
Ordered Weighted Averaging Operators 1988–2014:
A Citation-Based Literature Survey

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Abstract
This study surveys the Ordered Weighted Averaging (OWA) operator literature using a citation network analysis. The main goals are the historical reconstruction of scientific development of the OWA field, the identification of the dominant direction of knowledge accumulation that emerged since the publication of the first OWA paper and to discover the most active lines of research. The results suggest, as expected, that Yager (1988) [Yager, Ronald R. On ordered weighted averaging aggregation operators in multicriteria decision making. IEEE Transactions on Systems, Man, and Cybernetics, 18(1), 183–190.] is the most influential paper and the starting point of all other research using OWA. Starting from his contribution other lines of research developed and we describe them.

Keywords: Ordered weighted averaging (OWA); Aggregation operator; OWA Survey; OWA development

Introduction
The family of OWA operators was first introduced by Yager (1988) as a tool to deal with the problem of aggregating multicriteria to form an overall decision function. He described it as cumulative operators for membership aggregation. Following this conceptualization, the role of OWA weighting vector has been highlighted as a means for introducing the decision maker’s attitude (Yager, 1995a) and the OWA operator has

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received great attention and has been applied in different disciplinary contexts, for example, decision making under uncertainty (Yager and Kreinovich, 1999), fuzzy system and Information Retrieval System (IRS) (Kacprzyk and Zadrożny, 2001; Herrera-Viedma et al., 2003), data mining (Torra, 2004). It is widely recognised that the OWA operators have been applied to different research fields, but the present study is the first work depicting the OWA development scenario and describing its development path. This paper is the first systematic review of the growing literature on the OWA operator, it aims to trace the development of OWA research using Social Network Analysis (SNA) and presents a survey on the diffusion of the OWA in the literature over the last 26 years. Our main goals are:

- To identify the major publications/citations in the OWA field;
- To identify and illustrate the intellectual structure of this research domain;
- To describe the sub-area in which the OWA have been most applied.

To conduct this review we employ the data of ISI Web of Knowledge and elaborated them first with the HistCite software (Garfield et al., 1964; Garfield, 2009) to obtain the corresponding historiograph, secondly we analysed the data applying an algorithm widely used in the analysis of citation network, the Critical Path Method (CPM) (Kejžar et al., 2010). The historiograph displays how each paper has influenced other papers included in the panel provided by ISI (Garfield, 2003) and allows the chance to understand the role of key events (papers), people (authors) and journals in a field. This historiograph analysis is focused on the most influential contributions to the body of research on the OWA operators. Differently the CPM aims to trace the dominant direction of knowledge
accumulation. To identify the papers dealing with the OWA we first used the keyword “ordered weighted averaging” and obtained 537¹ results that include published academic paper (394) and proceedings (143).

The OWA operators: Background

The formulation of OWA, as proposed originally by Yager (1988), refers to the issue of aggregating criteria functions to form an overall decision function.

Definition: A mapping $F$ from $I^n \rightarrow I$ (where $I = [0,1]$)

is called an OWA operator of dimension $n$ if associate with $F$, is a weighting vector $W$,

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix}$$

such that

$$W_1 \in (0,1)$$

$$\sum_i W_i = 1$$

$$F(a_1, a_2, ..., a_n) = W_1 b_1 + W_2 b_2 + ... + W_n b_n,$$

where $b_1$ is the largest element in the collection $a_1, a_2, ..., a_n$. And an $n$ vector $B$ can be the ordered argument vector if each element $b_i \in [0,1]$ and $b_i \geq b_j$ if $j > i$. Given and OWA operator $F$ with weight vector $W$ and an argument tuple $(a_1, a_2, ..., a_n)$ we can associate with this tuple an ordered input vector $B$ is the vector consisting of the argument of $F$ put in descending order. Using this notation then

$$F(a_1, a_2, ..., a_n) = W'B,$$

¹ The full list of papers can be found in a supplementary document provided online.
the inner product of $W'$ and $B$. It is also possible to denote $F(a_1, a_2, \ldots, a_n)$ as $F(B)$ where $B$ is the highest associated ordered argument vector.

Furthermore, Yager (1988) points to the fact that the weights, the $W$'s, are associated with particular ordered position rather than a particular element, that is, $W_i$ is the weight associated with the i-th largest vector $B$ and any OWA operator $F$ with weighting vector $W$ that $0 \leq F(B) \leq 1$.

**Methodology**

The study of papers citation network by means of SNA has become very popular in the last few years as it allows to understand different dynamics such as collaboration among researchers (De Stefano et al., 2011; Lee et al., 2014); knowledge patterns and (Calero-Medina and Noyons, 2008) and emerging knowledge trends within disciplines (Ding et al., 2013; Liu et al., 2013). Two major contributions characterised this growing methodological approach, the pioneering study by Garfield et al. (1964) and the development of three algorithms proposed by Hummon and Doreian (Hummon and Doreian, 1989). The former seeks to shed light on the chronological representation of the development of a discipline focusing on the most cited authors and works to infer about their impact on the discipline's progress; while the latter shifts the attention from nodes to links allowing the so-called connectivity analysis.

