Modulation instability in high power laser amplifiers

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Abstract: The modulation instability (MI) is one of the main factors responsible for the degradation of beam quality in high-power laser systems. The so-called B-integral restriction is commonly used as the criteria for MI control in passive optics devices. For amplifiers the adiabatic model, assuming locally the Bespalov-Talanov expression for MI growth, is commonly used to estimate the destructive impact of the instability. We present here the exact solution of MI development in amplifiers. We determine the parameters which control the effect of MI in amplifiers and calculate the MI growth rate as a function of those parameters. The safety range of operational parameters is presented. The results of the exact calculations are compared with the adiabatic model, and the range of validity of the latest is determined. We demonstrate that for practical situations the adiabatic approximation noticeably overestimates MI. The additional margin of laser system design is quantified.

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1. Introduction

When the power of a laser beam propagating in nonlinear medium exceeds a critical value P_c, the transverse beam modulations (random or induced) begin to grow exponentially. This physical phenomenon is known as light beam modulation or self-focusing instability [1]. As a result of this instability, the beam quality degrades and the beam breaks into filaments with

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power of the order of P_c. Every such filament then experiences self-focusing up to the point at which either the high intensity produces the material breakdown, or the field collapse is arrested before breakdown by some other physical effect, depending on the specific configuration and the medium material.

The laser power in modern high-energy pulse laser systems often greatly exceeds P_c. For an example, in the National Ignition Facility (NIF) [2] the power is *millions* of times higher that the critical power. The beam quality in such advanced laser facility is maintained by keeping the cascaded system elements effectively short enough to prevent the dangerous development of the modulation instability [3]. Spatial filters are inserted into the system to clean the beam and to prevent the growth of the instability. For passive optical elements (lenses), the self-focusing instability can be treated within linear theory [1]. In amplifiers, where the intensity increases exponentially, an especially careful quantitative treatment is required. Direct numerical modeling of beam propagation can be applied, but this requires a great deal of computational time and is not convenient in multi-parametric system design optimization. Quantitative analytical models are very desirable as they can be used for design analysis before full-scale comprehensive modeling. In this paper, we present exact analytical results providing base for design guidance rules in such complex large-scale laser systems.

2. Physical model and basic equations

We start from the mathematical description of the problem. The nonlinear Schrödinger equation (NSE) governs the propagation of a high-power beam through the amplifier medium according to

$$i\frac{\partial\Psi}{\partial z}+\frac{1}{2n_0k_0}\Delta_\perp\Psi+k_0n_2\,|\Psi|^2\,\Psi=i\,\frac{g_0}{2}\Psi. \tag{1}$$

Here k_0 is the propagation vector in vacuum; n_0 and n_2 are the linear and nonlinear refractive indices, respectively; and g_0 is the amplifier gain. After the straightforward transformation $\Psi=\exp[g_0z/2]\times U$, this equation reads

$$i\frac{\partial U}{\partial z}+\frac{1}{2n_0k_0}\Delta_\perp U+k_0n_2(z)\,|U|^2\,U=0. \tag{1a}$$

Here $n_2(z)=n_2(0)\cdot\exp[g_0z]$, and the optical field propagates from $z=0$ to $z=L.$ Note that the problem of instability in an amplifier is mathematically similar to the problem of beam propagation in non-uniform media [4] with exponentially increasing nonlinear refractive index. The solution of Eq. (1a) that we are interested in is a plane wave with a z-dependent phase. We consider here a modulation instability of the continuous wave (CW) having the form: $U=U_0\exp[ik_0\,|U_0|^2\int n_2(z')dz']$. The evolution of small perturbations to the CW is given by:

$$U(z,r)=(U_0+a+ib)\times\exp[ik_0\,|U_0|^2\int n_2(z')dz',$$

$$\begin{aligned}&\frac{\partial a}{\partial z}+\frac{1}{2n_0k_0}\Delta_\perp b=0,\\&-\frac{\partial b}{\partial z}+\frac{1}{2n_0k_0}\Delta_\perp a+2k_0n_2(z)\,|U_0|^2\,a=0.\end{aligned} \tag{2}$$

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These two first-order equations can be combined into the following second-order equation in z:

$$\frac{\partial^2 a}{\partial z^2}+\frac{1}{2n_0k_0}\Delta_\perp\{\frac{1}{2n_0k_0}\Delta_\perp+2k_0n_2(z)|U_0|^2\}a=0.$$

