

Non-Stationary Stochastic Inventory Lot-Sizing with Emission and Service Level Constraints in a Carbon Cap-and-Trade System

Abstract

Firms worldwide are taking major initiatives to reduce the carbon footprint of their supply chains in response to the growing governmental and consumer pressures. In real life, these supply chains face stochastic and non-stationary demand but most of the studies on inventory lot-sizing problem with emission concerns consider deterministic demand. In this paper, we study the inventory lot-sizing problem under non-stationary stochastic demand condition with emission and cycle service level constraints considering carbon cap-and-trade regulatory mechanism. Using a mixed integer linear programming model, this paper aims to investigate the effects of emission parameters, product- and system-related features on the supply chain performance through extensive computational experiments to cover general type business settings and not a specific scenario. Results show that cycle service level and demand coefficient of variation have significant impacts on total cost and emission irrespective of level of demand variability while the impact of product's demand pattern is significant only at lower level of demand variability. Finally, results also show that increasing value of carbon price reduces total cost, total emission and total inventory and the scope of emission reduction by increasing carbon price is greater at higher levels of cycle service level and demand coefficient of variation. The analysis of results helps supply chain managers to take right decision in different demand and service level situations.

Keywords: Carbon emission; Lot-sizing; Non-stationary stochastic demand; Cycle service level; Carbon Cap-and-Trade System.

1. Introduction

Across the world, firms are under intense pressure from their major stakeholders including governments and customers, to cut carbon emissions generated during conduct of their business. Considering the detrimental impacts of carbon emission such as global warming and climate change, many countries are enforcing different carbon regulatory mechanisms such as carbon cap-and-trade and carbon tax or environmental standards such as ISO 14000, to control the carbon emission. Employing more energy efficient machines/equipments and facilities, and using energy generated by renewable sources are also commonly used but costlier solutions to reduce emissions. But, given the impact of supply chain decisions on carbon emissions, a potential solution could be achieved by incorporating emission concerns in the models to optimize these decisions (Huisinigh et al., 2015; Govindan et al., 2014; Toptal et al., 2014; Benjaafar et al., 2013). In addition, modeling of these supply chains considering of stochasticity and dynamic nature of demand process would provide results applicable to more realistic situations.

This paper deals with the inventory lot-sizing problem of a firm under non-stationary stochastic demand with carbon emission constraints. We consider cycle service level as customer service measure since the demand process is not deterministic. With the help of a mixed integer linear programming model we determine the optimal replenishment schedule that minimizes the system-wide cost, in advance of the planning horizon. The model explicitly accounts for the emissions generated due to purchasing, ordering and storage activities along with corresponding costs. Under the carbon cap-and-trade regulatory mechanism, the present study analyses the impacts of emission parameters such as ordering emission, carbon price and carbon cap, and product- and system- related features such as cycle service level, ordering cost, demand pattern, level of demand variability and demand coefficient of variation through extensive computational experimentations on the model. We consider carbon emission cap-and-trade policy since it is a market-based mechanism and generally accepted as an effective solution in curbing emission (Hua et al., 2011). Moreover, the other regulatory mechanisms such as carbon tax or environmental standard are not market controlled, but government controlled (Dobos, 2005). The European Union Emissions Trading System (EU-ETS) is one of the first and biggest emission trading systems which may lower carbon emissions by 21% than the 2005 levels by 2020 (Jaber et al., 2013). The results of the study cover general type business settings and not a specific scenario. Furthermore, we analyze the results to help supply chain manager to take right decision under a given business setting.

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2 The rest of the paper is organized as follows. In Section 2 we present the extant literature review
3 followed by Section 3 in which we give details of methodology, problem statement and
4 mathematical formulation. In Section 4 we present experimental design followed by detailed
5 analysis of experimental results. Finally we discuss the results in Section 5 and conclude the
6 paper with future research scope in Section 6.
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10 **2. Literature Review**

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12 The extant literature on sustainable supply chain management mostly focuses on product
13 recycling or re-use. The issue of sustainability has been addressed in several studies in the
14 context of reverse or closed-loop supply chain, but only few papers apply the sustainability
15 concerns in forward supply chains using quantitative modeling (Seuring, 2013; Gonzalez et al.,
16 2013). Many studies in this area conclude that supply chain decisions have significant impact on
17 carbon emissions (Huisingh et al., 2015; Govindan et al., 2014; Toptal et al., 2014; Benjaafar et
18 al., 2013). The study of Plambeck (2012) suggests that tremendous changes in supply chain
19 design and operation are required to avert climate change and explains through case studies that
20 how carbon emissions can be reduced profitably in supply chains. Furthermore, a few studies
21 integrate carbon emissions concerns in forward supply chain problems such as network design,
22 production planning, product mix and supplier selection (Jin et al., 2014; Zhang and Xu, 2013;
23 Chaabane et al., 2012; Shaw et al., 2012; Bai and Sarkis, 2010; Letmathe and Balakrishnan,
24 2005; Gong and Zhou, 2013; Shaw et al., 2012) and others investigate its impact on supply chain
25 structures and transportation mode selection (Cachon, 2011; Hoen et al., 2014). Some studies
26 incorporate emission concerns in inventory lot-sizing decision which not only influences total
27 system-wide cost, customer service levels and carbon emissions significantly, but also affects
28 other decisions such as packaging, waste and location (Bonney and Jaber, 2011).
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46 Most of the studies in the area of inventory lot-sizing with carbon emissions concerns, consider
47 continuous demand (He et al., 2014; Hua et al., 2011; Wahab et al., 2011; Arslan and Turkey,
48 2010; Bouchery et al., 2012; Toptal et al., 2013; Chen et al., 2013; Battini et. al., 2014) or time-
49 varying deterministic demand (Kantas et al., 2015; Absi et al., 2013; Helmrich et al., 2012). Only
50 the work of Song and Leng, (2011) considers stochastic continuous demand. But in real life,
51 demand process is non-stationary stochastic type due to shrinking product life cycles, frequent
52 new product launches and use of promotion schemes (Martel et al., 1995; Neale and Willems,
53 2009; Choudhary and Shankar, 2014; Choudhary and Shankar, 2015). But it has been ignored
54 while addressing the emission concerns in inventory lot-sizing decision.
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2 In this line, our research aims to model the inventory lot-sizing problem with emission and cycle
3 service level constraints. Unlike Benjaafar et al. (2013) who address a simple lot-sizing problem
4 with deterministic demand, this paper attempts to address single item inventory lot-sizing
5 problem under non-stationary stochastic demand.
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10 **3. Methodology**

