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Variations of local heat transfer coefficient
in piped flow of viscous liquids.

Submitted for the degree
of Doctor of Philosophy.

SUMMARY.

Measurements were carried out to determine local coefficients of heat transfer in short lengths of horizontal pipe, and in the region of a discontinuity in pipe diameter. Laminar, transitional and turbulent flow regimes were investigated, and mixtures of propylene glycol and water were used in the experiments to give a range of viscous fluids.

Theoretical and empirical analyses were implemented to find how the fundamental mechanism of forced convection was modified by the secondary effects of free convection, temperature dependent viscosity, and viscous dissipation.

From experiments with the short tube it was possible to determine simple empirical relationships describing the axial distribution of the local Nusselt number and its dependence on the Reynolds and Prandtl numbers. Small corrections were made to account for the secondary effects mentioned above. Two different entrance configurations were investigated to demonstrate how conditions upstream could influence the heat transfer coefficients measured downstream.

In experiments with a sudden contraction in pipe diameter the distribution of local Nusselt number depended on the Prandtl number of the fluid in a complicated way. Graphical data is presented describing this dependence for a range of fluids indicating how the local Nusselt number varied with the diameter-ratio. Ratios up to 3.34:1 were considered.

With a sudden divergence in pipe diameter, it was possible to derive the axial distribution of the local Nusselt number for a range of Reynolds and Prandtl numbers in a similar way to the 'convergence' experiments. Difficulty was encountered in explaining some of the measurements obtained at low Reynolds numbers, and flow visualization
techniques were used to determine the complex flow patterns which could lead to the anomalous results mentioned.

Tests were carried out with divergences up to 1:3.34 to find the way in which the local Nusselt number varied with the diameter ratio, and a few experiments were carried out with very large ratios up to 14.4.

A limited amount of theoretical analysis of the 'divergence' system was carried out to substantiate certain explanations of the heat transfer mechanisms postulated.
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Further thanks are due to the Senate and Staff of the University of Aston in Birmingham for providing the extensive facilities which made the work possible.
In preparing this thesis it was decided that particular consideration should be given to the background of the reader who, in some cases, may be familiar with the terminology of heat-transfer but have a practical rather than formal appreciation of the subject. It is hoped that part 2. of the thesis will present a sufficient background to the underlying problems without being pedantic.

The theoretical aspects of the work are presented in a single section, part 11. of the thesis. In practice the analytical work was carried out in small sections, each supporting some particular aspect of the experimental investigations. To position each section of the theory within the text at the appropriate stage of an empirical argument would have meant a loss of continuity in the development of the theory. The method of presentation chosen leads to a certain number of unavoidable cross-references when reading the thesis.

The figures appropriate to a particular part of the text are contained at the end of that part, and are denoted with a number; which is the same as the part number, followed by a period then a further number, which gives the position in sequence. Occasionally a figure is placed within the text where appropriate. A similar numbering system is used for the most important equations within each part of the thesis.

As far as possible, the numerical data given has been specified in metric units. As a guide to the International System of Units, the following publication was referred to:

B.S.I. publication number PD5686.

A series of data sheets have been made available by the Engineering Sciences Data Unit which provide the designer with
information on a range of topics connected with heat transfer in tubes. Preferred results have been correlated from a review of the literature, and relationships for calculating coefficients of heat transfer in tube configurations are proposed, together with an estimate of their reliability. These references provide a much more satisfactory source than the older text books, but have not been considered in this thesis because they do not represent an original source of experimental data, and because the results of this work are supplementary to them -Refs: K.14.
NOMENCLATURE.

The following definitions are generally applicable, but sometimes the text contains symbols which have a local definition and these may or may not appear in the list below. All fluid properties are evaluated at the local bulk temperature of the fluid unless otherwise stated.

c, C  Specific heat capacity at constant pressure.
D  Inside diameter of tube.
e  Electrical resistivity of tube material.
E  Mechanical dissipation parameter \((\mu \Omega_2)/(\chi x_w)\).
f  Friction factor (see text)
F  Volume flow-rate.
g  Gravitational acceleration.
Gr  Grashof number \(= (8 \cdot 10^7 (t_w - t_b) r_w^3)/(\nu^2)\).
h  Local coefficient of heat transfer \(q/(t_w - t_b)\).
h_{\infty}  Value of \(h\) at large axial distances.
h_{\infty}  Mean value of \(h\) from \(L\), \(\int_0^L h \, dx\).
H  Specific Enthalpy.
I  Electrical current in tube wall (A)
k, K  Thermal conductivity.
l  Prandtl's mixing length.
L  An axial (scalar) length of tube.
m  Mass flow-rate.
M  The ratio of two viscosities (see text).
M_{wall}  The local ratio of (viscosity of fluid at bulk temp)/ (viscosity at tube wall). \((\nu/I)/\nu_w\).
Nu  Local Nusselt number.
Nu_{\infty}  Nusselt number at large axial distances.
Nu_{\infty}  Mean Nusselt number from \(L\), \(\int_0^L \text{Nu} \, dx\).
Nu_{\infty}  Defined in text, but generally the value of Nu for a corresponding fluid having constant viscosity and density, and no mechanical dissipation.
p  Static pressure.
P  Electrical power generation.
Pr  Prandtl number \((\nu C/\kappa)\)
q  Heat flux at tube wall.
Q  Rate of heat transfer.
Q'  Rate of heat transfer per unit length.
r, R  Radius from tube axis.
Re  Reynolds number \( (\bar{u} D/r) \).
s  Circumferential distance around tube wall.
S  Diameter ratio \( (D_2/D_1) \).
t, T  Temperature.
\( u, \bar{u} \)  (x-wise) axial velocity, and mean axial velocity.
\( v \)  (r or y-wise) velocity along a radius.
V  Applied voltage.
w  (z-wise) circumferential velocity.
x  axial distance downstream direction measured from either onset of heating or from change in tube diameter.
X  Dimensionless axial distance. Either \( x/2r_w \) or \( x/r_w / (RePr) \) (always defined in text).
y  Distance from tube wall \( r_w - r \).
Z  Circumferential distance.