More specifically, Hummon and Doreian (1989) algorithms, Search Path Link Count (SPLC), Search Path Node Pairs (SPNP) and Node Projection Pairs Count (NPPC), capture the level of connectivity of each citation (a link between two nodes) and are based on sequences of links and nodes called “search path”. Recently, Batagelj (2003) elaborated the Search Path Count (SPC) algorithm, which is considered the best development of Hummon and Doreian algorithms and overcome some limitations. In this works citations are considered proxies for knowledge flows, thus if the author 'A' cites author’s 'B' we assume there is a knowledge flows between them, more precisely, 'A' work relies to some extent on 'B' contribution (Figure 1).
Figure 1. Example of citation network

In this study we combine the two citations-based methodologies, to investigate the OWA literature. As outputs we provide first the historiograph (Garfield et al., 1964) of the related discipline to study the chronological development of the discipline, then we apply the Critical Path Method (CPM) which is based on the SPC that calculates traversal weights on arcs, and finally we analyse co-citation network of most cited publications to highlight similarities between these works. Traversal weights measure the importance of path linking entry vertices in a network to exit ones. Entry vertices are those not cited within the dataset, while exit vertices do not cite others within the dataset. The CPM algorithm determines the path from entry vertices to exit vertices with the largest total sum of weights on the arcs and provides a visual display of a broader longitudinal connectivity then the SPC output. We apply it to map the knowledge underlying the evolution of the main direction the field. We consider this as the backbone of the discipline.

The analysis of the historiography was first introduced by Garfield et al. (1964), which described the historiography as a chronological map allowing the historical reconstruction of scientific development of a field and its chronological representation. Typically it shows only a portion of the most cited works within the field. Thus, it is a genealogical approach to the study of a discipline, showing when it starts and what its descendants are. We choose to provide the historiograph of the OWA field (Figure 3) as output as this paper is the first review of the scientific development of this discipline.

CPM captures the dominant direction of knowledge accumulation that emerged over the whole time period covered by this analysis, namely the backbone of the field of interest. By
computing the total number of paths linking the oldest vertices in a citation network to the most recent ones, the algorithm maps all possible streams of cumulative growth of knowledge, and selects the most important one. CPM determines the source-sink path(s) with the largest total sum of weights and identifies the path from entry vertices to exit vertices with the largest total sum of weights on the arcs. We conduct the CPM to highlight the intellectual structure underpinning the scientific development of the field of interest and complement this analysis with insights from a co-citation perspective. In fact the analysis of references of published articles allows to highlight if any two references are commonly co-cited, that is referenced together. Is a set of references is commonly co-cited, it can be argued that they constitute the intellectual structure of the field (Leydesdorff and Vaughan, 2006). Data have been analysed with two major software: HistCite and Pajek.

**Data and basic statistics**

We adopt ISI Web of Science (WoS) as the data source of this study. OWA papers have been searched and retrieved through the use of the keyword “ordered weighted averaging”. We first obtained 540 results, 3 of these were not imported as they do not belong to the ‘Core Collection within ISI Web Science, thus the procedure ended up with 537 results, 674 authors and 249 journals. A main issue to handle when searching for OWA papers is the right ‘search key’, we opt to use “ordered weighted averaging” instead of the abbreviation OWA to avoid potential misunderstanding. As first goal we identify the major publications and authors in the OWA field. The growing attention received by this topic is shown in Figure 2, which depicts the trend of publications since Yager’s first OWA paper in 1988.

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3 The data to be analysed with HistCite software should belong to the Web of Science Core Collection. The 3 results not included are papers published in the Journal of Environmental Systems by Smith, P. N.
We ranked authors and journals using the Total Local Score (TLC), which refers to how many times the author's papers included in this collection have been cited by other papers also in the collection and the Total Global Citation Score (TGCS), which refers to how many times the author's papers included in this collection have been cited. This score is calculated from the Times Cited score retrieved from the Web of Science. Thus, considering TGCS means accounting also for the influence that authors' publication has outside the discipline's borders. However it is based only on the materials included in the ISI WoS database, which constitute the main limitation of this kind of study.

The first visual representation of our analysis is the historiograph depicted in Figure 3 that provides a citation-based graphical representation of how core papers have influenced one another. The figure depicts only the top 30 most cited papers as shown in Table 1. The decision to set a threshold of 30 papers is arbitrary, however is usually suggested as reasonable to get first information about most influential works. A key indicator of influence is relative circle size, which reflects the extent of an article’s influence over the development of the core body of knowledge concerning the OWA research domain. As expected the Yager
(1988) paper shows the biggest shape as it is recognised as the starting and most influential contribution.

Table 1. First 30 most cited papers ranked according to Global Citation Score (GCS)

<table>
<thead>
<tr>
<th>ID</th>
<th>Authors</th>
<th>LCS</th>
<th>GCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yager RR, 1988, IEEE Transactions on Systems</td>
<td>451</td>
<td>2029</td>
</tr>
<tr>
<td>8</td>
<td>Filev D, 1995, Information Sciences</td>
<td>50</td>
<td>92</td>
</tr>
<tr>
<td>9</td>
<td>Herrera F, 1995, Information Sciences</td>
<td>36</td>
<td>213</td>
</tr>
<tr>
<td>17</td>
<td>Herrera F, 1996, Fuzzy Sets and Systems</td>
<td>43</td>
<td>264</td>
</tr>
<tr>
<td>30</td>
<td>Yager RR, 1999, IEEE Transactions on Systems</td>
<td>130</td>
<td>284</td>
</tr>
<tr>
<td>46</td>
<td>Fuller R, 2001, Fuzzy Sets and Systems</td>
<td>78</td>
<td>127</td>
</tr>
<tr>
<td>49</td>
<td>Xu ZS, 2002, International Journal of Intelligent Systems</td>
<td>55</td>
<td>144</td>
</tr>
<tr>
<td>50</td>
<td>Xu ZS, 2002, International Journal of Intelligent Systems</td>
<td>33</td>
<td>95</td>
</tr>
<tr>
<td>45</td>
<td>Fuller R, 2003, Fuzzy Sets and Systems</td>
<td>65</td>
<td>106</td>
</tr>
<tr>
<td>60</td>
<td>Yager RR, 2003, Fuzzy Sets and Systems</td>
<td>56</td>
<td>117</td>
</tr>
<tr>
<td>86</td>
<td>Wang YM, 2005, Information Sciences</td>
<td>48</td>
<td>73</td>
</tr>
<tr>
<td>110</td>
<td>Xu ZS, 2006, Information Fusion</td>
<td>28</td>
<td>112</td>
</tr>
<tr>
<td>162</td>
<td>Xu ZS, 2007, IEEE Transactions on Fuzzy Systems</td>
<td>34</td>
<td>219</td>
</tr>
<tr>
<td>251</td>
<td>Merigó JM, 2009, Information Sciences</td>
<td>75</td>
<td>144</td>
</tr>
<tr>
<td>254</td>
<td>Wu J, 2009, Computers &amp; Industrial Engineering</td>
<td>26</td>
<td>49</td>
</tr>
<tr>
<td>259</td>
<td>Merigó JM, 2009, International Journal of Intelligent Systems</td>
<td>44</td>
<td>75</td>
</tr>
<tr>
<td>285</td>
<td>Merigó JM, 2010, Cybernetics and Systems</td>
<td>27</td>
<td>47</td>
</tr>
<tr>
<td>317</td>
<td>Merigó JM, 2010, Computers &amp; Industrial Engineering</td>
<td>41</td>
<td>69</td>
</tr>
<tr>
<td>324</td>
<td>Merigó JM, 2010, Information Sciences</td>
<td>50</td>
<td>87</td>
</tr>
<tr>
<td>360</td>
<td>Merigó JM, 2011, Computers &amp; Industrial Engineering</td>
<td>28</td>
<td>53</td>
</tr>
</tbody>
</table>
Figure 3. The historiograph showing the top 30 most cited OWA papers
**Researcher statistics**