11 *3.1 Problem statement*

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13 We consider a supply chain as shown in Figure 1, in which a buyer firm fulfills the dynamic and
14 uncertain demands of a single product over a planning horizon of T periods with a cycle service
15 level of α . The planning horizon with all possible order cycles has been depicted in Figure 1
16 (inside the box) where time line shows starting and ending of adjacent periods with thick dark
17 circles. An arc represents an order cycle where starting of the first period is connected to the
18 ending of last period of the cycle. The random demands d_i in discrete time periods $i \in \{1, \dots, T\}$
19 are mutually independent and normal distributed with known means and standard deviations that
20 may vary over time. We assume that these quantities are the outcome of a forecasting procedure.
21 Moreover, we account for carbon emissions generated by different activities of the firm such as
22 ordering (e.g. transportation emission), holding (emissions due to energy spent on storage) and
23 purchasing (emissions due to handling). We consider a carbon cap-and-trade emission regulatory
24 mechanism in which the total emissions due to all activities over the planning horizon cannot
25 exceed a carbon cap as imposed by the regulator. The firm can buy or sell the balance carbon
26 credits from the open carbon trade market at a given carbon price if the generated emissions are
27 greater or less than the carbon cap.
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43 [Insert Figure 1]

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45 The firm needs to decide the replenishment schedule that minimizes the total costs which include
46 ordering and holding costs along with the cost (revenue) of purchasing (selling) balance carbon
47 credits. As the quantification of shortage penalty cost is very difficult, we assume that the firm
48 meets target cycle service level and backorders the unfilled demand. The cycle service level
49 specifies a minimum probability α which ensures that at the end of every period the net inventory
50 will not be negative. The near optimal solution for stochastic dynamic lot-size problem has been
51 given in Bookbinder and Tan (1988). They consider three strategies of lot-sizing decision, named
52 “static uncertainty”, “dynamic uncertainty” and “static-dynamic uncertainty”. The replenishment
53 timings and order sizes are decided at the beginning of the planning horizon in case of “static
54 uncertainty” strategy whereas these decisions are taken in every period in case of “dynamic
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2 uncertainty”. We assume “static-dynamic uncertainty” strategy in this paper where
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4 replenishment timings and corresponding stock levels are fixed at beginning of the planning
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6 horizon and the order sizes for coming periods are determined after realization of the demands of
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8 previous periods. The replenishment timings and order sizes are crucial in achieving desired
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10 cycle service level and determining the total costs and emissions generated.

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12 Following the mixed integer programming formulation proposed by Tarim and Kingsman (2004)
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14 we develop the non-stationary stochastic lot-sizing model by integrating carbon emission and
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16 cycle service level constraints, as described in the following section. The model selects the series
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18 of those order cycles from all possible $T(T + 1)/2$ order cycles in a T periods planning horizon
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20 problem which minimizes the total cost and satisfies the carbon emission constraint.

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22 Furthermore, we use full factorial experimental design considering emission parameters such as
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24 ordering emission, carbon price and carbon cap, and product- and system- related features such
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26 as cycle service level, ordering cost, demand pattern, level of demand variability and demand
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28 coefficient of variation. Through extensive computational experimentations using the developed
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30 model, we produce results for all experimental settings. For each experimental set-up, it is also
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32 required to calculate some quantities offline, as described later in the illustration section. The
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34 results are further analyzed to identify the impacts of problem parameters on supply chain
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36 performance in terms of costs and emissions.

37 *3.2 Model development*

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39 In the following paragraphs, we explain the details of objective function and constraints of the
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41 model. The model determines the optimum replenishment schedule, defined by a series of
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43 adjacent replenishment order cycles which minimizes the total cost.

44 45 1. Objective function

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47 The objective function in Equation (1) minimizes the total cost over the planning horizon of T
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49 periods. The first term represents the ordering cost where o is ordering cost per order and Z_i is a
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51 binary variable indicating whether the order in period i is placed or not. The second term
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53 indicates holding cost of the expected net inventory $E(I_i)$ in each period where h is holding cost
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55 per item per period. The purchasing cost is calculated in the third term where v and $E(X_i)$ are unit
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57 purchasing cost and expected lot-size ordered in period i , respectively. The fourth term
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59 represents cost or revenue related to carbon credit purchased (e_i^p) or sold (e_i^n) at a carbon price p
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61 in the open carbon trade market.

$$\text{Min} = \sum_{i=1}^T [oZ_i + hE(I_i) + vE(X_i) + p(e_i^p - e_i^n)] \quad (1)$$

2. Inventory balance constraints

The Equations (2) calculate expected end-period inventory by subtracting expected demand from the expected order-up-to-level $E(R_i)$ of the period i . The Equations (3) calculate expected lot-size $E(X_i)$ in any period i which is the difference of order-up-to-level of the present period and net inventory level of the previous period. The Constraints (4) ensure that the expected order-up-to-level of a period must be greater than the ending inventory of previous periods.

$$E(I_i) = E(R_i) - E(d_i), \quad i = 1, \dots, T \quad (2)$$

$$E(X_i) = E(R_i) - E(I_{i-1}), \quad i = 1, \dots, T \quad (3)$$

$$E(R_i) \geq E(I_{i-1}) \quad i = 1, \dots, T \quad (4)$$

3. Ordering cost charging constraints

The Constraints (5) ensure charging of ordering cost if an order is placed in period i where M is a large number.