Subscripts:

1, 2  Upstream, downstream of diameter change.
b, bulk  Evaluated at mixed mean temperature.
eff  Effective value of parameter.
i  At inner surface.
m, mean  A mean value of the parameter.
o  Evaluated at conditions at the inlet of tube; a reference value of the parameter; or value at outer surface.
t  Turbulent component of parameter.
w, wall  Evaluated at the wall temperature.
x  Value at distance x.
\( \beta \) Temperature coefficient of thermal expansion.

\( \Gamma \) The Gamma factorial function.

\( \varepsilon \) Signifies a parameter having a small numerical value.

\( \theta \) Dimensionless temperature function 
\((t - t_n)/(qr_w/K)\) where \( t_n \) is some reference temperature i.e. \( t_b \) or \( t_o \).

\( \mu \) Viscosity.

\( \varepsilon_s \) Viscosity variation parameter whose value depends on fluid properties and heat flux.

\( \rho \) Density.

\( \sigma \) Prandtl number.

\( \tau \) Shear stress.

\( \omega \) Vorticity.

Other Greek symbols appearing in the text are defined locally, for example \( \phi \) can be a mechanical dissipation parameter, or a general mathematical function. \( \gamma \) can be the stream function or a general mathematical function.
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INTRODUCTION

1.1. INTERNAL FLUID FLOWS WITH HEAT TRANSFER.

Heat transfer by convection is encountered frequently in engineering processes, and often plays a fundamental part in the operation of plant and machinery. The term 'convection' is used to describe the mode of energy transfer by which heat is transported in a fluid through the motion of the fluid particles. The process is one of 'free' or 'natural convection' when the movement is caused by density differences, and of 'forced convection' when the motion is imparted by an external source of friction force. In practice, these mechanisms are seldom independent of each other, but in many applications 'forced convection' predominates, and this is the case with the experiments described in this thesis.

Internal fluid flows constitute a particularly important class of forced convection systems in engineering. The design of mechanisms often depends crucially on the existence of reliable heat transfer information, and in some cases considerable capital investments are represented by large heat exchangers, so that overspecification is expensive. A list of typical applications serves to illustrate the scope of this subject - heat exchangers in generating plant, cooling passages in electrical machinery, boiler tubes, gas turbine cooling passages, space heating systems, exhaust systems for internal combustion engines, hydraulic controls, bearings, and so on.

This work is concerned with internal fluid flows of the most common type, namely those bounded by circular tubes, where the tube is maintained at a higher temperature than the entrained fluid.
A wide range of fluids is utilised in forced convection applications. Most of these may be classified as being characteristically gaseous, aqueous or viscous; typical examples for instance could be air, water and oil, respectively. The outstanding physical property which distinguishes these fluids is the viscosity, and the relevant orders of magnitude might be .015, 1.0, and 50 centipoise (cP).

From a review of research into heat transfer in tubes (as in part 3) it became apparent that considerable attention has been given in the past to air and water, whereas comparatively little work has been done using viscous fluids; no doubt this is because of the natural abundance of air and water. Since the rate of heat transfer by forced convection is known to be highly dependent on the viscosity of the fluid, it is not possible to extrapolate experimental data derived using low viscosity fluids into the highly viscous region reliably.

One of the main objectives of this work has been the investigation of forced convection in highly viscous fluids, and much of the experimental data is unique in this respect. In many ways this may be viewed as a logical extension of the work of Ede (Ref: H.1, H.2) who studied similar experimental configurations using air and water as the heat transfer media.

In the course of these investigations a particular kind of internal fluid flow pattern was encountered; this may be briefly described as being a localised region of the fluid in which the flow is stalled and disturbed. The phenomenon is usually termed 'separation' and is well known, however, despite the considerable volume of publications on the subject of heat
transfer in separation, no previous works could be found dealing with viscous fluids. It was intended that this research would help to extend the state of the art in this direction, as well as providing a detailed investigation of some particular tube configurations.

1.3. THE TUBE GEOMETRY.

An extensive range of circular ducts is encountered in heat transfer applications, the simplest and most important being long, straight lengths of uniform diameter tube. Most arrangements also incorporate other shapes, which may be described generally as embodying changes in cross-section or direction. Additionally the straight, uniform duct may be included in very short lengths.

A considerable amount of research effort has been applied to heat transfer in long tubes, and notwithstanding this, publications continue to appear regularly on this topic. At the present time more productive research may be carried out with the other duct geometries mentioned, particularly if the experimental configuration is kept simple and possesses near similitude with a variety of practical systems.

The tube geometries chosen for these investigations were the short, straight tube, and the tube with a change in cross-section. In the latter case a sudden, discontinuous increase or decrease in diameter was considered, since it is known that in this kind of disturbed flow region extremely high rates of heat transfer are possible. In practice, situations of this type arise in the cooling passages of electrical machinery or hydraulic controls, and in multifarious heat exchangers there can be found component parts which at least approximate to the configuration mentioned.