The 20 most cited authors have been ranked in Table 2 according to the number of total citation score excluding self-citations, which are less indicative of influence on others. As expected Yager is the most cited author, followed by Xu, Merigó and Filev in the top 5 positions.

Table 2. Most cited authors ranked by TLCSx (Total citations score excluding self-citation).

<table>
<thead>
<tr>
<th>Authors</th>
<th>Number of record</th>
<th>TLCS</th>
<th>TLCS/t</th>
<th>TLCSx</th>
<th>TGCS</th>
<th>TGCS/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yager RR</td>
<td>40</td>
<td>1078</td>
<td>57.21</td>
<td>995</td>
<td>3669</td>
<td>185.11</td>
</tr>
<tr>
<td>Xu ZS</td>
<td>27</td>
<td>431</td>
<td>48.85</td>
<td>364</td>
<td>1572</td>
<td>180.39</td>
</tr>
<tr>
<td>Merigó JM</td>
<td>62</td>
<td>524</td>
<td>112.27</td>
<td>191</td>
<td>958</td>
<td>206.92</td>
</tr>
<tr>
<td>Filev DP</td>
<td>3</td>
<td>193</td>
<td>11.24</td>
<td>180</td>
<td>412</td>
<td>24.06</td>
</tr>
<tr>
<td>Herrera F</td>
<td>10</td>
<td>191</td>
<td>14.68</td>
<td>161</td>
<td>1110</td>
<td>88.39</td>
</tr>
<tr>
<td>Herrera-Viedma E</td>
<td>14</td>
<td>191</td>
<td>14.68</td>
<td>161</td>
<td>1246</td>
<td>103.17</td>
</tr>
<tr>
<td>Majlender P</td>
<td>3</td>
<td>166</td>
<td>13.29</td>
<td>161</td>
<td>266</td>
<td>21.20</td>
</tr>
<tr>
<td>Fuller R</td>
<td>3</td>
<td>145</td>
<td>11.24</td>
<td>140</td>
<td>240</td>
<td>18.78</td>
</tr>
<tr>
<td>Da QL</td>
<td>5</td>
<td>163</td>
<td>14.11</td>
<td>129</td>
<td>428</td>
<td>36.27</td>
</tr>
<tr>
<td>Casanovas M</td>
<td>22</td>
<td>245</td>
<td>51.65</td>
<td>89</td>
<td>441</td>
<td>93.18</td>
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<tr>
<td>Verdegay JL</td>
<td>4</td>
<td>99</td>
<td>5.20</td>
<td>88</td>
<td>618</td>
<td>32.39</td>
</tr>
<tr>
<td>Ahn BS</td>
<td>14</td>
<td>116</td>
<td>16.94</td>
<td>84</td>
<td>162</td>
<td>23.60</td>
</tr>
<tr>
<td>Wang YM</td>
<td>4</td>
<td>85</td>
<td>9.68</td>
<td>80</td>
<td>145</td>
<td>17.93</td>
</tr>
<tr>
<td>Chiclana F</td>
<td>11</td>
<td>99</td>
<td>11.08</td>
<td>79</td>
<td>522</td>
<td>63.48</td>
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<td>Gil-Lafuente AM</td>
<td>18</td>
<td>183</td>
<td>37.83</td>
<td>73</td>
<td>338</td>
<td>70.73</td>
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<td>Liu XW</td>
<td>17</td>
<td>88</td>
<td>12.27</td>
<td>64</td>
<td>182</td>
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<td>Alonso S</td>
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<td>7.11</td>
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<td>432</td>
<td>51.65</td>
</tr>
<tr>
<td>Filev D</td>
<td>1</td>
<td>50</td>
<td>2.50</td>
<td>49</td>
<td>92</td>
<td>4.60</td>
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<td>Emrouznejad A</td>
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<td>55</td>
<td>8.55</td>
<td>46</td>
<td>91</td>
<td>14.42</td>
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<tr>
<td>Malczewski J</td>
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<td>52</td>
<td>5.92</td>
<td>46</td>
<td>156</td>
<td>19.30</td>
</tr>
</tbody>
</table>

**Journal statistics**

Table 3 shows the top 20 most active journals that publish OWA papers. The top five journals in this area are International Journal of Intelligent Systems, Information Science, Fuzzy Sets, Systems and Expert Systems with Applications and Computers & Industrial Engineering. Journals are ranked considering the TGCS and considering time (TGCS/t).