$$E(X_i) \leq MZ_i, \quad i = 1, \dots, T \quad (5)$$

4. Cycle service level constraints

The Constraints (6) ensure achieving a cycle service level (α) in every order cycle, ending in period i and has a cycle length of j periods. The term $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$ is the inverse of cumulative probability distribution function and represents order-up-to-level at the beginning of the order cycle. The binary variable P_{ij} equals one if the order cycle ending in period i and having a cycle length of j periods is carried out, otherwise it is zero.

$$E(I_i) \geq \sum_{j=1}^i \left[G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha) - \sum_{k=i-j+1}^i E(d_k) \right] P_{ij}, \quad i = 1, \dots, T \quad (6)$$

5. Constraints to uniquely identify optimum replenishment schedule

With the help of Equations (7) and Constraints (8), the binary variables P_{ij} identify the optimum replenishment schedule uniquely. They select the adjacent order cycles from $T(T+1)/2$ number of possible order cycles for a T periods planning horizon so that total costs are minimized. The Equations (7) ensure termination of only one replenishment cycle in a period i while it may start from any period before period i .

$$\sum_{j=1}^i P_{ij} = 1, \quad i = 1, \dots, T \quad (7)$$

$$P_{ij} \geq Z_{i-j+1} - \sum_{k=i-j+2}^i Z_k, \quad i = 1, \dots, T \quad j = 1, \dots, i \quad (8)$$

6. Emission constraints

Constraint (9) controls carbon emissions under carbon cap-and-trade regulatory mechanism applied over the planning horizon. The first term in Constraint (9) accounts for carbon emissions produced due to ordering where o_e is the ordering related emission per order. The second and third terms account for variable and storage related emissions where v_e and h_e denote variable emission per item and holding emission per item per period respectively. On the right hand side of the Constraint (9), the first term denotes carbon cap $CAP_{horizon}$ applied over the horizon whereas the second term includes e_i^p and e_i^n denoting the amount of carbon credits the organization buys or sells, respectively in any period. Buying or selling carbon credits help in relaxing the carbon cap imposed over the horizon. Constraints (10) correspond to binary and non-negativity constraints

$$\sum_{i=1}^T (o_e Z_i + v_e E(X_i) + h_e E(I_i)) \leq CAP_{horizon} + \sum_{i=1}^T (e_i^p - e_i^n), \quad (9)$$

$$P_{ij}, Z_i \in \{0, 1\}, \quad E(I_i), E(R_i), E(X_i) \geq 0 \quad i \in [1, T], \quad j \in [1, i], \quad (10)$$

3.3 An illustration

The solution of integer linear programming model, as described by expressions (1) to (10), requires offline calculation of inverse cumulative distribution function $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$ for all possible order cycles. In this section, this offline calculation procedure is explained. Through a numerical example, as detailed in next paragraph, we also explain the steps involved in solving the integer linear programming model along with its theoretical framework.

A buyer firm faces random demands of a product over a planning horizon of six periods and wants to fulfill it with a cycle service level of 90%. The demand in each period is assumed to be mutually independent and normally distributed with mean d_i and a coefficient of variation (CV) of 0.3. Table 1 shows the expected mean demand and its standard deviation in each period. We assume holding cost and ordering cost to be 1 and 200 respectively. Replenishment lead time and unit purchasing cost are ignored. Let emission parameter such as ordering emission, holding

emission and unitary carbon emission are 400, 1 and 2 respectively. We also assume carbon price to be 5 and carbon cap to be 3000.

[Insert Table 1]

Let an order cycle ending in period i and having a cycle length of j periods is carried out, therefore it starts in period $(i - j + 1)$, as shown in Figure 1. It is required to find the order-up-to-level R_{i-j+1} at the beginning of the cycle so as to achieve targeted cycle service level of α . As suggested in Tarim and Kingsman (2004), the order-up-to-level R_{i-j+1} is given by the inverse

cumulative distribution function $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$ which ensure that $P\left\{R_{i-j+1} \geq \sum_{k=i-j+1}^i d_k\right\} \geq \alpha$,

i.e. probability of having order-up-to-level R_{i-j+1} greater than expected total demand in the cycle, to be greater than cycle service level α . This implies that the cumulative probability of having total expected demand of the cycle less than the starting order-up-to-level, is greater than cycle service level, i.e. $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}(R_{i-j+1}) \geq \alpha$. With the help of following steps, we can

calculate inverse cumulative distribution function $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$ for all possible cycles.

Step 1: The inverse cumulative distribution function is given as,

$$G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha) = \sum_{k=i-j+1}^i E\{d_k\} + Z_\alpha * CV * \sqrt{\left(\sum_{k=i-j+1}^i E^2\{d_k\}\right)}$$

For an order cycle covering the demands of periods 2 to 5, the value of $i = 5$ and the length of cycle $j = 4$. Therefore,

$$G_{d_2+d_3+d_4+d_5}^{-1}(0.90) = \sum_{k=2}^5 E\{d_k\} + 1.282 * 0.3 * \sqrt{\left(\sum_{k=2}^5 E^2\{d_k\}\right)}$$

$$G_{d_2+d_3+d_4+d_5}^{-1}(0.90) = (170 + 185 + 200 + 215) + 1.282 * 0.3 * \sqrt{(170^2 + 185^2 + 200^2 + 215^2)} \approx 919$$

For all possible cycles the $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$ values are shown in Table 2.

[Insert Table 2]

Step 2: Based on $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$ values calculated offline and the parameters values assumed earlier, the optimum replenishment schedule can be determined by solving the model as defined in Equations (1) – (10) on any commercial solver. For the given example, we obtain order up-to-

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2 levels $R_1 = 413$, $R_3 = 490$ and $R_5 = 566$ with total expected costs equal to 11728 and total
3 emission equal to 4980, as detailed in Table 3.