With a little appreciation of the underlying mechanisms of heat transfer it becomes apparent that the sudden convergence or divergence in diameter bears close similarity to other realistic applications, such as a stepped plate, or an exterior surface with
certain types of sharp projection. It may even be possible to consider an external, axial flow around a convergence as being qualitatively similar to an internal flow through a divergence. The implication here is that an appreciation of a much wider range of practical, heat transfer problems may be gained than is first realised, as a result of investigating the discontinuous tube described.

Six values of the ratio upstream to downstream diameter were selected in the range 3.3 to 0.3. No special significance was attributed to the actual values used, provided the effects of this ratio upon heat transfer could be determined. Hence, it appeared reasonable to choose diameter ratios at least near to some of those encountered in similar work on gaseous and aqueous fluids, thus facilitating a direct comparison of the results obtained. The range of ratios could be extended as the investigations proceeded, should this become desirable, but the ratio 3.3 was a reasonably practical maximum to impose, even though some applications are bound to exist outside the range considered. This point is taken up later in the thesis.

The unique diameter ratio of unity corresponds to the short, uniform tube or the long, uniform tube depending on the system of heating i.e. for a tube having a discontinuity in the axial distribution of the heat source the situation is analogous to the short tube. In these experiments only the short tube was to be considered.

In selecting experimental tubes three factors have to be considered, the type of material, the quality of the tube surfaces or roughness, and the dimensions of the tube. The material is determined largely by practical considerations, and does not influence the convection process. The tube dimensions are also chosen on practical grounds, but the diameter may well be related to the convection process in some way which can only become clear with hindsight. The actual sizes chosen were of the same order
of magnitude as would be encountered in a typical heat exchanger -
insofar as one can generalise. The roughness may affect the fluid
flow pattern; however, it is sufficient at this stage to say that
the tube surfaces were hydraulically smooth, as is the case in the
majority of real applications. These points will be further
discussed in part 5.

1.4. TRANSFER MECHANISMS OF HEAT AND MOMENTUM.

Any discussion on convective heat transfer must of necessity
include a description of the fluid flow. It is usual for heat to
be transferred from a solid boundary to an adjacent fluid medium,
and the mechanism is simple to illustrate qualitatively. Moving
particles of fluid change position relative to the boundary and
pass through regions of different temperature level. Heat is gained
or lost by a particle through the process of conduction, the amount
depending on the velocity and direction of the particle relative to
the boundary. The greater the rate of interchange of particles
between the space near to and distant from the boundary, the greater
the amount of heat transferred.

It is evident that a highly disturbed flow will transfer
heat more effectively than one which moves smoothly, parallel to
the boundary. Furthermore, a fast moving flow will be more effective
in transferring heat than a slow one.

Steady, forced flows may be defined as characteristically
'laminar' or 'turbulent.' The former refers to those in which the
molecules of fluid slide smoothly over each other and the motion
appears not to change with time. Turbulent flows are very disturbed,
and the molecules move randomly throughout the fluid. At a given
point in the flow, the velocity is time dependent in magnitude and
direction, however, it is usually possible to consider the actual
velocity as being made up of a mean velocity with superimposed
velocity fluctuations. This means the main, mean velocity component has magnitude and direction which is not time dependent.

For a given geometry at the boundary, both laminar and turbulent flow may exist, but the existence of either depends largely on the magnitude of the mean velocity. Generally, turbulence occurs in 'fast' flows and lamination in 'slow' flows. For a given fluid, and above a certain mean velocity, the turbulent regime can be sustained. Near to this critical velocity there is often a 'transitional' flow regime in which regions of the flow oscillate between turbulence and lamination.

In studying a particular forced convection system it is necessary to specify the type of heating, so in an effort to generalise on the temperature conditions at the boundary two realistic situations are found useful. First, the boundary may be maintained at a constant, uniform temperature throughout the entire surface, and second, a uniform rate of heating per unit area (or uniform heat-flux) may be used as the constraint. These conditions are somewhat idealised, and in reality something between the two is to be expected, however, experience has indicated that there is little to choose between either with regard to the applicability of the experimental results. The first constraint is similar to the kind of boundary condition found on a tube in an evaporator or condenser, and the second kind appears in equipment where the heat source is electrical, radiant or nuclear. In the present work the uniform heat flux condition was selected, primarily because of certain advantages in the construction and operation of the experimental apparatus.

1.5. A DESIRABLE RANGE OF FLOW RATE.

From a previous knowledge of flow in plain tubes it was possible to estimate the minimum velocity at which full turbulence
was likely to occur. This may be given approximately by the formula

\[
\text{Velocity} = 10^4 \times \frac{\text{Viscosity}}{\text{Density} \times \text{Diameter}}.
\]

It was desirable that heat transfer measurements be made in the laminar, transitional and turbulent regimes. In practice the range of flow-rate must be limited by the maximum, attainable pressure drop in the tube, at the other end of this range no obvious, practical limitation exists. The pressure drop is known to be proportional to the square of velocity, so that when viscous fluids are used the system pressure is likely to be high. From the above equation it is seen that by increasing the viscosity by a factor of 20, and for a similar range of experimental results, with respect to the flow-regime, the system pressure must be increased by 400 times.

