} y_{jk} - y_{ij} + \Lambda_j + \sqrt{2\sigma_j^2}\operatorname{erf}^{-1}(2p-1) + \zeta_j \geq 0 \tag{8}$$

The recursion relation between VFEs can be solved iteratively between neighbours to find an optimal solution. The VFE constitutes the message to be passed between neighbours; however, it is difficult to pass messages that are continuous functions and so [8] suggests passing instead, the first and second derivatives of the VFE assuming that it can be represented well using these derivatives. These are exact if the VFE is a Gaussian and an approximation otherwise. Rewriting it as the Taylor expansion we obtain:

$$F_{ij} = \sum_{k\neq i}\mathcal{A}_{jk}\left(\left(A_{jk}^{<>} + \phi'_{jk}\right)\varepsilon_{jk} + \frac{1}{2}\left(B_{jk}^{<>} + \phi''_{jk}\right)\varepsilon_{jk}^2\right) + \Theta\left(-\sum_{all}\bar{\Lambda}_{all} - \Lambda_j\right)\frac{\alpha_j\zeta_j^2}{2} + \mu_{ij}\left(\sum_{k\neq i}(y_{jk} + \varepsilon_{jk}) - y_{ij} + \Lambda_j + \sqrt{2}\sigma_j \mathrm{erf}^{-1}(2p-1) + \zeta_j\right) \tag{9}$$

where μ_{ij} is a Lagrange multiplier which ensures the constraint is satisfied and ε_{jk} is a small adjustment to edge y_{jk}; ϕ'_{jk} and ϕ''_{jk} indicate the first and second derivative of ϕ with respect to the edges y_{jk} and $A_{jk}^{<>}$ and $B_{jk}^{<>}$ are the first and second derivative of the expected VFE with respect to the edge y_{jk}. In addition, the sum over all indicates the sum of all nodes in the network excluding j, this component ensures that load shedding is only done when generation does not meet demand. Solving these equations results in the messages (10) and (11) where

$$x = y_{ij} - \sum_{k\neq i}\left(y_{jk} - \frac{A_{jk}^{<>} + \phi'_{jk}}{B_{jk}^{<>} + \phi''_{jk}}\right) - \sqrt{2}\sigma_j \mathrm{erf}^{-1}(2p-1)$$

and at each iteration a backwards message $y_{jk} \leftarrow y_{jk} - \frac{A_{jk}^{<>} + \phi'_{jk} + \langle\mu_{ij}\rangle}{B_{jk}^{<>} + \phi''_{jk}}$ is also sent to each descendant. The messages are passed iteratively for a new j and set of neighbours until convergence. The converged values correspond to the pseudo-posterior values and an approximate solution.

$$A_{jk}^{<>} = \begin{cases} \frac{1}{2}\dfrac{\left(\mathrm{erf}\left(\frac{x-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)+1\right)(x-\bar{\Lambda}_j)+\frac{2\sigma_j^2}{\sqrt{2\pi\sigma_j^2}}e^{-\left(\frac{x-\bar{\Lambda}_j}{\sqrt{2\sigma_j^2}}\right)^2}}{\sum_{k\neq i}\mathcal{A}_{jk}\frac{1}{B_{jk}^{<>}+\phi''_{jk}}+\frac{\Theta\left(-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j\right)}{\alpha_j}}, & \text{if } x \le -\sum_{all}\bar{\Lambda}_{all} \\ \frac{1}{2}\dfrac{\left(\mathrm{erf}\left(\frac{-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)+1\right)(x-\bar{\Lambda}_j)+\frac{2\sigma_j^2}{\sqrt{2\pi\sigma_j^2}}e^{-\left(\frac{-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j}{\sqrt{2\sigma_j^2}}\right)^2}}{\sum_{k\neq i}\mathcal{A}_{jk}\frac{1}{B_{jk}^{<>}+\phi''_{jk}}+\frac{\Theta\left(-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j\right)}{\alpha_j}} + \\ \frac{1}{2}\dfrac{\left(\mathrm{erf}\left(\frac{x-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)-\mathrm{erf}\left(\frac{-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)\right)(x-\bar{\Lambda}_j)+\frac{2\sigma_j^2}{\sqrt{2\pi\sigma_j^2}}\left(e^{-\left(\frac{x-\bar{\Lambda}_j}{\sqrt{2\sigma_j^2}}\right)^2}-e^{-\left(\frac{-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j}{\sqrt{2\sigma_j^2}}\right)^2}\right)}{\sum_{k\neq i}\mathcal{A}_{jk}\frac{1}{B_{jk}^{<>}+\phi''_{jk}}}, & \text{if } x > -\sum_{all}\bar{\Lambda}_{all} \end{cases} \tag{10}$$

$$B_{jk}^{<>} = \begin{cases} \frac{1}{2}\dfrac{\mathrm{erfc}\left(\frac{x-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)+1}{\sum_{k\neq i}\mathcal{A}_{jk}\frac{1}{B_{jk}^{<>}+\phi''_{jk}}+\frac{\Theta\left(-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j\right)}{\alpha_j}}, & \text{if } x \le -\sum_{all}\bar{\Lambda}_{all} \\ \frac{1}{2}\dfrac{\mathrm{erfc}\left(\frac{x-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)-\mathrm{erf}\left(\frac{-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j}{\sqrt{2}\sigma_j}\right)}{\sum_{k\neq i}\mathcal{A}_{jk}\frac{1}{B_{jk}^{<>}+\phi''_{jk}}+\frac{\Theta\left(-\sum_{all}\bar{\Lambda}_{all}-\bar{\Lambda}_j\right)}{\alpha_j}}, & \text{if } x > -\sum_{all}\bar{\Lambda}_{all} \end{cases} \tag{11}$$