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6 [Insert Table 3]

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8 Expected total emission over the planning horizon = Ordering emission + Holding emission +

9 Unitary emission = 4980

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12 Carbon credit purchased = $4980 - 3000 = 1980$

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15 Expected total cost = Total order cost + Expected total holding cost + cost of carbon credits

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17 purchased = $3*200 + 1*1228 + 5*1980 = 11728$

18 19 20 **4. Numerical Experiments**

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22 In this section, we conduct extensive computational experiments to investigate the impacts of
23 emission parameters, product- and system-related features on supply chain performance
24 measures such as reduction in emissions, costs and inventory levels. The emission parameters
25 include ordering emission, carbon price and carbon cap whereas product- and system- related
26 features include cycle service level, ordering cost, demand pattern, level of demand variability
27 and demand coefficient of variation.

28 29 30 *4.1 Experimental design*

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32 To cover general type business settings and not a specific scenario, we design a test set involving
33 a variety of demand patterns, system-and emission- related parameters. We carry out full
34 factorial experiments using the model described in Section 2.3 and the following sets of
35 parameters: demand patterns $DP \in \{STAT, RAND, SIN1, SIN2, LCY1, LCY2\}$, ordering cost o
36 $\in \{200, 400, 900\}$, cycle service level $\alpha \in \{0.9, 0.95, 0.99\}$, demand coefficient of variation $CV \in$
37 $\{0.1, 0.4, 0.7\}$, ordering emission $o_e \in \{200, 400, 900\}$, carbon cap $CAP_{horizon} \in \{10000, 25000\}$
38 and carbon price $p \in \{1, 5\}$. In all the experiments, we set inventory holding cost $h = 1$ and
39 holding related emission $h_e = 1$. We assume zero initial inventory and unit purchasing cost, as
40 these values do not affect the solution. The planning horizon consists of 18 periods of equal time
41 duration.

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2 SIN1 and SIN2 patterns as well as the LCY1 and LCY2 patterns are at lower and higher levels of
3 demand variability coefficient respectively. Demand variability coefficient is a measure of the
4 variability of a demand pattern and defined as the ratio of variance of demand per period to the
5 square of average demand per period. The demand patterns are so designed that the average
6 demand is same for each pattern thus avoiding any effect that may occur because of variation in
7 total demand. The total mean demand is 3600 for each demand pattern.
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14 [Insert Figure 2]

15 16 4.2 Analyzing the impact of variation in problem parameters

17
18 With the help of full-factorial experimental design we create 1944 test instances by varying
19 problem parameters such as DP , o , α , CV , o_e , $CAP_{horizon}$ and p . The results produced after
20 conducting experiments on these test instances are shown in Figure 3-6.
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24
25 [Insert Figure 3]

[Insert Figure 4]

26
27 Figure 3-4 reveal that the demand pattern strongly influences the amount of emission produced
28 and total cost in a supply chain. Under the RANDOM demand pattern, the emissions and costs
29 are always higher as compared to SIN and LCY patterns. Moreover, at a lower level of demand
30 variability, the LCY pattern gives the lowest value of emission and costs while at higher levels,
31 the SIN pattern gives minimum costs and emissions. With the increase in demand variability, the
32 gap in the values of total emissions and total costs among different demand patterns would go on
33 decreasing. Meaning thereby, a product's demand pattern plays a significant role in determining
34 total cost and emission when demand variability is low.
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42
43 [Insert Figure 5]

44
45 In Figure 4, it is evident that the cycle service level also has significant impacts on both the costs
46 and emissions. The increase in cycle service level would increase the required level of inventory
47 in the system accompanied with an increase in frequency of replenishment as shown in Figure 7.
48 It results in increase in ordering costs and emissions as well as increase in inventory holding
49 costs and emissions. Therefore, when cycle service level increases from 0.90 to 0.95 total cost,
50 total inventory and total emission would increase significantly. Moreover, these increments are
51 even steeper when cycle service level increases from 0.95 to 0.99 as the rate of increase in
52 inventory levels is higher.
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2 [Insert Figure 6]
3

4 [Insert Figure 7]

[Insert Figure 8]

6 Likewise, the demand coefficient of variation influences total costs and emissions significantly.
7
8 The variations in total inventory and order frequency with demand coefficient of variation are
9 shown in Figure 8. It reveals that total inventory as well as order frequency increase with
10 increase in coefficient of variation. Therefore, total costs and emissions would also increase with
11 the increase in demand coefficient of variation since the costs and emissions related to inventory
12 holding would increase at a higher level of demand uncertainty. But at a lower level of demand
13 uncertainty, the required inventory is low and the firm would make revenue by selling extra
14 emission credits especially when the emission cap is higher. Therefore, total costs are shown
15 negative in Figure 6.
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23 [Insert Figure 9]

[Insert Figure 10]

24 The impacts of ordering cost and ordering emission are shown in Figure 9 and Figure 10. The
25 total cost increases with both the ordering parameters, though the rate of increase is higher at
26 higher values of ordering cost and emission. But the change in total emission is not
27 unidirectional with the change in ordering cost, though it keeps on increasing with ordering
28 emission. The results show that the variation in carbon cap has no effect on total emission and
29 inventory level but it affects total cost significantly. This result is in line with Benjaafar et al.
30 (2013).
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40 *4.3 Analyzing effects of carbon price*