It was decided initially that the maximum flow rate required be given approximately by twice that value necessary for full turbulence, the fluid being ten times as viscous as water. The maximum system pressure thus obtained was of the order 700kN/m².
2. FUNDAMENTALS AND HISTORICAL BACKGROUND.

2.1. INTRODUCTION.

It is desirable at this stage to give an account of the earlier works concerned with heat transfer in tubes, the purpose being to introduce some of the fundamental concepts which arise in work of this nature, and to present the background from which the current research has evolved. This section begins with an explanation of some important principles encountered in convective heat transfer (2.2.).

2.2. EXPERIMENTAL AND THEORETICAL FUNDAMENTALS.

2.2.(i) DIMENSIONLESS GROUPS.

In forced convective heat transfer, analysis may proceed on theoretical or empirical lines. In general however, the information required is obtained by conducting experiments, then determining correlations between various groups of significant parameters. Usually, measurements of the coefficient of heat transfer, \( h \), are carried out, this variable being defined by Newton's Law of cooling, \( q = h \Delta t \). In the latter expression \( q \) is the heat flux, and \( \Delta t \) the temperature difference between the heated surface and the fluid medium.

The following dimensionless groups of parameters are encountered frequently in fluid mechanics and heat transfer, and will occur throughout this thesis.

- Nusselt number \( \text{Nu} = \frac{hD}{k} \)
- Reynolds number \( \text{Re} = \frac{\rho u D}{\mu} \)
- Prandtl number \( \text{Pr} = \frac{\mu c_p}{k} \)
- Grashof number \( \text{Gr} = \frac{\rho g \beta D^3 \Delta t}{\mu^2} \)

Dimensionless distance \( X = x/D \),

where \( \Delta t = q/h \) or \( (t_w - t_b) \),

\( t_w \) = the temperature at the inner tube surface,
\[ t_b = \text{bulk, or mixed mean temperature of the fluid} \]
\[ = 2 \int_0^1 \frac{u}{u} \left( \frac{r}{r_w} \right) t \, d\left( \frac{r}{r_w} \right), \]
\[ u, \bar{u} = \text{axial velocity of fluid, and the mean velocity,} \]
\[ \bar{u} = 2 \int_0^1 \left( \frac{r}{r_w} \right) u \, d\left( \frac{r}{r_w} \right), \]
\[ D = 2r_w \text{ or inner diameter of the tube,} \]
\[ x = \text{distance from the onset of heating.} \]

In convective heat transfer the value of the dependent variable \( \text{Nu} \) can often be obtained from an expression of the kind
\[ \text{Nu} = f(Re, Pr, Gr, X) \quad (2.1) \]

where the functionality is determined experimentally.

The uniqueness of this kind of relationship depends on the validity of the initial assumptions made regarding the significance of certain experimental variables. In this research, consideration was given to frictional heating in the fluid, the temperature dependence of the viscosity, and the way in which these phenomena could affect the value of \( \text{Nu} \). The above expression excludes any such dependence.

2.2.(ii) THE ENERGY AND MOMENTUM EQUATIONS.

The derivations of the energy and momentum (or Navier-Stokes) equations are given in many standard text books (e.g. Ref.K1). For steady, incompressible, axi-symmetrical flow through a tube with axi-symmetrical temperature distribution, these equations are stated
\[ \rho \frac{\partial H}{\partial x} + \mu \frac{\partial H}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{r}{K} \frac{\partial T}{\partial r} \right] - \frac{1}{r} \frac{\partial}{\partial x} \left[ K \frac{\partial T}{\partial x} \right] - \mu \phi - u \frac{\partial \phi}{\partial r} - \nu \frac{\partial \phi}{\partial r} = 0 \quad (2.2) \]
where
\[ \phi = 2 \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial r} \right)^2 \right) + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial r} \right)^2 - \frac{2}{3} \left( \frac{\partial v}{\partial r} + \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \right)^2 \quad (2.3) \]
and
\[ \rho \frac{\partial u}{\partial x} + \mu \frac{\partial u}{\partial r} = -\frac{\partial \phi}{\partial x} + 2 \frac{\partial}{\partial x} \left[ \mu \frac{\partial u}{\partial x} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \right) \right] \quad (2.4) \]
with
\[ \rho \frac{\partial v}{\partial r} + \mu \frac{\partial v}{\partial r} = -\frac{\partial \phi}{\partial r} + 2 \frac{\partial}{\partial x} \left[ \mu \frac{\partial v}{\partial x} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x} \right) \right] + 2 \frac{\mu}{r} \frac{\partial v}{\partial r} \quad (2.5) \]
The equation of continuity is
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0. \] (2.6)

Some of the theoretical work carried out in this research required the use of the above equations, but the well known boundary-layer simplifications of Prandtl were applied for the analyses associated with unidirectional flows. The energy equation, unlike that of Prandtl, retains a first order approximation for the dissipation function \( \beta \), as follows:
\[ \left( \frac{u}{dx} \frac{\partial t}{\partial x} + \frac{v}{dr} \frac{\partial t}{\partial r} \right) = \frac{k}{c} \frac{\partial r}{\partial r} \left( \frac{\partial^2 u}{\partial r^2} \right)^2. \] (2.7)

The single momentum equation becomes
\[ \left( \frac{u}{dx} \frac{\partial u}{\partial x} + \frac{v}{dr} \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{mr}{\partial r} \right) \right). \] (2.8)

The specific heat capacity is assumed to be constant, which is a reasonable approximation for most liquids.