3. Results

To demonstrate the efficacy of the algorithm we first tested it on a simple synthetic IEEE benchmark network [16] with two generator nodes of fixed capacities Λ=4, with the remaining nodes defined as consumers of fixed capacities Λ=-2, resulting in an overall power deficit of -16. To focus on load shedding performance of the algorithm we do not consider fluctuations in this example. Fig. 1 shows pie charts where the colour of each slice represents the different nodes and their importance weights I are given on the circumference which we vary to show the efficacy of the algorithm (replacing the α notation for I). The pie chart represents the total amount of deficit within the network and how it is divided among the nodes. Any slices detached indicate a node which still has positive power (surplus generation or satisfied). We see that when importance weights for all nodes are small (top left) the power loss objective function becomes relatively more important and some power remains at the generators. Top right shows how when the importance of all nodes is large and equal the power deficit is equally divided and effects of minimising power loss are negligible. Bottom left is an example of a case where one node is less important than others and the algorithm assigns the majority of the power deficit to it. Bottom right shows that alternatively, if one node is much more important that others the power deficit assigned to it is negligible.

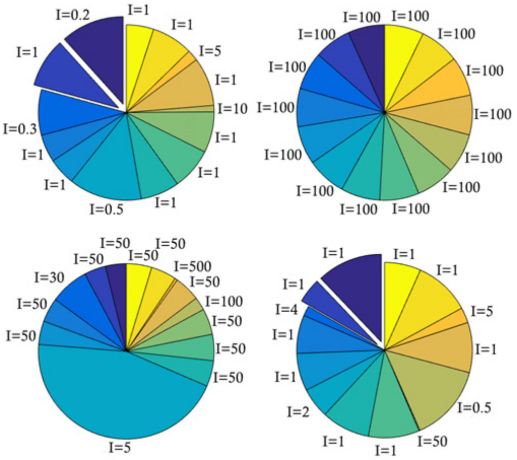

Fig. 1. Pie charts with colours representing different nodes; I values represent the importance value α for each node. Examples are taken from a 14-Bus synthetic IEEE benchmark network. There are two generator nodes with a mean capacity $\overline{\Lambda} = 4$, and the remaining consumer nodes have $\overline{\Lambda} = -2$. All node capacities are fixed, $\sigma = 0$. The confidence interval is $p = 0.1$. Slices detached from the chart indicated power surplus at the node.

Fig. 2 shows histograms of the power distribution supplied over the edges once a solution is found. It can be seen that when importance is zero, minimising power loss is the most important objective and no power is distributed. As the importance increases the power distribution histogram tends towards a bi-modal distribution with means at 0.043 and 0.33; representing the power sent to minimise the deficit of neighbouring nodes. The peaks presumably characterise more and less central edges with high and low power flow, respectively. They become less emphasised and more spread out as resources primarily aim to reduce load shedding irrespective of power loss, equalising the deficit. Each graph also includes the average power loss per solution (PL), which increases as importance values increase, while the corresponding average load shedding cost (LS) decreases. When there is inevitable failure in real power networks due to power deficit, reducing the deficit at the most important nodes becomes a priority over transportation costs.

Fig. 3 shows the relation between power excess at the node and the average importance, where importance is calculated by randomly drawing values from the Gaussian distribution $\mathcal{N}(m, 1)$, where m is 0.5 (×), 1 (•), 1.5 (○) and 2 (+), respectively. We see that when the importance is smaller, the excess at nodes varies the most; and as the average importance increases, the value of excess at each node tends to become more uniform as transportation costs become less significant. It can also be seen for each m that importance per node effects the remaining deficit.

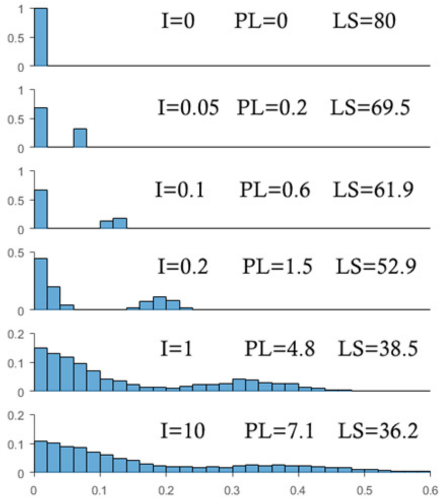

Fig. 2. The average, normalised distribution of the power values at each edge after convergence. The importance given to each node in the network, with capacities given in Fig. 2, is indicated above each of the histograms.