Table 3. Top 20 most influential journals in the OWA field ranked according to their TLCS/t

<table>
<thead>
<tr>
<th>Journals</th>
<th>Number of record</th>
<th>TLCS</th>
<th>TLCS/t</th>
<th>TGCS</th>
<th>TGCS/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>International Journal of Intelligent Systems</td>
<td>42</td>
<td>429</td>
<td>51.27</td>
<td>1086</td>
<td>124.59</td>
</tr>
<tr>
<td>Information Sciences</td>
<td>19</td>
<td>371</td>
<td>46.31</td>
<td>1105</td>
<td>120.16</td>
</tr>
<tr>
<td></td>
<td>Journal</td>
<td>Volume</td>
<td>Pages</td>
<td>Impact Factor</td>
<td>Total Cites</td>
</tr>
<tr>
<td>---</td>
<td>---------------------------------------------</td>
<td>--------</td>
<td>-------</td>
<td>---------------</td>
<td>-------------</td>
</tr>
<tr>
<td>3</td>
<td>Fuzzy Sets and Systems</td>
<td>20</td>
<td>517</td>
<td>37.77</td>
<td>1476</td>
</tr>
<tr>
<td>4</td>
<td>Expert Systems with Applications</td>
<td>27</td>
<td>157</td>
<td>37.73</td>
<td>366</td>
</tr>
<tr>
<td>5</td>
<td>Computers &amp; Industrial Engineering</td>
<td>12</td>
<td>167</td>
<td>32.06</td>
<td>308</td>
</tr>
<tr>
<td>6</td>
<td>IEEE Transactions on Systems Man and Cybernetics</td>
<td>1</td>
<td>451</td>
<td>16.70</td>
<td>2029</td>
</tr>
<tr>
<td>7</td>
<td>IEEE Transactions on Fuzzy Systems</td>
<td>18</td>
<td>154</td>
<td>16.06</td>
<td>669</td>
</tr>
<tr>
<td>8</td>
<td>International Journal on Fuzzy Systems</td>
<td>6</td>
<td>66</td>
<td>13.42</td>
<td>136</td>
</tr>
<tr>
<td>9</td>
<td>IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics</td>
<td>9</td>
<td>157</td>
<td>12.22</td>
<td>488</td>
</tr>
<tr>
<td>10</td>
<td>International Journal of Approximate Reasoning</td>
<td>10</td>
<td>88</td>
<td>11.59</td>
<td>234</td>
</tr>
<tr>
<td>11</td>
<td>European Journal of Operational Research</td>
<td>12</td>
<td>96</td>
<td>11.05</td>
<td>370</td>
</tr>
<tr>
<td>12</td>
<td>International Journal of Uncertainty Fuzziness and Knowledge-based Systems</td>
<td>17</td>
<td>97</td>
<td>10.54</td>
<td>262</td>
</tr>
<tr>
<td>13</td>
<td>Group Decision and Negotiation</td>
<td>8</td>
<td>32</td>
<td>7.19</td>
<td>185</td>
</tr>
<tr>
<td>14</td>
<td>International Journal of Computational Intelligence Systems</td>
<td>3</td>
<td>33</td>
<td>7.15</td>
<td>80</td>
</tr>
<tr>
<td>15</td>
<td>Journal of Systems Engineering and Electronics</td>
<td>7</td>
<td>35</td>
<td>6.75</td>
<td>61</td>
</tr>
<tr>
<td>16</td>
<td>Information Fusion</td>
<td>5</td>
<td>48</td>
<td>6.68</td>
<td>162</td>
</tr>
<tr>
<td>17</td>
<td>Cybernetics and Systems</td>
<td>4</td>
<td>35</td>
<td>6.33</td>
<td>65</td>
</tr>
<tr>
<td>18</td>
<td>International Journal of General Systems</td>
<td>9</td>
<td>87</td>
<td>5.93</td>
<td>203</td>
</tr>
<tr>
<td>19</td>
<td>Knowledge-based Systems</td>
<td>10</td>
<td>22</td>
<td>4.96</td>
<td>71</td>
</tr>
<tr>
<td>20</td>
<td>Economic Computation and Economic Cybernetics Studies and Research</td>
<td>4</td>
<td>15</td>
<td>4.25</td>
<td>30</td>
</tr>
</tbody>
</table>

**OWA knowledge accumulation using Critical Path Method**

Figure 4 shows the result of CPM, which captures the evolution and direction of knowledge accumulation. The graph aims at showing the sequence of knowledge contributions and differently from the historiograph here we do not have differences in shapes dimension to mean a different influence played by one on another. Here the emphasis is on the evolution of the discipline and its direction.
Figure 4. Critical Path Method of OWA development

After examining the title, abstract, and keywords\textsuperscript{4} of these papers (Table 4) we describe the development of this discipline and its major areas of research. The analysis of the content reveals the efforts of researchers focused on two major directions.

Table 4. Papers on the CPM

<table>
<thead>
<tr>
<th>Id</th>
<th>Authors</th>
<th>Title</th>
<th>Journal</th>
<th>Keywords</th>
<th>Year published</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yager, R.R.</td>
<td>On ordered weighted averaging operators in multicriteria decision-making</td>
<td>IEEE Transactions on Systems Man and Cybernetics</td>
<td>Ordered weighted averaging operators, decision making</td>
<td>1988</td>
</tr>
<tr>
<td>4</td>
<td>Yager, R. R.</td>
<td>Families of OWA operators</td>
<td>Fuzzy Sets and Systems</td>
<td>Aggregation; fuzzy sets; averaging operators; linguistic quantifiers; logical operators</td>
<td>1993</td>
</tr>
<tr>
<td>5</td>
<td>Yager, R. R.; Filev, D R.</td>
<td>Parameterized and-like and or-like OWA operators</td>
<td>International Journal of General Systems</td>
<td>Aggregation operators; decision making; averaging operators; fuzzy set theory; fuzzy logic</td>
<td>1994</td>
</tr>
</tbody>
</table>