2.2.(iii) THE LAMINAR REGIME.

In the investigation of laminar flows (in a uniform tube), laminar flow is known to be present when \( \text{Re} < 2,500 \) the axial velocity distribution in the fluid must be specified at the tube entrance. For most applications an adequate description of this flow condition would be that the velocity profile is 'developed' or 'undeveloped'. The former condition implies a parabolic radial distribution, and the latter implies uniform distribution. During laminar heat-transfer the thermal conductivity is assumed to be constant, since variations with temperature are of the order \( \frac{1}{3} \) per Kelvin.

2.2.(iv) THE TURBULENT REGIME.

For Reynolds numbers above 2,500 in a straight tube, the flow is usually turbulent. It is well understood (Ref: K4.) that full turbulence can be sustained when \( \text{Re} \) is approximately
4,000 to 10,000, and that for intermediate Re a transitional regime occurs. When the geometry of the system is altered the numbers quoted have little relevance, so when discontinuous tubes are encountered, as in this research, the possibility of a more complicated flow regime with different, critical Reynolds numbers, must be considered.

In the theoretical analysis of turbulent flow, it is usual to treat the system as a laminar one, for convenience, but to replace the viscosity and thermal conductivity by 'effective' properties which have an added turbulent component: viz:

\[ \mu'(\text{effective}) = \mu'(\text{laminar}) + \mu'(\text{turb}), \quad \kappa'(\text{effective}) = \kappa'(\text{laminar}) + \kappa'(\text{turb}) \]

or \[ \mu'_{\text{eff}} = \left( \mu'_{\text{turb}} / \mu'_{\text{laminar}} \right), \quad \kappa'_{\text{eff}} = \left( \kappa + \kappa_{\text{turb}} \right) \].

The Reynolds analogy can be used to determine the relationship between \[ \mu'_{\text{turb}} \] and \[ \kappa_{\text{turb}} \]. This states that the rate of transport of heat by turbulence is the same as the rate of transport of momentum (for unit heat flux and stress). Therefore \[ \frac{1}{\eta_x} \frac{\partial}{\partial y} \left( \mu' \frac{\partial u}{\partial y} \right) = \frac{1}{\kappa} \frac{\partial}{\partial y} \left( \kappa' \frac{\partial \mu}{\partial y} \right) \]

or \[ \left( \frac{\mu'_{\text{turb}} C}{\kappa_{\text{turb}}} \right) = 1 \].

A model of turbulent flow near to a heated surface was developed in 1910 by Prandtl, and successive improvements by Von Karman in 1939 (Ref: C.5) and later by Martinelli, 1941 (Ref: C.6) produced a workable theory which enabled coefficients of heat transfer to be calculated for fluids with different Prandtl numbers. The theory was applied to long tubes and was reliable for moderately high Prandtl numbers (\[ \approx \]). The model referred to made allowance for the fact that turbulent eddies are damped by viscous action near to a surface. A laminar sub-layer was hypothesised, and a fully turbulent mainstream. In between these two extremes; a
buffer layer was inserted which was only partially turbulent.

In 1925 Prandtl proposed the 'mixing length theory' which has been widely used in the analysis of turbulent flows. The properties $\lambda_{\text{turb}}$ and $K_{\text{turb}}$ are calculated from the following formula:

$$\left( \frac{\lambda_{\text{turb}}}{\lambda} \right) = \left( \frac{K_{\text{turb}}}{K} \right) = \ell^2 \left| \frac{du}{dy} \right|,$$

where $\ell$ is proportional to the width of the mixing region, or width of the boundary layer.

The equations of momentum and energy become greatly simplified for turbulent flow near a wall, because streamlines are almost parallel. Hence:

$$\left( \frac{u \partial \lambda}{\partial x} \right) = \frac{d}{dr} \left( K_{\text{eff}} \frac{\partial \lambda}{\partial r} \right) + \lambda'_{\text{eff}} \left( \frac{\partial u}{\partial r} \right)^2 \quad (2.9)$$

with

$$\frac{dp}{dx} = \frac{d}{dr} \left( \lambda'_{\text{eff}} \frac{du}{dr} \right). \quad (2.10)$$

2.2.(v) SEPARATED FLOWS.

Flow in tubes may often be considered as boundary-layer flow since the axial velocity component usually predominates. If the flow is disturbed by a discontinuity in the tube's geometry a localised region of the fluid may experience an adverse pressure gradient which causes large transverse velocity components. The axial velocity may be reversed locally, and a standing eddy (indicated by closed streamlines) formed downstream of the discontinuity. This kind of flow is termed 'separated', and separation can have a considerable effect on the local coefficient of heat transfer.

In general, the boundary-layer equations cannot be used to analyze this sort of problem, only a solution of the full equations will suffice.

The early experiments of Boelzer (Ref: H.15) on forced
convection with air in tubes having different entrance configurations, gave evidence as to the general character of heat transfer during separation. The local coefficient of heat transfer becomes very high, about four times greater than its non-separated value. The location of the maximum value is probably near to the point of boundary-layer reattachment, i.e., the point downstream of the separation 'bubble' at which the direction of the flow reverses on the heat-transfer surface, and the entire domain becomes a boundary layer. The magnitude of the local coefficient of heat transfer falls away sharply as the distance from the position of the maximum value increases.