41 The experimental results are summarized in Table 4 for the cases when carbon price is increased
42 from $p=1$ to $p=5$. The average reduction in total cost, total inventory and total emission are
43 2493.98, 88.16 and 202.28 respectively as shown in the last row of Table 4. In order to minimize
44 cost, the firm tends to save carbon emission with the increase in carbon price by reducing the
45 required inventory level. Though it increases ordering frequency, but the corresponding increase
46 in ordering cost and ordering emission is lesser than the savings achieved. The decrease in total
47 holding cost and reduction in total emission and corresponding cost result into net decrease in
48 total cost. The saving of emission credits achieved by increasing carbon price at the higher
49 carbon cap can be sold in the open carbon trade market to earn revenue.
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59 [Insert Table 4]
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2 The increase in carbon price would reduce total cost, inventory level and emission, but its impact
3 on these reductions varies while having interaction with other problem parameters. The
4 reductions in performance measures along with interaction with other problem parameters are
5 shown in Table 4. For example, when cycle service level (α) or demand CV increases along with
6 rise in carbon price, the reduction in total cost would go on decreasing and becomes negative, but
7 the reduction in emission behaves contrary i.e. it increases with cycle service level (α) /demand
8 CV . Total cost is -2083 at $\alpha=0.99$ and -8157 at demand $CV=0.70$. Therefore, total cost increases
9 with an increase in carbon price at higher levels of cyclic service level or demand CV but it
10 decreases in their lower levels. The scope of emission reduction by increasing carbon price is
11 greater at higher levels of cycle service level (α) and demand CV . Moreover, the impact of
12 carbon price rise on total cost reduction is greater when the cycle service level (α) and demand
13 CV are at lower levels. Furthermore, the reduction in total inventory decreases with increasing
14 value of cycle service level (α) but it is not unidirectional with demand CV variation. The carbon
15 price rise would achieve greater total inventory reduction at higher level of α and mid range
16 values of demand CV .

17
18 We observe from Table 4 that the variation in total cost reduction with carbon price rise is more
19 sensible to variation in ordering emission than to ordering cost. It decreases with increasing
20 values of ordering cost/emission and even becomes negative when ordering emission is 900. The
21 reduction in total emission keeps decreasing with increase in ordering emission but it is not
22 unidirectional in case of increase in ordering cost. The total cost reduction as well as total
23 emission reduction with the rise in carbon price is greater at lower levels of ordering
24 cost/emission. The reduction in inventory with an increase in carbon price keeps decreasing with
25 increase in ordering emission level, but keeps increasing with increase in ordering cost. Even at
26 the highest level of ordering emission (= 900) or at the lowest level of ordering cost (=200), the
27 inventory level increases with carbon price as extra emissions generated by increased inventory
28 are lesser than the saved emission due to less frequent ordering.

29
30 The demand patterns also have significant impacts on performance measures with the rise in
31 carbon price. As shown in Table 4, at low demand variability when carbon price increases, the
32 stationary demand pattern gives lowest reductions in both total cost and total emission while
33 these reductions are highest for life cycle demand pattern. Furthermore, at higher demand
34 variability these reductions are lowest for random demand pattern whereas highest reduction in
35 total cost happens for life cycle demand pattern and highest reduction in total emission happens
36 for sinusoidal demand pattern, as shown in Table 4.

5. Discussion

The results of a detailed numerical study suggest that at a lower level of demand variability coefficient the demand pattern factor has greater impact. Total costs and emission are lower for life cycle (LCY) demand pattern at low level of demand variability but are lower for sinusoidal pattern at higher level. However, random (RAND) demand pattern results in higher costs and emissions for both the levels. The results also show that both cycle service level and demand coefficient of variation affect total cost and emission significantly. But the changes are even steeper for their higher levels. We also observe that the increasing value of carbon price reduces total cost, total emission and total inventory. But the increasing value of carbon price along with an increase in cycle service level (α) or demand CV would decrease the total cost reduction and increase the total emission reduction. The scope of emission reduction by increasing carbon price is greater at higher levels of cycle service level (α) and demand CV. The effect of carbon cap on total emission reduction is negligible, which is in line with Benjaafar et al. (2013). The results also show that the variation in total cost reduction with carbon price rise is more sensible to variation in ordering emission than to ordering cost.

6. Conclusions

The increasing pressure of governments and other stakeholders has been forcing the firms worldwide to review the supply chain decisions and incorporate emission concerns. This study addresses the inventory lot-sizing problem under non-stationary stochastic demand with emission and cycle service level constraints. It explicitly accounts for emissions due to ordering and storage activities along with emission per unit purchased. We aim to investigate the impacts of product features, system- and emission-related parameters on the emissions, costs and inventory levels of a supply chain. In practice, firms need to achieve several objectives such as service level, total cost and total emissions. Our study would help managers as it establishes how problem parameters affect the performance measures and it would be easier to make decisions according to their specific business environment.

This paper evaluates the impacts of different problem parameters on costs and emissions . The results of this study are applicabe to general type business settings not to any specific scenario. It is evident in the analysis that the demand pattern of a product has a significant impact on costs and emissions generated when demand variability is low. Moreover, a product with random demand pattern would always have higher costs and emissions as compared to a product with sinusoidal or life cycle demand pattern. Likewise, if a firm raises its service level, it would incur higher costs and emissions but the rate of increase are higher at higher cycle service levels.

1
2 Therefore, it is required to strike a trade-off among service level, total costs and emissions.
3
4 Similarly, the analysis also reveals that meeting demand of a product with higher demand
5
6 uncertainty is costlier and more emission intensive as compared to meeting demand of a product
7
8 with stable demand. The increase in carbon price always motivates firms to reduce emissions as
9
10 reflected in the results. The scope of emission reduction is greater for those organizations who
11
12 keeps very high service level or deal with products with high demand uncertainty. Except for
13
14 such organizations or for those which incur high ordering emission, the rise in carbon price
15
16 would also get reduction in total cost. Similarly, the products with life cycle demand pattern
17
18 would have greater reduction in total costs with increasing carbon price whereas the items with
19
20 sinusoidal or life cycle demand pattern would see higher reduction in total emissions.

21 The present study assumes carbon price to be constant. Future research may consider variation in
22
23 carbon price with other factors to investigate its impact on emission and cost reduction.

24
25 Additional research, based on empirical data drawn from real life cases, is worthwhile. Future
26
27 studies with different supply chain structures may further help in understanding the impact of
28
29 system parameters on cost and emission reduction objectives.