In what follows, for simplicity, we will skip using indices $k_\perp$ indicating that $a(z)$ is the Fourier mode. When $n_2(z)=const$, analysis of the Fourier modes $a \propto \exp[ik_z z+i\vec{k}_\perp\vec{r}_\perp]$ leads to the standard modulation instability relation [1]:

$$k_z^2=\frac{\vec{k}_\perp^2}{2n_0k_0}[\frac{\vec{k}_\perp^2}{2n_0k_0}-2k_0n_2|U_0|^2]. \quad (3)$$

It is seen from (3) that the spatial growth k_z is increased with $k_\perp$ for small values of the latter. It reaches a maximum at

$$\frac{k_\perp^2}{2n_0k_0}=k_0n_2|A|^2 \quad (4)$$

and it turns to zero at the point

$$\frac{k_\perp^2}{2n_0k_0}=2k_0n_2|A_0|^2. \quad (5)$$

The maximal wave-number k_z related to the increment of the modulation instability is given by the expression $\mathrm{Im}(k_z)=-k_0n_2|A_0|^2$. In the case of $n_2(z)=const$ after propagation over a distance L, the initial perturbation increases as

$$a(0),b(0)\exp[k_zL]=a(0),b(0)\exp[k_0n_2|A_0|^2 L]=a(0),b(0)\exp[B].$$

Here B is the so-called B integral – a nonlinear phase shift acquired after propagation through the system. To guarantee the beam quality, laser system designers typically require B to be smaller than 2-3. This means that not only perturbations starting from noise, but even induced modulations due to stray light or coating defects, cannot degrade the beam quality. For example, even a perturbation as large as 1% perturbation will not grow to more than 10%. In the case of a long laser (with B larger than 3), the typical design solution is to insert spatial filters into the system to clean the beam and to reduce effectively the B integral [3]. A knowledge of the speed of growth of small perturbations is crucial for design of high power laser facilities (see e.g. [2]).

3. The modulation instability in amplifiers

From the design consideration, it is important to be able to identify safe zones within the space of operational parameters. An example would be the amplifier length such that instabilities would not develop to a dangerous level. To start with, one can estimate from the introduced above equations the perturbation growth due to modulation instability, using the following simple inequality bounding the mode amplitude increase for propagation in the amplifier medium. The evolution of the modes $a,b \propto \exp[i\vec{k}_\perp\vec{r}_\perp]$ is given by:

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$$\frac{\partial a}{\partial z}=\frac{\vec{k}_\perp^2}{2n_0k_0}b,\ \frac{\partial b}{\partial z}=-\frac{\vec{k}_\perp^2}{2n_0k_0}a+2k_0n_2(z)\,|U_0|^2\,a.$$

It can be shown by straightforward manipulations that the z-dependence of the summed amplitudes satisfies $|a(z)|^2+|b(z)|^2\le\exp[2k_0\,|U_0|^2\int_0^z n_2(z')dz']\times(|a(0)|^2+|b(0)|^2).$

This expression gives an exact upper estimate of the perturbation growth. However, a more accurate analysis is desirable for design purpose. Note that in amplifiers, where the initial intensity increases during propagation as $I=I_0e^{g_0z}$, the most unstable scale of perturbation decreases during propagation due to the exponential increase of intensity, as seen in (5). A more accurate, and at the same time simple and safe, criteria for good beam quality can be introduced based on the assumption that the maximal growth is given by the mode with $k_z=k_0n_2\,|A(z)|^2$. Then the corresponding B-integral for an amplifier of length L is defined as in [3]:

$$B'(L)=k_0n_2(0)\int_0^L|U_0|^2\,\exp[g_0z]dz=k_0n_2(0)\,|U_0|^2\,\frac{\exp[g_0L]-1}{g_0}=q\ \ \frac{\exp[g_0L]-1}{4},\tag{6}$$

$$q=4k_0n_2(0)\,|U_0|^2\,/\,g_0.$$

The typical practical requirement is that B' is smaller then 3. This is a safe range, but it is still too conservative an estimate, because one assumes that the maximal growth corresponds to the same $k_\perp^2$. In reality, during propagation, the higher $k_\perp^2$ becomes most unstable. Therefore, it is likely that values of B' larger than 3 can be tolerable. We would like to point out that the difference in the accurate estimate of the B-integral is not just an academic interest, but it translates into very practical factors such as, for example, an overall cost of the laser system. Therefore, we further elaborate on different approaches to the finding the safe operational zones, before presenting our results.