The length of the separated region remains reasonably constant at high Re and is about seven times the height of the discontinuity causing separation when this is a sharp projection or downward step in the flow direction. At low Re, though how low cannot be defined, the length of the region could well increase with reducing Reynolds number, but this must remain intuitive for the present.

Separation may also be caused by an upward step in the flow direction, or by various forms of cavity. Even smooth directional changes such as occur in a diffuser, around a bend, or in cross-flow, over a cylinder can cause separation. The latter was the subject of many early heat-transfer researches (see Knudsen and Katz Ref: K.3) but such configurations have little relevance in this work and will not be discussed.

Because of its importance in aerodynamic applications, the study of separated heat transfer has tended to be limited to gaseous media and in particular with supersonic velocities. Some recent investigators (Ref: H.1, H.9, H.11, H.17) have used water, but no previous data is available for viscous fluids.
2.2.(vi) THE PRANDTL NUMBER AND VISCOS FLUIDS.

Before proceeding with a discussion on heat transfer in tubes, it should be pointed out that viscous fluids generally have a high Prandtl number. Since it is sometimes convenient to utilise dimensionless groups, the terms 'viscous fluids' and 'fluids with high Prandtl number' are often used synonymously in this sense.

2.3. THE LONG STRAIGHT TUBE.

2.3.(i) DESCRIPTION.

The long straight tube is defined as one in which the axial velocity gradient is zero, and the axial temperature gradient is constant. In practice, this implies a length of at least 100 diameters in the turbulent regime, or (0.05 Re Pr) diameters in the laminar regime (e.g. see Keys Ref: K.5)

2.3.(ii) LAMINAR HEAT TRANSFER.

This elementary problem has been solved theoretically yielding $N_u = 4.36$, and experimental corroboration has been obtained (Ref: C.1).

2.3.(iii) TURBULENT HEAT TRANSFER.

Considerable interest has been shown in turbulent heat-transfer in long tubes. Theoretical and experimental analyses have been carried out, some of which will be discussed.

Numerous papers on the subject were published prior to 1936. McAdams (Ref: K.2) gives a comprehensive review of these. Most of the experimental data was correlated with equations of the form $N_u = B_1 Re^m Pr^n$, where $B_1, m$ and $n$ are constants. Some controversy existed as to the best reference temperature for determining the fluid properties, but this was of little consequence since the temperature differences $(t_w - t_b)$ employed were usually small, and the physical properties varied insignificantly. The values of the experimental constants which best suited the data
available were found by McAdams to be $B = 0.023$, $m = 0.8$ and $n = 0.4$, for the following conditions: $Re > 10,000$, $0.7 < Pr < 100$. The value of $Nu_\infty$ can be estimated with $\pm 30\%$ accuracy from this exponential relationship, which is remarkable when consideration is given to the comparatively crude apparatus utilised in the early researches.

Little data existed at the time for experiments with viscous fluids. In 1928 Morris and Whitman (Ref: A.5) did some tests on oils with viscosities up to 55 cP. The mean rate of heat transfer was determined for a steam-heated tube 150 diameters long. The temperature differences $(t_w - t_b)$ were in the range 15 to 70 K. It was recommended that the following constants be used, $m = 0.83$, $n = 0.37$, and it was pointed out that with viscous liquids, the high dependence of viscosity on temperature made the choice of reference temperature an important issue.

In 1932 Sherwood (Ref: A.7) conducted some experiments similar to those of Morris and Whitman, incorporating a range of viscous fluids. The bulk temperature was used for determining the properties, and it was concluded that the value of $Nu_\infty$ should be given by $Nu_\infty = 0.023 \, Re^{0.8} \, Pr^{0.4}$. This is in agreement with the equation already quoted, which was based largely on data obtained with non-viscous fluids. Sherwood suggested that the exponent of $Re$ could possibly increase slightly with increasing $Pr$.

In 1933 Colburn made some recommendations (Ref: A.8) on how to correlate the experimental results for a long tube with heat transfer, and supported his hypothesis with existing data. $Nu_\infty$ was supposed to be proportional to $Pr^{0.5}$, and the reference temperature for determining the physical properties was taken as the average of the wall temperature and the bulk temperature. It was further suggested that the viscosity-temperature dependence be
taken into account by making $N_u$ a function of $(\text{viscosity at bulk temp.})/(\text{viscosity at reference temp.})$.

In 1936 Sieder and Tate (Ref: D.1) reconsidered some earlier experimental data obtained from tests to determine the mean rate of heat transfer to oils in a long steam-heated tube. Colburn's conclusions were simplified for practical reasons and the ensuing relationship proposed was $N_u = 0.027 \, R_e^{0.8} \, P_r^{0.8} \, K_{\text{wall}}^{0.14}$, where $K_{\text{wall}} = (\text{viscosity at the wall})/(\text{viscosity at bulk temp.})$, and all other properties were estimated at the bulk temperature. This expression has gained wide acceptance by virtue of its simplicity and is still in use today despite the fact that experimental deviations of ± 30% were evident in the original correlation. A graph of the Sieder and Tate results is given in figure 2.1. The authors could not substantiate the selection of the exponent 0.14 in the turbulent regime but conjectured its value from results obtained with laminar flow.