\textsuperscript{4} Some journals such as International Journal of Intelligent Systems and IEEE Transactions on Systems Man and Cybernetics, do not provide keywords. In these cases we propose keywords as recurrent words along the papers and use Italic font to highlight them.
<table>
<thead>
<tr>
<th>ID</th>
<th>Author(s)</th>
<th>Title</th>
<th>Journal</th>
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<tr>
<td>6</td>
<td>Yager, R.R.</td>
<td>Measures of entropy and fuzziness related to aggregation operators</td>
<td>Information Sciences</td>
<td>Entropy measures</td>
<td>1995</td>
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<td>18</td>
<td>Yager, R. R.</td>
<td>Constrained OWA aggregation</td>
<td>Fuzzy Sets and Systems</td>
<td>Fuzzy mathematical programming; linguistic quantifiers; constrained optimization; OWA operators</td>
<td>1996</td>
</tr>
<tr>
<td>21</td>
<td>Yager, R. R.</td>
<td>On the analytic representation of the Leximin ordering and its application to flexible constraint propagation</td>
<td>European Journal of Operational Research</td>
<td>Aggregation; constraint propagation; fuzzy sets; OWA operators; Leximin; mathematical programming</td>
<td>1997</td>
</tr>
<tr>
<td>35</td>
<td>Mitchell, H B.; Schaefer, P. A.</td>
<td>Multiple priorities in an induced ordered weighted averaging operator</td>
<td>International Journal of Intelligent Systems</td>
<td>Multiple fuzzy priorities</td>
<td>2000</td>
</tr>
<tr>
<td>49</td>
<td>Xu, Z.S.; Da, Q. L.</td>
<td>The uncertain OWA operator</td>
<td>International Journal of Intelligent Systems</td>
<td>Internal numbers; uncertain OWA operator</td>
<td>2002</td>
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<td>50</td>
<td>Xu, Z.S.; Da, Q. L.</td>
<td>The ordered weighted geometric averaging operators</td>
<td>International Journal of Intelligent Systems</td>
<td>Ordered weighted geometric averaging operators</td>
<td>2002</td>
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<td>57</td>
<td>Herrera, F.; Herrera-Viedma, E.; Chiclana, F.</td>
<td>A study of the origin and uses of the ordered weighted Geometric operator in multicriteria decision making</td>
<td>International Journal of Intelligent Systems</td>
<td>Ordered weighted geometric operator; multicriteria decision making</td>
<td>2003</td>
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<td>59</td>
<td>Ogryczak, W.; Sliwinski, T.</td>
<td>On solving linear programs with the ordered weighted averaging objective</td>
<td>European Journal of Operational Research</td>
<td>Equity; lexicographic maximin; linear programming; multiple criteria; ordered weighted averaging</td>
<td>2003</td>
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<td>60</td>
<td>Yager, R. R.</td>
<td>Induced aggregation operators</td>
<td>Fuzzy Sets and Systems</td>
<td>IOWA operator; OWA aggregation operators; best yesterday models</td>
<td>2003</td>
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<td>61</td>
<td>Xu, Z. S.; Da, Q. L.</td>
<td>An overview of operators for aggregating information</td>
<td>International Journal of Intelligent Systems</td>
<td>Survey; aggregation operators</td>
<td>2003</td>
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<td>68</td>
<td>Liu, X. W.; Chen, L. H.</td>
<td>On the properties of parametric geometric OWA operator</td>
<td>International Journal of Approximate Reasoning</td>
<td>OWA operator; geometric OWA operator; maximum entropy OWA operator</td>
<td>2004</td>
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<td>76</td>
<td>Xu, Z. S.</td>
<td>EOWA and EOWG</td>
<td>International Journal</td>
<td>Group decision</td>
<td>2004</td>
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</table>
operators for aggregating linguistic labels based on linguistic preference relations of Uncertainty Fuzziness and Knowledge-based Systems making; multiplicative linguistic preference relations; additive linguistic preference relations; extended ordered weighted averaging (EOWA) operator