A more regular approach based on the so-called adiabatic approximation was used in [5, 6]. One can assume that the transversal perturbation grows faster than the intensity in the amplifier. In particular, this is certainly true for sufficiently high intensities. In this case, one can assume that the perturbations growth follows the intensity adiabatically and (3) is applicable with a z-dependent intensity. We have then the overall amplification factor

$$\gamma_a=\int_0^L \operatorname{Im}k_z dz=0.5q\cdot f(\omega,g_0L),\ \ \omega^2=\frac{\vec{k}_\perp^2}{4n_0k_0^2n_2(0)\,|U_0|^2},$$

where the function f is defined as

$$\begin{aligned}
f(\omega,g_0L)&=\int_0^{g_0L}[(e^x-\omega^2)\omega^2]^{1/2}dx:\omega^2<1\\
f(\omega,g_0L)&=\int_{\ln\omega^2}^{g_0L}[(e^x-\omega^2)\omega^2]^{1/2}dx:\omega^2>1
\end{aligned}\tag{7}$$

The second expression takes into account that, with the intensity growth, some of initially stable perturbations become unstable. The integrals are straightforward and yield:

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$$f(\omega, g_0 L) = 2\omega\{(e^{g_0 L} - \omega^2)^{1/2} - \sqrt{1-\omega^2} - \omega \tan^{-1}\frac{\sqrt{e^{g_0 L} - \omega^2}}{\omega} + \omega \tan^{-1}\frac{\sqrt{1-\omega^2}}{\omega}\};\ \omega^2 < 1$$

$$f(\omega, g_0 L) = 2\omega\{(e^{g_0 L} - \omega^2)^{1/2} - \omega \tan^{-1}\frac{\sqrt{e^{g_0 L} - \omega^2}}{\omega}\};\ \omega^2 > 1$$

However, the applicability of the adiabatic approximation is not evident and its accuracy can hardly be controlled *a priori*. Therefore, in order to determine the safe margin of amplifier operation, we now consider the exact solution of the problem, making corresponding comparison with the results of the adiabatic approach.

In an inhomogeneous medium, the instability evolution is described by equation (2), or

$$\frac{d^2 a}{dz^2} + \frac{\vec{k}_\perp^2}{2n_0 k_0}\left[\frac{\vec{k}_\perp^2}{2n_0 k_0} - 2k_0 n_2(z)|U_0|^2\right] a = 0. \tag{8}$$

The initial conditions are: $a(0) = f_0$, $\frac{da}{dz}|_{z=0} = f_1$, and, in the figures below we use $f_1 = 0$. For $n_2(z) = n_2(0)\exp[g_0 z]$, introducing $\omega^2 = \frac{\vec{k}_\perp^2}{4n_0 k_0^2 n_2(0)|U_0|^2}$, $\nu = q\omega^2 = \frac{\vec{k}_\perp^2}{n_0 k_0 g_0}$, and using $q = 4\ k_0\ n_2(0)|U_0|^2\ /g_0$, we can write the equation as:

$$\frac{d^2 a}{dz^2} + \frac{q^2\omega^2 g_0^2}{4}(\omega^2 - \exp[g_0 z])a = 0.$$

After the substitution $x = q\omega\exp[g_0 z/2]$, this equation can be re-written as:

$$x^2\frac{d^2 a}{dx^2} + x\frac{da}{dx} + (\nu^2 - x^2)a = 0. \tag{9}$$

A similar equation was derived by M. Karlsson [7] in the context of studies of modulation instability in lossy fibres. We follow here his very important work in many technical aspects. However, apart from an obvious difference in physics, there are also mathematical differences in our analysis compared to [7]. The key difference is that, while in [7] only the decaying solution of (8) was considered, here we are interested in the opposite case of a growing solution. The formal solutions of the equation (9) are the Bessel functions $I_{i\nu}(x)$ and $I_{-i\nu}(x)$. Therefore the full solution of the initial problem (8) $a(z)$ can be written as:

$$a(z) = i\frac{f_0 q\omega\pi}{2\sinh[\nu\pi]}\cdot\left\{I'_{-i\nu}(q\omega)\cdot I_{i\nu}\left(q\omega e^{g_0 z/2}\right) - I'_{i\nu}(q\omega)\cdot I_{-i\nu}\left(q\omega e^{g_0 z/2}\right)\right\} + \tag{10}$$

$$i\frac{f_1\pi}{g_0\sinh[\nu\pi]}\cdot\left\{I_{i\nu}(q\omega)\cdot I_{-i\nu}\left(q\omega e^{g_0 z/2}\right) - I_{-i\nu}(q\omega)\cdot I_{i\nu}\left(q\omega e^{g_0 z/2}\right)\right\}.$$

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Here the constants are chosen to satisfy the initial conditions: $a(0) = f_0$, $\frac{da}{dz}|_{z=0} = f_1$.