In the theoretical analysis of turbulent heat-transfer in long tubes, the work of Martinelli (Ref: C.G.) in 1941 is outstanding as the first refined analysis. It was no doubt based on the earlier analyses of Prandtl and Von Karman (mentioned in 2.2.(iv)) which were fairly crude attempts. Martinelli made use of Prandtl's mixing length theory, and after an approximate integration of the momentum and energy equations calculated $N_u$ to be given by

$$N_u = \left(\frac{f}{2}\right)^{0.5} \frac{R_e \, P_r}{5 \, D \left[ P_r + \log(1 + 5 \, P_r) + 0.5 \, P_r \log \left( \frac{R_e}{50} \left(\frac{f}{2}\right)^{0.2} \right) \right]}$$

where all the properties are determined at the bulk temperature, and

$f = \tau_{\text{wall}} / (\bar{u}^2)$, the friction factor,

$\bar{u} = \left( \frac{\text{wall temp} - \text{bulk temp}}{(\text{wall temp} - \text{temp at centre of tube})} \right)$

Martinelli claimed ± 20% accuracy in the range $R_e > 10^5, 0.5 < P_r < 350$
It is possible that this was an overoptimistic estimate of reliability particularly when high values of Pr are encountered. As the diagram (Figure 2.2) indicates, with Pr = 100, 95.5% of the temperature drop \( (t_w - t_b) \) occurs in the region of the laminar sublayer. Some special consideration should be given in such an extreme case.

2.3(iv) TRANSITIONAL HEAT TRANSFER.

In the transitional region \( (2,500 < \text{Re} < 10,000) \) comparatively little research has been carried out. This is probably due to the erratic behaviour of the tube temperature, which is a consequence of the very disturbed flow pattern and accompanies transition. McAdams (Ref: K.2) made some tentative suggestions but Colburn (Ref: A.8) attempted the first significant correlation of experimental data.

Colburn showed that the mean value of \( \text{Nu}_\infty \), with respect to time, changed smoothly from the characteristically low values associated with laminar flow, to the characteristically high values associated with turbulent flow, as the Reynolds number increased from 2,500 to 10,000. It is interesting to note that the data for fluids having a low viscosity did not correlate nearly as well as the data for viscous fluids.

Sieder and Tate (Ref: D.1) included much experimental data for the transition region in their work on viscous liquids. They showed that plotting

\[
\left( \frac{\text{Nu}_\infty}{\text{Pr}^{\frac{3}{4}}} \right) \text{ versus Re}
\]

gave a unique relationship when the tube length was great.

The experiments of Norris and Sims (Ref: A.9) dealt with cooling, rather than heating, in the transitional region. It was shown that for fluids with Pr in the range 35 to 140, the value of \( \text{Nu}_\infty \) could be estimated from \( \text{Nu}_\infty = 0.0067 \text{ Re Pr}^{0.2} \), when \( 3,500 < \text{Re} < 11,000 \).
The maximum experimental errors were approximately 5%, and the tests carried out were repeatable. This work indicated that useful heat-transfer information can be obtained with transitional flow, despite the fluctuating temperatures.

2.4. THE SHORT STRAIGHT TUBE.

2.4.(i) DESCRIPTION.

In the early research, little attention was given to the measurement of local coefficients of heat transfer, and consequently when short lengths of tube were encountered mean Nusselt numbers, dependent on the overall tube length, were postulated. These will be written \( \text{Nu}_m \) in this thesis. In general, the local Nusselt number, \( \text{Nu} \), is dependent on the axial position, and the measurement of this axial profile is more complicated than the measurement of \( \text{Nu}_m \).

Consider a fluid entering a uniformly heated section of tube. At onset of heating, \( x = 0 \), there is a heat flux but no heat has penetrated the fluid. This implies that \( \text{Nu} = \infty \) by definition. Further downstream at \( x = x_1 \), a temperature profile has developed within the fluid therefore \( \text{Nu} = \text{Nu}_1 \), which is less than infinite. Hence, \( \text{Nu} \) must fall with increasing axial distance.

The function \( \text{Nu}(x) \), must be dependent, amongst other things, on whether a developed or undeveloped velocity profile is present at the onset of heating.

2.4.(ii) LAMINAR HEAT TRANSFER.

Theoretical analyses, for the short tube with laminar flow, have been thoroughly investigated since Graetz solved the boundary-layer equations in 1885 (e.g. see Knudsen Ref: K.3.) for a tube at constant temperature, with fully developed velocity. The resulting eigenvalue series in \( X, Re \) and \( Pr \) converges slowly at high Prandtl numbers and small distances, as do the solutions.
for other boundary conditions, such as uniform flux with undeveloped velocity at the inlet (Ref: C.13, C.14, C.15, C.16, C.17).

The above-mentioned solutions have been found useful with gaseous media, but unsuitable for viscous media. In 1928 Leveque (Ref: C.18) discovered an asymptotic solution to the uniform tube-temperature problem, for high Pr,

$$\text{Nu} = 1.077 \left( \frac{2r_w^2 \text{Re Pr}}{x} \right)^{\frac{3}{5}}$$

The simplifying assumption made was that the velocity profile could be linearized near the wall. This was founded on the observation that 'temperature boundary-layers' are always much thinner than 'velocity boundary-layers' at high Prandtl numbers. The uniform flux problem was solved much later (Ref: C.13).

Most of the early experimental data is quoted in terms of \( \text{Nu}_m \), and can only be compared with the local value Nu through the following integral.

$$\text{Nu}_m = \frac{1}{L} \int_0^L \text{Nu}(x) \, dx.$$ 

Some of the references already quoted, - Sherwood, Colburn and Sieder - have made use of \( \text{Nu}_m \), and the difference should be appreciated as this discussion progresses.