<p>| 77 | Xu, Z. S. | Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment | Information Sciences | Aggregation; multiple attribute group decision making; uncertain linguistic ordered weighted averaging (ULOWA) operator; uncertain linguistic hybrid aggregation (ULHA) operator |
| 86 | Wang, Y. M.; Parkan, C. | A minimax disparity approach for obtaining OWA operator weights | Information Sciences | OWA operator; Operator weights; Degree of orness; Minimax |
| 10 | Xu, Z. S. | On generalized induced linguistic aggregation operators | International Journal of General Systems | Generalized induced linguistic aggregation operators, linguistic variable, uncertain linguistic variable, operational laws |
| 11 | Amin, G. R., Emrouznejad, A. | An extended minimax disparity to determine the OWA operator weights | Computers &amp; Industrial Engineering | OWA operator weights; duality of linear programming |
| 15 | Wang, Y. M.; Luo, Y.; Hua, Z. | Aggregating preference rankings using OWA operator weights | Information Sciences | Preference ranking; preference aggregation; OWA operator weights; orness degree |
| 15 | Llamazares, B. | Choosing OWA operator weights in the field of Social Choice | Information Sciences | Ordered weighted averaging operators; aggregation operator weights; majority rules |
| 16 | Xu, S. Z. | Intuitionistic fuzzy aggregation operators | IEEE Transactions on Fuzzy Systems | Intuitionistic fuzzy hybrid aggregation, intuitionistic fuzzy ordered weighted averaging (IFOWA) |
| 25 | Merigó, J. M.; Gil-Lafuente, A. M. | The induced generalized OWA operator | Information Sciences | Aggregation operators; OWA operators; generalized mean; quasi-arithmetic mean; decision-making |</p>
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<td>4</td>
<td>Merigó, J. M.; Casanovas, M.</td>
<td>The fuzzy generalized OWA operator and its application in strategic decision making</td>
<td>Cybernetics and Systems</td>
<td>Aggregation operators; decision making; fuzzy OWA operator; selection of strategies</td>
<td>2010</td>
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<td>0</td>
<td>Zhao, H.; Xu, Z.; Ni, M.; Liu, S.</td>
<td>Generalized aggregation operators for intuitionistic fuzzy sets</td>
<td>International Journal of Intelligent Systems</td>
<td>Generalized intuitionistic fuzzy weighted averaging operator</td>
<td>2010</td>
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<td>8</td>
<td>Merigó, J. M.; Casanovas, M.</td>
<td>Fuzzy generalized hybrid aggregation operators and its application in fuzzy decision making</td>
<td>International Journal of Fuzzy Systems</td>
<td>Aggregation operators; fuzzy numbers; hybrid averaging; OWA operator; decision making</td>
<td>2010</td>
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<td>6</td>
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<td>Fuzzy decision making with immediate probabilities</td>
<td>Computers &amp; Industrial Engineering</td>
<td>Decision-making; immediate probabilities; OWA operator; fuzzy numbers; strategic selection</td>
<td>2010</td>
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<td>1</td>
<td>Merigó, J. M.; Casanovas, M.</td>
<td>Induced and heavy aggregation operators with distance measures</td>
<td>Journal of Systems Engineering and Electronics</td>
<td>It is called the induced heavy ordered weighted averaging (OWA) distance (IHOWAD) operator.</td>
<td>2010</td>
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<td>3</td>
<td>Merigó, J. M.; Gil-Lafuente, A. M.</td>
<td>New decision-making techniques and their application in the selection of financial products</td>
<td>Information Sciences</td>
<td>Decision making; OWA operator; selection of financial products; hamming distance</td>
<td>2010</td>
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<td>7</td>
<td>Merigó, J. M.; Casanovas, M.</td>
<td>Decision making with distance measures and linguistic aggregation operators</td>
<td>International Journal of Fuzzy Systems</td>
<td>Linguistic ordered weighted averaging distance (LOWAD) operator</td>
<td>2010</td>
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<td>9</td>
<td>Merigó, J. M.; Casanovas, M.</td>
<td>Decision-making with distance measures and induced aggregation operators</td>
<td>Computers &amp; Industrial Engineering</td>
<td>Decision-making; OWA operator; distance measures; induced aggregation operators</td>
<td>2011</td>
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<td>9</td>
<td>Merigó, J. M.; Casanovas, M.</td>
<td>Induced aggregation operators in the Euclidean distance and its application in financial decision making</td>
<td>Expert Systems with Applications</td>
<td>Induced aggregation operators; Euclidean distance; decision making; selection of investment</td>
<td>2011</td>
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<td>5</td>
<td>Merigó, J. M.; Gil-Lafuente, A. M.; Gil-Aluja, J.</td>
<td>Soft computing techniques for decision making with induced aggregation operators</td>
<td>Information-An International Journal</td>
<td>Induced aggregation operators; induced ordered weighted averaging; induced ordered weighted averaging adequacy</td>
<td>2011</td>
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<td>Page</td>
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<td>37</td>
<td>Merigó, J. M.; Gil-Lafuente, A. M.</td>
<td>Fuzzy induced generalized aggregation operators and its application in multi-person decision making</td>
<td>Expert Systems with Applications</td>
<td>Aggregation operator; OWA operator; fuzzy numbers; multi-person decision making</td>
<td>2011</td>
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<td>38</td>
<td>Merigó, J. M.</td>
<td>A unified model between the weighted average and the induced OWA operator</td>
<td>Expert Systems with Applications</td>
<td>Weighted average; OWA operator; aggregation operators; multi-person decision making</td>
<td>2011</td>
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<td>38</td>
<td>Merigó, J. M.</td>
<td>Fuzzy multi-person decision making with fuzzy probabilistic aggregation operators</td>
<td>International Journal of Fuzzy Systems</td>
<td>Multi-person decision making; Fuzzy probabilistic OWA</td>
<td>2011</td>
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<td>40</td>
<td>Zeng, S. Z.; Su W.</td>
<td>Linguistic induced generalized aggregation distance operators and their application to decision making</td>
<td>Economic Computation and Economic Cybernetics Studies and Research</td>
<td>Linguistic variables; OWA operator; distance measure; decision making; human resource management</td>
<td>2012</td>
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<tr>
<td>48</td>
<td>Merigó, J. M.; Xu, Y.; Zeng, S.</td>
<td>Group decision making with distance measures and probabilistic information</td>
<td>Knowledge-based Systems</td>
<td>Decision making; selection of policies; probability; Hamming distance; aggregating operators</td>
<td>2013</td>
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<td>50</td>
<td>Zeng, S.; Merigó, J. M.; Su, W.</td>
<td>The uncertain probabilistic OWA distance operator and its application in group decision making</td>
<td>Applied Mathematical Modelling</td>
<td>Probability; OWA operator; distance measures; uncertainty; group decision-making</td>
<td>2013</td>
</tr>
<tr>
<td>52</td>
<td>Su, W.; Li, W.; Zeng, S.</td>
<td>Atanassov’s intuitionistic linguistic ordered weighted averaging distance operator and its application to decision making</td>
<td>Journal of Intelligent &amp; Fuzzy Systems</td>
<td>Distance measures, OWA operator, Atanassov’s intuitionistic linguistic variables, multi-person decision making</td>
<td>2014</td>
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</table>