The analogue of the increment of the spatial growth rate in our situation will be the value $\gamma = \ln\left[a(L)/a(0)\right]$. For a large ratio $a(L)/a(0)$, this value is practically independent of boundary conditions. The asymptotic expansion of (10) at large index values coincides with the results of the adiabatic approximation [6]. However, the key issue here is a quantitative analysis. It is not clear a priori in what range of parameters and with what accuracy the adiabatic approximation works and we will discuss this issue in more details below.

The exact solution is the function of three parameters q, g_0L and ω. The parameter ω^2 in the uniform case is the ratio of $k_\perp^2$ to its maximal unstable value. For fixed values of gain g_0L and an effective nonlinearity q we must find the maximal value of γ as a function of ω. In a uniform media ($g_0L = 0$), the most unstable mode corresponds to $\omega^2 = 1/2$ and the cut-off to $\omega^2 = 1$. In the amplifying medium the growing solution does exist in the interval $0 \le \omega \le e^{g_0L/2}$ and the growth starts when propagation distance is larger than $z^* = \ln[\omega^2]/g_0$. This should be taken into account when calculating increment of instability. The most unstable value of $k_\perp^2$ increases in amplifier during the propagation.

We plot in Figs 1-6 solutions of Eq. (10) (shown at the point L=1) with the boundary conditions: $f_0 = 1$, $f_1 = 0$ at $z = 0$ for $0 < \omega \le 1$, and $f_0 = 1$, $f_1 = 0$ at $z^* = \ln[\omega^2]/g_0$ for $1 \le \omega \le e^{g_0L/2}$. Figure 1 shows the function $\gamma(\omega)$ for selected values of q and for $g_0L = 3$, while Figure 2 shows this function for $q = 1$ and for selected values of g_0L. The black lines in these graphics correspond to the adiabatic approximation. As expected, the maximum of ω shifts up with increasing g_0L. However, for considered solutions the cut-off takes place at $\omega_{cut\_off} = e^{g_0L/2} > 1$ and the most unstable mode corresponds to $\omega > 1$. This, in particular, means that the most dangerous modes initially were stable ones and start to grow only later downstream. The result for the adiabatic approximation is plotted for only a single value of q because of the simple q dependence given by (7). It is seen that the adiabatic approximation visibly overestimates the peak value of $\gamma(\omega)$. Note that definition of cut-off frequency is somewhat not well defined as near the cut-off the growth of the mode is not large enough to avoid impact of the initial conditions. An important advantage of the exact analytical solution is that the influence of the initial conditions can be traced directly.

Figure 3 shows γ for selected values of $\omega = 0.5, 1$ and g_0L, as a function of q, for both the exact solution and the adiabatic approximation. One can see that the curves converge, but only at very high and impractical values of γ. For parameters of practical interest, the adiabatic approximation systematically overestimates the growth of the modulation instability. This is one of the most important practical results of this work. Fig. 4 shows ω corresponding to the maximal value of γ as a function of q, for selected values of g_0L. The dashed lines on these pictures represents the results of the adiabatic approximations (7).

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As was indicated above, the maximum of ω shifts up with increasing g_0L, the cut-off takes place at $\omega > 1$, and the most unstable modes corresponds to $\omega > 1$. This means that the most dangerous modes initially were stable and started to grow only later downstream.

For fixed initial intensity, the increase of q for fixed g_0L is equivalent to an increase of the amplifier length for the same final intensity. For higher q, more instability growth takes place at low intensities, which one can see in a decrease of the most unstable ω with increasing q. For smaller values of gain, the situation is closer to the uniform case and the shift of ω with change of q is less visible (Fig. 4). In all cases the maximal value of ω is higher then the uniform value $1/\sqrt{2}$.

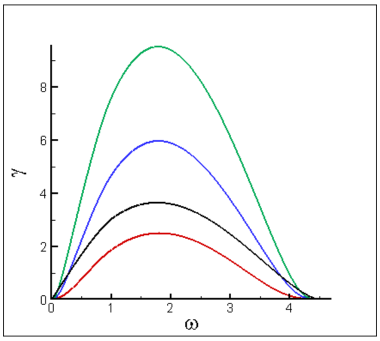

Fig.1. γ as function of transversal perturbation wave-number ω; L=1. Red line q=1, blue q=2, green q=3, black adiabatic approximation with $q=1$, $g_0L=3$.