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Dear Editor-in-Chief of *Journal of Cleaner Production*,

We have addressed all the comments of learned reviewers. Please find enclosed revised manuscript, entitled “Non-Stationary Stochastic Inventory Lot-Sizing with Emission and Service Level Constraints in a Carbon Cap-and-Trade System”, by Purohit et al., which we would like to submit for publication as a Research Paper in *Journal of Cleaner Production*. Details of paper are as below:

No. of words including tables and figure captions	: 6352
No. of tables	: 4
No. of figures	: 10

We believe that our study would serve as a reference for production and supply chain managers to take right decision in different business settings under stochastic dynamic demand conditions while meeting economic and environmental aspects of sustainability, as a wide-reaching journal publishing original research on all aspects of inventory lot-sizing and sustainable operations.

Finally, this paper is our original unpublished work and it has not been submitted to any other journal for reviews or publication.

With kind regards.

Yours sincerely,

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Non-Stationary Stochastic Inventory Lot-Sizing with Emission and Service Level Constraints in a Carbon Cap-and-Trade System

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Research highlights

- Models inventory lot-sizing problem with emission concerns
- Considers non-stationary stochastic demand with cycle service level
- Analyzes effects of emission parameters under carbon cap-and-trade regulatory policy
- Analyzes effects of product and system related features

Figure

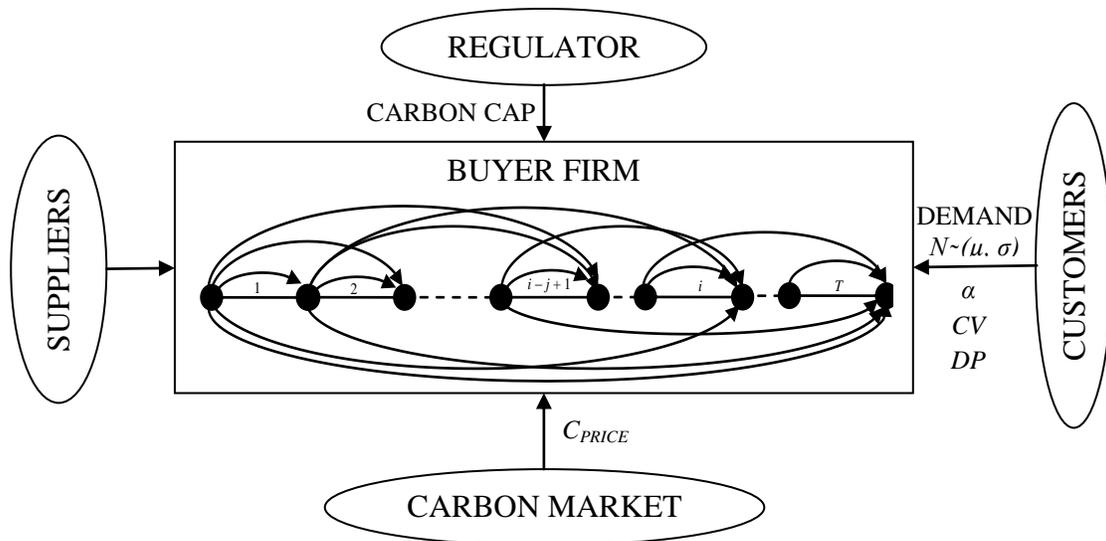


Figure 1 System at a glance with graphical representation of all possible order cycles