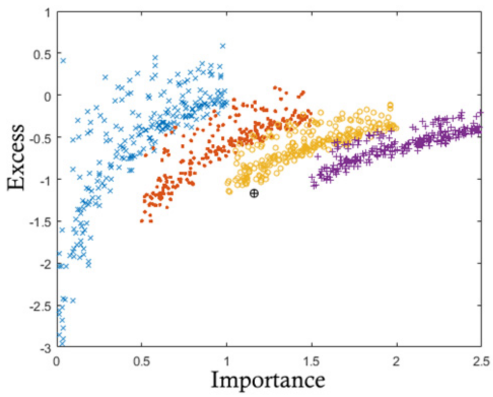

Fig. 3. Scatter graph showing examples of importance values selected randomly from a Gaussian distribution with means 0.5 (×), 1 (•), 1.5 (○) and 2 (+) and variance 1. Each symbol represents a node in the 100-node randomly connected network, plotted according to its power deficit (or negative excess) and the importance weight it was given. One fifth of nodes in the network are given mean capacity $\overline{\Lambda} = 1$, and the remaing consumers $\overline{\Lambda} = -1$, all variances are fixed at $\sigma = 0.5$.

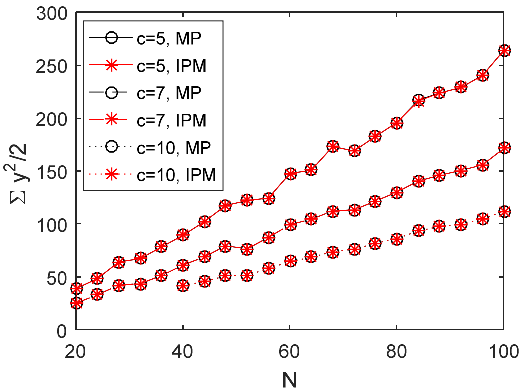

Fig. 4. Line graph of the cost of transportation when increasing N when using a message passing algorithm (black) and the interior point method (red) algorithm from the Matpower program [17], for different connectivity's on a simple network without fluctuations. One fifth of nodes are generators with $\overline{\Lambda} = 10$, two fifths are consumers of $\overline{\Lambda} = -3$ and the rest are substations of $\overline{\Lambda} = 0$.

Fig. 4 shows transportation cost as N increases, calculated using message passing (black) and the interior point method [17] (red) for $c = 5, 7$ and 10 (for $c = 10$ we only tested larger networks with $N \geq 40$ to reduce the likelihood of small loops) on a simplified network. Comparing the performance of our method against the interior point method in the simple conditions we find that they return exactly the same costs. Fig. 5 indicates the average time taken (in seconds) to optimise the networks in Fig. 4 as N increases. The figure shows that message passing takes longer and increases linearly with the system size, while the computing time interior point method remains small. The linear increase in computational cost with the system size is consistent with the analysis and since we advocate message passing as a method for economic dispatch at the 15-60 min time-scale it remains a competitive method that converges in seconds. We also expect that message passing could also be optimised to reduce the associated computational time. The Matpower program has been professionally coded to reduce computation time but is less flexible in the type of objective functions and uncertainties it can accommodate; computational cost is likely to grow much faster for more complex scenarios. The main advantage of the message passing method is in its flexibility and ability to accommodate different objective functions, uncertainties and constraints.

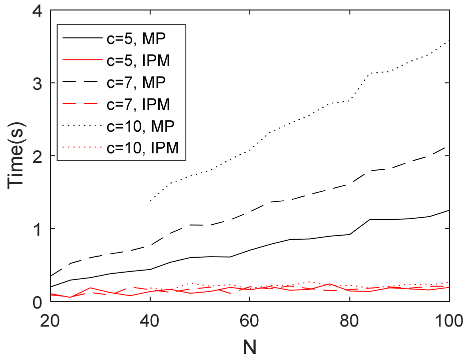

Fig. 5. The average time taken to run simple networks from Fig. 4 for increasing N using a message passing algorithm (black) and the interior point method (red) algorithm, for different connectivity's.

4. Conclusions

The paper shows how message passing is capable of optimising power distribution in a network while considering multiple objective functions and accommodating fluctuations in both generation and demand. The paper also shows that nodes can be weighed according to importance so that in the event of inevitable failure due to power shortage the power provision is prioritised. Simulation results, carried out over a synthetic IEEE benchmark network, have shown that the importance weights can have a significant effect on the distribution of power deficit. Developing alternative derivations of load shedding algorithms could include adding a maximum deficit constraint (so that generators cannot have negative deficit and consumers' deficit is limited by their capacity). Additionally, nodes could be made to reach maximum power deficit one at a time, so that other nodes remain unaffected. Comparing MP against as existing method we see that message passing can find optimal solutions in a short time period. When considering fluctuations, we expect our method to provide similar or better results as the MP algorithm is able to inherently consider fluctuations, this will be examined in later studies.

More generally, message passing techniques could be applied to address a number of other distributed optimisation problems in power grids. Future research directions may include adding bandwidth restrictions to fluctuating networks, minimising generation costs and adjusting voltages at nodes to fit a DC optimal power flow.

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