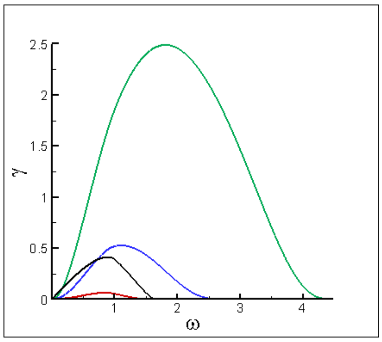

Fig.2 γ as function of transversal perturbation wave-number ω. Red line $g_0L=1$, blue $g_0L=2$, green $g_0L=3$, black adiabatic approximation $g_0L=1$, $q=1$.

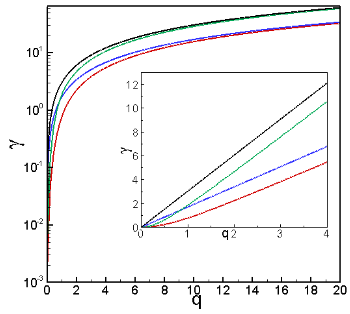

Fig.3 γ as function q for $\omega=0.5$ (red line — exact solution, blue – adiabatic approximation) and for $\omega=1$ (green line – exact solution, black – adiabatic approximation), $g_0L=3$.

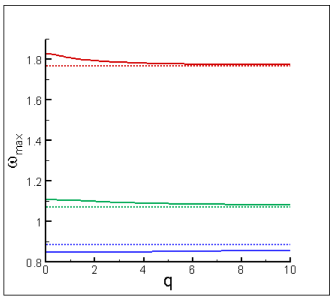

Fig.4 $\omega_{\max}$ as function q for different values of g_0L. Red line $g_0L=3$, green $g_0L=2$, blue $g_0L=1$. The dashed lines are results of the adiabatic approximation.

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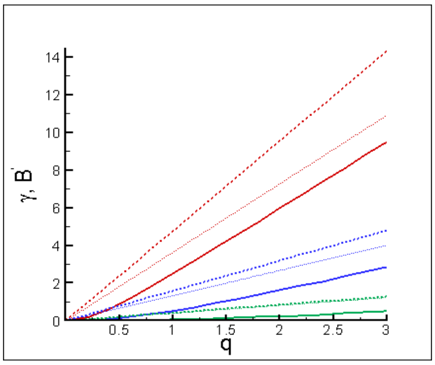

Fig.5 Maximum γ (solid line) and B-integral (dashed line) line as function of q for different values of g_0L. Red line $g_0L=3$, blue $g_0L=2$, green, $g_0L=1$. Dotted lines correspond to the adiabatic approximation.

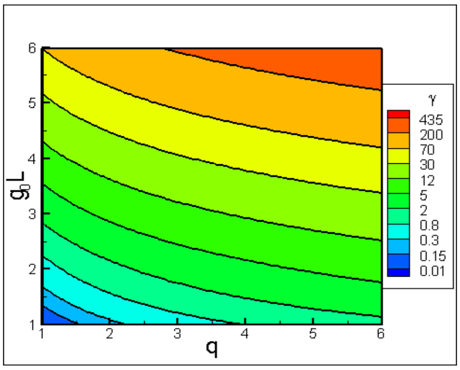

Fig.6 Counterplot $\gamma_{\max}$ as function q and g_0L.

Figure 5 presents the most important result from a practical point of view, which is the instability growth rate as a function of q. For comparison, on the same graph we show the value of the B' integral given by Eq. (6). The results of the adiabatic approximation also presented here. One can see that the adiabatic approximation noticeably overestimates the growth of the modulation instability. More careful estimate given by the exact analytical solution can be important for practical design considerations. One can see that the real growth is smaller than (6), (7), and that the difference is larger for the high gain situation. For the case $g_0L=3$ and $B^{'}=5$ the result is the still tolerable value of $\gamma \approx 2$. The general view of a maximum value of the increment $\gamma_{\max}(q, g_0L)$ is shown in Fig. 6.

4. Conclusions

We have presented a theory that accurately describes the growth of the transversal modulations of an optical beam in a laser amplifier. We found the most unstable mode and calculated the growth rate for such mode. In a typical situation, the most unstable modes are stable at the amplifier input and become unstable during the propagation. We demonstrated that the commonly used B'-integral estimate (6), and the more regular adiabatic approximation (7), both substantially overestimate the effects of modulation instability for practically used parameters. The range of applicability of adiabatic approximation is determined. An important advantage of the exact analytical solution is that it can be used to study impact of the initial conditions in the situations when the growth is not large enough to minimize dependence on the initial perturbations. Our results can be used in laser designs to prevent the degradation of beam quality. The important conclusion is that the real system can be less sensitive to perturbations induced by various optics and coatings defects. Our results might also have applications beyond the field of laser science [8-10].

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