The first works by Yager (1988, 1993, 1995b) and Yager and Filev (1994) constitute the knowledge base over which future works developed and further applied the OWA method. They lay out the foundation of this research topic. Yager (1988) deals with the problem of aggregating multiple criteria to form an overall decision function and introduces the ‘orness’,
which refers to the ‘and-like’ or ‘or-like’ aggregation result of an OWA operator. Thus the operator lies between two extremes, 1 (‘and-like’) and 0 (‘or-like’), the former relates to the situation in which all criteria are satisfied. Differently the latter refers to the situation in which at least one of the criteria has to be satisfied. The eleven values between 0 and 1 depend on the decision maker expertise and are suppose to reflect his degree of optimism. The ‘orness’ concept itself received great attention and further specification (Marichal, 1999; Fernández Salido and Murakami, 2003; Yager, 2004). Two lines of research depart from this knowledge base, mainly dealing with different approaches to obtain the associated weights. On the one hand we identify a branch of literature including a group of works that generalize the OWA operator to include the case of real-number and fuzzy ranks (Mitchell and Estrakh, 1998); use a multiple priority induced OWA operator (Mitchell and Schaefer, 2000); propose new classes of aggregation operators such as the ordered weighted geometric averaging (OWGA) operators (Xu and Da, 2002a) investigate the uncertain OWA operator in which the associated weighting parameters cannot be specified, but value ranges can be obtained and each input argument is given in the form of an interval of numerical values (Xu and Da, 2002b); investigate the ordered weighted geometric (OWG) operator and its relationship to the OWA operator in multi-criteria decision making (MCDM) (Herrera et al., 2003). Within this area we can find two other works of Yager. A paper dealing with fuzzy methods to model nearest neighbour rules (Yager, 2002) and a second one about induced OWA operators (IOWA) (Yager, 2003) that receive further attention in this sub area identified and great development later as we will show. Xu and Da propose the induced ordered weighted geometric averaging (IOWGA) operator (2003) as new aggregator and the generalized induced linguistic aggregation operators (Xu, 2006). Other two papers of Xu and Da extend the OWA proposing the (EOWA) operator and the uncertain linguistic ordered weighted
averaging (ULOWA) operator and the uncertain linguistic hybrid aggregation (ULHA) operator.

The subsequently line focuses on fuzzy aggregation and fuzzy-set theory. Within this group the CPM highlights the following as the most significant contributions. Xu (2007) propose an intuitionistic fuzzy version of the OWA operator (IFOWA); Zhao et al. (2010) paper extends the generalized OWA operators introduce by Yager (2004) to the intuitionistic fuzzy information. Merigó and Casanovas (2010a) present a series of operators, the fuzzy generalized hybrid averaging (FGHA) operator, the fuzzy induced generalized hybrid averaging (FIGHA) operator, the Quasi-FHA operator and the Quasi-FIHA operator, with the advantage of generalize a wide range of fuzzy aggregation operators so that can be used in different applications such as decision making problems.

On the other hand we see Yager (1996) paper on the problem of maximizing an OWA aggregation of a group of variables interrelated and constrained by a collection of linear inequality. In this paper, Yager proposes to model this problem as a linear programming (LP) problem. Subsequently the OWA operator is proposed to as analytic formulation for the Leximin method, overcoming its lack of analytic formulation (Yager, 1997). Following these conceptualizations researchers worked on the linear programming formulations with the OWA objective functions (Ogryczak and Śliwiński, 2003; Liu and Chen, 2004; Wang and Parkan, 2005; Amin and Emrouznejad, 2006). However there are differences among various approaches using the linear programming. According to Ogryczak and Śliwiński (2003) the LP problem with the OWA objective can be performed as a standard linear problem and two alternative LP formulations are introduced the max-min and the deviation model. Liu and Chen (2004) propose the concept of parametric geometric OWA operator (PGOWA) and parametric maximum entropy OWA operator (PMEOWA) showing the consistence of the orness level and the aggregation value for an aggregated elements with PGOWA. The
equivalence between PGOWA and PMEOWA is also proven. Wang and Parkan (2005) paper represents the first attempt to propose the minimax disparity approach as a method to identify OWA operator weights using LP under a give level of ‘orness’. According to this approach OWA operator weights have been determined by minimizing the maximum difference between two adjacent weights, under a give level of ‘orness’. Within this line of research, Amin and Emrouznejad (2006) extend the minimax disparity to determine the OWA model based on LP and introduce the minimax disparity approach between any distinct pairs of the weights. Drawing on this works, the sub area that we find between 2007 and 2009 (Llamazares, 2007; Wang et al., 2007; Merigó and Gil-Lafuente, 2009) make a step further in this direction developing models that slightly different the previous ones. More specifically, Wang et al. (2007) paper deals with the determination of the weights of different ranking places. Their model allows the weights associated with different ranking places to be determined in terms of a decision maker (DM)’s optimism level, which is characterized by an orness degree. Llamazares (2007) aims to determine the OWA operator weights that allow to extend, through the OWA operator, some classes of majority rules obtained when individuals do not grade their preferences between two alternatives. Subsequently we find Merigó and Gil-Lafuente (2009), which can be seen as a bridge between the previous areas of research. This new area relies on both lines of previous research and comprises works mainly dealing with induced and fuzzy OWA operators. Merigó and Gil-Lafuente (2009) and Merigó and Casanovas (2010b) build on the previous line of research to introduce the induced generalized ordered weighted averaging (IGOWA) operator. It is a new aggregation operator that generalizes the OWA operator, including the main characteristics of both the generalized OWA and the induced OWA operator. They propose the application of the IGOWA in a financial decision-making problem. Merigó (2010) develop a decision-making model with probabilistic information and use the concept of the immediate probability to aggregate the
information and applies it to the selection of strategies. Merigó and Gil-Lafuente (2010) introduce the ordered weighted averaging distance (OWAD) operator and the ordered weighted averaging adequacy coefficient (OWAAC) operator to the selection of financial products. This line of research has been further exploited by Merigó and hi co-authors that successfully applied the proposed models to other disciplinary context, such as strategic and business decision making (Merigó and Casanovas, 2010c, 2011a). Within this line of research they develop also decision-making model with distance measures by using linguistic aggregation operators. In doing so they propose linguistic ordered weighted averaging distance (LOWAD) operator and apply it to support decision makers in human resource management (Merigó and Casanovas, 2010d). Subsequently they further developed a OWA model based using distance measures and induced aggregation operators (Merigó and Casanovas, 2011b). This model provides a parameterized family of distance aggregation operators between the maximum and the minimum distance based on a complex reordering process that reflects the complex attitudinal character of the decision-maker. The fuzzy induced generalized aggregation operators (FIGOWA) has been also proposed in strategic multi-person decision making (Merigó and Gil-Lafuente, 2011). Merigó also work on a model that uses the weighted average (WA) and the induced ordered weighted averaging (IOWA) operator in the same formulation and apply it in multi-person decision-making in political management (Merigó, 2011).