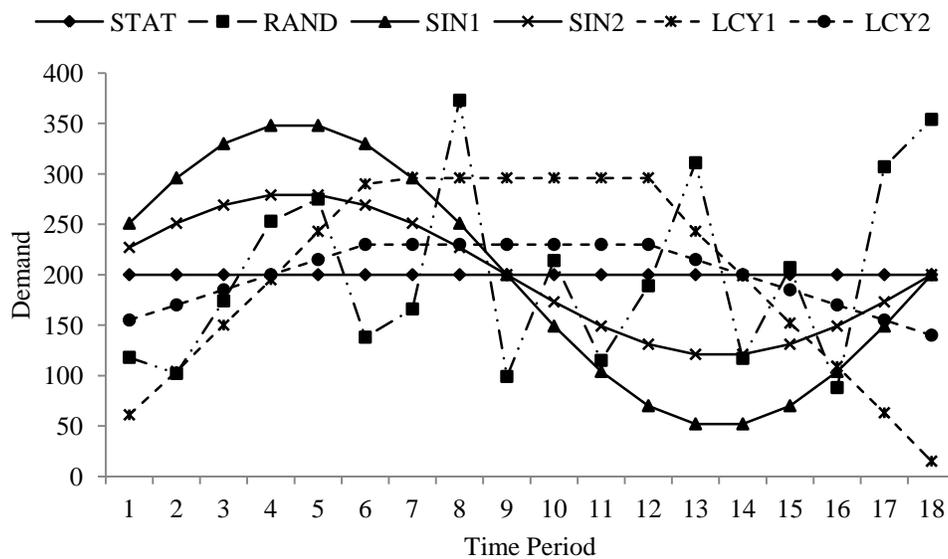


Figure 2 End-customer demand patterns

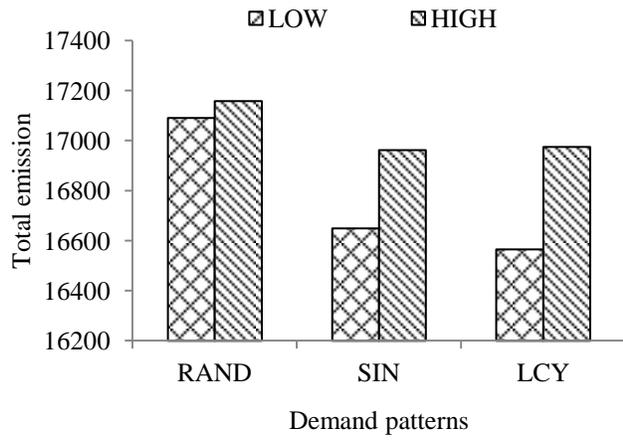


Figure 3 Variation in total emission with demand variability and demand pattern

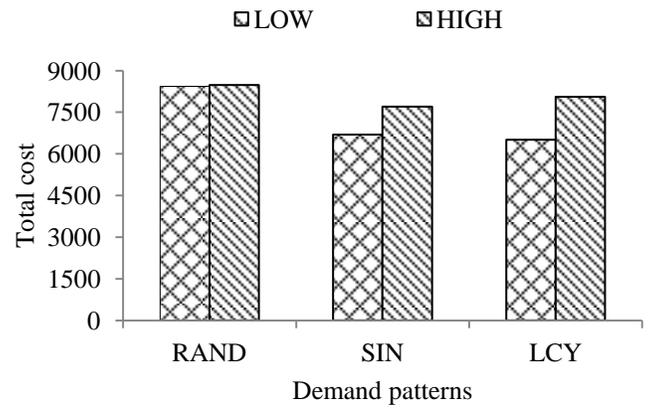


Figure 4 Variation in total cost with demand variability and demand pattern

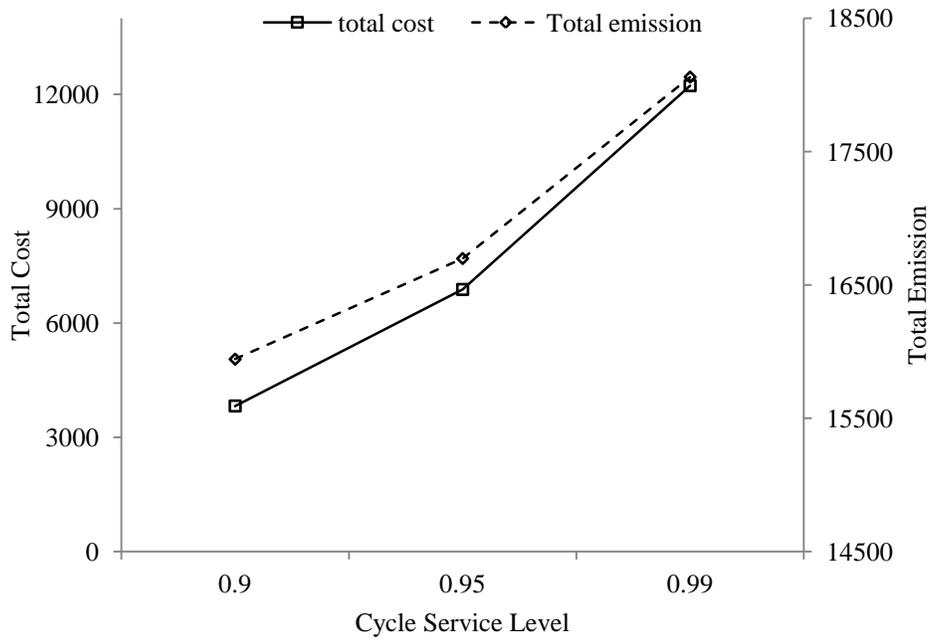


Figure 5 Effect of cycle service level on total cost and total emission

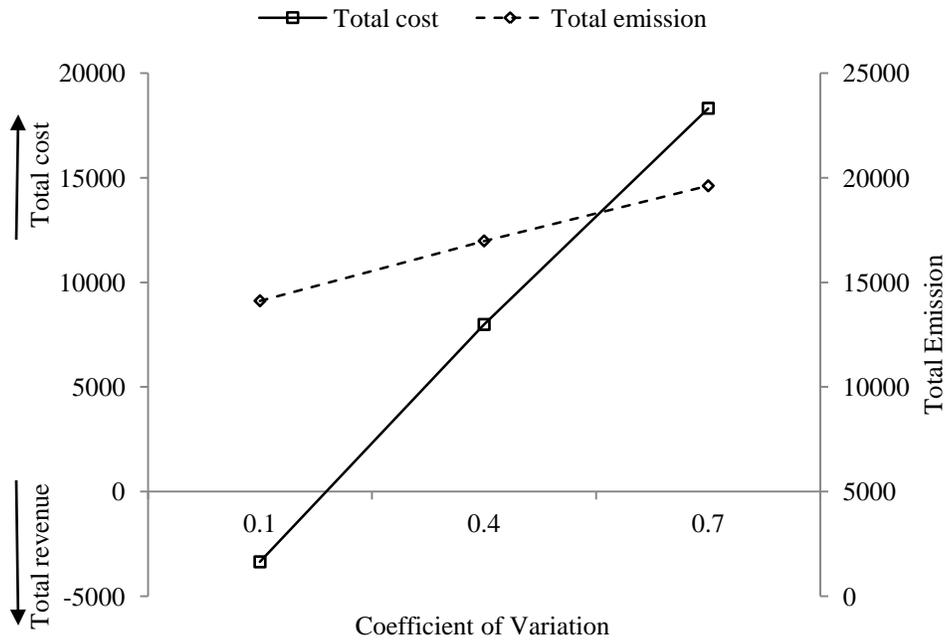


Figure 6 Effect of coefficient of variation on total cost and total emission

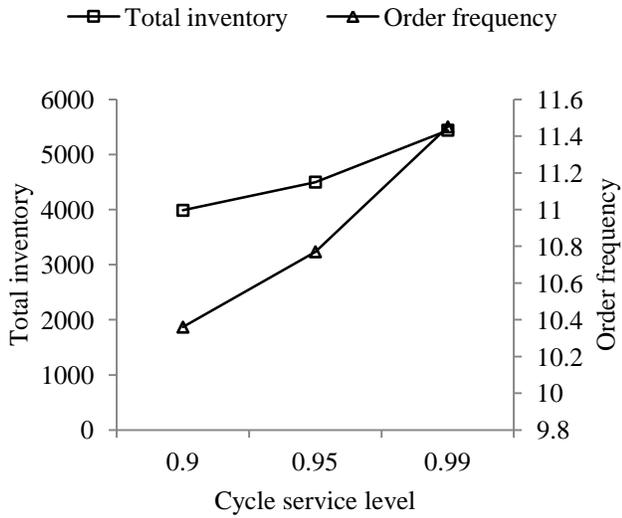


Figure 7 Effects of cycle service level on total inventory and order frequency

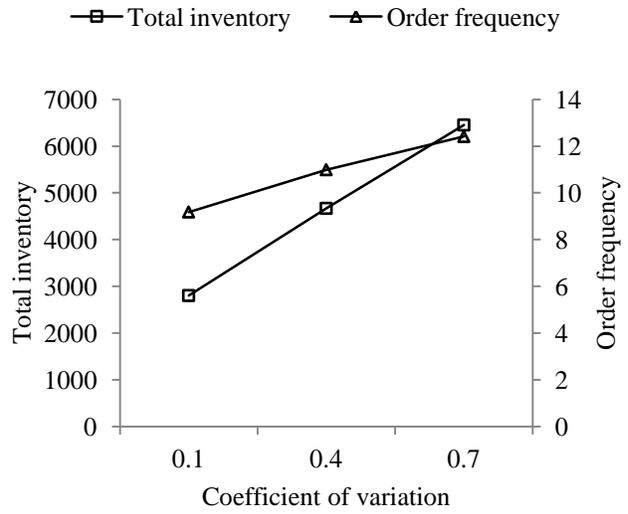


Figure 8 Effects of demand CV on total inventory and order frequency

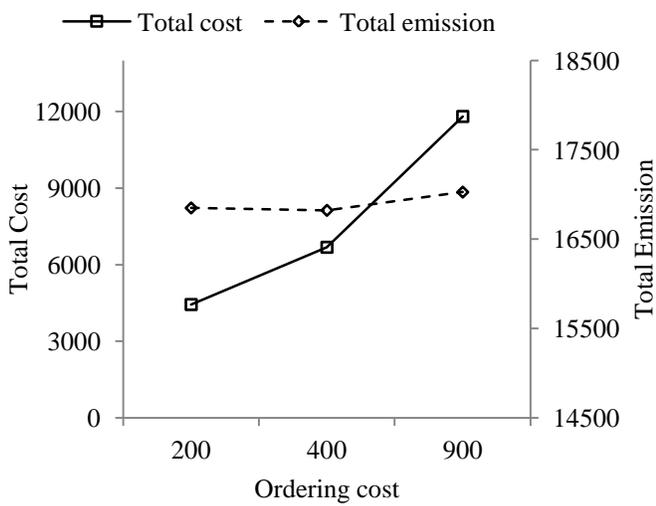


Figure 9 Effects of ordering cost on total cost and emission

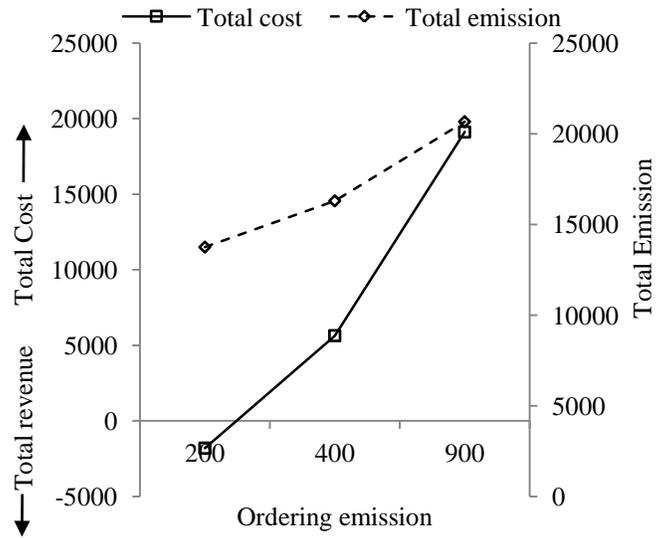


Figure 10 Effects of ordering emission on total cost and emission

Table 1 Demand data for the numerical example

Period (i)	1	2	3	4	5	6
Demand (d_i)	155	170	185	200	215	230
$\sigma_i = CV * d_i$	46.5	51	55.5	60	64.5	69

Table 2 Inverse cumulative distribution function $G_{d_{i-j+1}+d_{i-j+2}+\dots+d_i}^{-1}(\alpha)$

i/j	1	2	3	4	5	6
1	215					
2	235	413				
3	256	452	623			
4	277	490	678	847		