The 50 most frequently co-cited publications have been listed in table 5. Yager’s first OWA paper is the most frequently co-cited with other references. It is often co-cited with his other papers (Yager, 1993; 1996; 1999) and with the following publications, Filev and Yager (1998); Xu and Da (2003); Xu (2005); Merigo and Gil-Lafuente (2009).

Table 5. Most frequent reference citation and associated highest co-citations

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5 In this table we use only first author’s name to indicate the publication.
<table>
<thead>
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<th>Publication</th>
<th>Co-cit value</th>
<th>Publication most co-cited with</th>
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</table>
Figure 5 helps in understanding the intensity of such co-citation frequency. Old papers appear on the left while the newer ones are located to the right. The right side shows a higher degree of concentration and a higher number of ties. This informs about the most co-cited publications, while the biggest shape indicates the highest number of total citations received.

In fact Yager (1988) is the most cited, but also the most co-cited.

![Figure 5. Map of most co-cited publications](image-url)
Conclusion

This study investigates the dominant direction within the OWA literature. As it is the first systematic review of this topic, we focus on the dominant direction instead of describing the several areas of applications of the OWA. Despite this, we have been also able to identify within the dominant direction some sub areas of research that are strongly represented within the OWA CPM result and for this reason we expect to be further exploited by researchers in the future development of the discipline.

First we show the historiograph to provide a descriptive and chronological reconstruction of publications dealing with this topic. The second step of the analysis consists with the description of the CPM results that give a more fine-grained picture of the evolution of studies using the OWA operators, allowing us to suggest future line of research.

Major efforts have been dedicated by scholars to determining the OWA operator weights. While over the first 22 years two clear lines of research emerged and have been developed by different authors, the last 4 years, as mapped by the CPM algorithm, do not show a clear path of research but remark the previous two. Furthermore the most recent applications of OWA operators are in different disciplines, from financial to strategic decision-making and human resource management (Merigó and Gil-Lafuente, 2010; Merigó and Casanovas, 2011a; Zeng et al, 2013).

The OWA research is growing in different fields ranging from computer science to operational research to and economics. A great part of the literature deals with different approaches proposed to obtain the associated weights.

It is worth noting that scholars active in this area of research belong mainly to two main disciplinary areas, operational research and computer science on the one hand and economics on the other.
The analysis of core papers along the intellectual trajectory of the OWA field shows that among the most active journals, two published the most important papers in terms of core knowledge contributors, International Journal of Intelligent Systems and Information Science.

Supplement
List of all 537 references are available in the online supplement document.

References


Supplement document


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196. Li, X., et al. 2010. A METHOD FOR MULTIPLE ATTRIBUTE GROUP DECISION MAKING BASED ON LINGUISTIC-VALUED AGGREGATION OPERATORS.


262. Merigo, J. M. M. Casanovas 2008. FUZZY INDUCED AGGREGATION OPERATORS IN DECISION MAKING WITH DEMPSTER-SHAFER BELIEF STRUCTURE.


266. Merigo, J. M. M. Casanovas 2010. DEALING WITH UNCERTAIN INFORMATION IN THE INDUCED PROBABILISTIC OWA OPERATOR.


272. Merigo, J. M. M. Casanovas 2010. USING DISTANCE MEASURES IN HEAVY AGGREGATION OPERATORS.


284. Merigo, J. M. 2010. INDUCED GENERALIZED AGGREGATION OPERATORS IN THE WEIGHTED AVERAGE.


411. Wei, G. H. Yao 2006. Extended IOWA operator and its application to group decision making with linguistic information.


423. Wu, Y.-h., et al. 2009. A New Method for Fish disease Diagnosis System Based on Rough Set and Classifier Fusion.


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262. Merigo, J. M. M. Casanovas 2008. FUZZY INDUCED AGGREGATION OPERATORS IN DECISION MAKING WITH DEMPSTER-SHAFER BELIEF STRUCTURE.

263. Merigo, J. M. M. Casanovas 2008. THE GENERALIZED HYBRID AVERAGING OPERATOR AND ITS APPLICATION IN FINANCIAL DECISION MAKING.


266. Merigo, J. M. M. Casanovas 2010. DEALING WITH UNCERTAIN INFORMATION IN THE INDUCED PROBABILISTIC OWA OPERATOR.


272. Merigo, J. M. M. Casanovas 2010. USING DISTANCE MEASURES IN HEAVY AGGREGATION OPERATORS.


284. Merigo, J. M. 2010. INDUCED GENERALIZED AGGREGATION OPERATORS IN THE WEIGHTED AVERAGE.


411. Wei, G. H. Yao 2006. Extended IOWA operator and its application to group decision making with linguistic information.
423. Wu, Y.-h., et al. 2009. A New Method for Fish disease Diagnosis System Based on Rough Set and Classifier Fusion.