5	298	528	733	919	1085	
6	318	566	788	990	1173	1338

Table 3 Optimal replenishment schedule

Period (i)	1	2	3	4	5	6
Replenishment-up-to level	413		490		566	
Expected opening inventory	413	258	490	305	566	351
Expected demand	155	170	185	200	215	230
Expected closing inventory	258	88	305	105	351	121
Expected order quantity	413		402		461	
Ordering emission	400		400		400	
Holding emission	258	88	305	105	351	121
Unitary emission	826		804		922	

Table 4 Summary of experimental results when the carbon price increases from $p=1$ to $p=5$

Independent Variables	Total cost reduction	Total inventory reduction	Order frequency reduction	Total emissions reduction
Cycle service level (α)				
0.90	6156.8	131.5	-0.43	183
0.95	3401.9	85	-0.32	198
0.99	-2083.0	48.7	-0.33	225
Demand coefficient of variation (CV)				
0.10	13453.6	-40.8	-0.2	156
0.40	2180.1	166.3	-0.49	187
0.70	-8157.0	139.6	-0.44	263
Ordering emission				
200	15277.4	681.3	-1.87	325
400	4711.5	86.6	0.04	118
900	-12514.0	-502.8	0.76	165
Ordering cost				
200	2778.3	-464.5	0.99	150
400	2571.8	-53.3	-0.3	98
900	2162	776.9	-1.77	360
Demand Pattern (DP)				
<i>STAT</i>	1774.6	6.1	-0.2	186
<i>RAND</i>	1573.7	-2.7	-0.19	173
<i>SIN1</i>	3614.1	56.8	-0.28	218
<i>SIN2</i>	1795	160.6	-0.48	209
<i>LCY1</i>	3943.4	160.7	-0.44	225
<i>LCY2</i>	2251.5	150.9	-0.54	202
Average	2493.98	88.16	-0.36	202.28