In 1948 Cholette (Ref: B.14) provided some of the earliest measured local coefficients of heat transfer in the laminar regime. The rate of heat transfer to air was measured for a tube heated with a series of small, steam compartments. These rather crude measurements are given in figure 2.4, which indicates the variation in the local coefficient of heat transfer, \( h \), with axial distance from the start of heating, \( X \), for different values of \( \text{Re} \). For \( \text{Re} < 2,500 \) the values of \( h \) fall rapidly with increasing axial distance, from a high initial value. After approximately 40
When viscous fluids are considered, it is necessary to take account of the viscosity-temperature dependence in estimating $\text{Nu}$. Figure 2.3 illustrates how the axial velocity profile is affected by variations in the viscosity. In 1933 Colburn's synthesis (Ref: A.8) of a variety of experimental data resulted in a viscosity correction of the form

$$\frac{\text{Nu}_m (\text{variable})}{\text{Nu}_m (\text{constant})} = \left( \frac{\mu}{\mu_{ref}} \right)^{\frac{3}{2}}$$

where $\mu_{ref}$ is the viscosity at the average of the wall and bulk temperatures. No rigorous test was made of this proposition, and the work of Sieder and Tate (Ref: B.1) superseded it. Sieder proposed the relationship already discussed in 2.3.(iii) in connection with turbulent flow:

$$\frac{\text{Nu}_m (\text{variable})}{\text{Nu}_m (\text{constant})} = m_{wall}^{0.14}$$

This correction to the Nusselt number gave reasonable results when $m_{wall}$ was varied up to 10. Graphical representation of this relationship is given in figure 2.1.

The way in which free convective effects modify the flow through a tube is illustrated qualitatively in figure 2.3. In assessing the contribution of free convection to the value of $\text{Nu}_m$, Colburn realised the significance of the Grashof number. A tentative estimate of $\text{Nu}_m$ was given by $\text{Nu}_m (\text{with free convect.}) = \text{Nu}_m (\text{without free convect.}) (1 + 0.015 \text{Gr}^{3})$. The properties contained in Gr were evaluated at Colburn's reference temperature, and the equation was considered valid for $\text{Gr} > 25,000$. Sieder and Tate made a similar suggestion but chose to evaluate properties at the bulk temperature. In 1943 Korn and Othmer pointed out the limitations (Ref: E2) in such expressions by investigating free and forced convection in horizontal tubes with three oils. The main conclusion reached was that previous correlations were inadequate, and the right hand side of Colburn's equation should contain a function of Reynolds number.
2.4. (iii) TURBULENT HEAT TRANSFER.

McAdams (Ref: K.2) survey of the early research into short tubes with turbulent flow, led to the conclusion that $N_u_m$ could be determined from 

$$N_u_m = N_u \left(1 + C \frac{L}{2x_w}\right),$$

where $L$ is the overall tube length, and $C$ is a coefficient, the value of which depends largely on conditions upstream of the tube entrance, but is also a weak function of Reynolds number. Such relationships were reasonable substitutes in the absence of measured local coefficients of heat transfer.

Cholette's work (described in the preceding section) constituted one of the earliest attempts at measuring local coefficients of heat transfer, $h$. Figure 2.4 shows how $h$ varied with axial distance for air in a short tube fitted with a 'bellmouth' entrance to establish uniform velocity. In the initial part of the tube the flow developed as a laminar boundary layer, which broke down into turbulence at some distance from the inlet. The characteristic 'dip' in the 'h versus distance' curves can be attributed to the transition from laminar to turbulent flow.

In 1948 Boelter, Young and Iverson (Ref: H.15) measured local values of $h$ for a similar experimental configuration to that of Cholette. In this case the measurements also included data obtained with a 'calming length' fitted to the inlet, which established a fully developed velocity distribution. Figure 2.5 illustrates some of the results, and indicates how the absence of velocity development causes a smooth reduction in $h$, from a high initial value, to low final value, $h_\infty$, after approximately 40 diameters distance.

No investigations were carried out by the early researchers into the effects of free convection, or viscosity-temperature dependence, on the value of $h$. 
2.5. THE DISCONTINUOUS TUBE.

Boelter, Young and Iverson (Ref. H.15) passed air through a steam-heated tube, which was constructed so that local coefficients of heat transfer could be measured. Different devices were fitted to the inlet of the tube, so that \( h \) could be determined downstream of bends, bellmouths, orifices, sharp edged inlets, calming lengths and other configurations. Two of the arrangements have close similarity with the sudden divergence and convergence in diameter, the subjects of this research, and these were described as "the orifice type entrance" and "the right-angle edge type entrance". The studies of Boelter et al. did much to stimulate the current interest into heat transfer with regions of separated flow, and in tubes with complicated forms.

Figure 2.6 shows '\( h \) versus axial distance' for \( Re = 27,000 \). Despite the crude nature of the measurements, the very high coefficients of heat transfer arising near to the inlet of the tube highlighted the value of having data on local coefficients. Where thermal stress in the tube material, or economy of size are important criteria, the design process is more effective when local, rather than mean, values for \( Nu \) are available.

The presence of a maximum in \( h \) as a function of length was pronounced in the case of the orifice-entrance, but more difficult to explain in the case of the right-angle entrance. Boelter reasoned a priori that a stagnant pocket of air, near to the inlet, caused low coefficients of heat transfer by insulating the mainstream from the tube. Slightly downstream, the flow was supposed to develop in a similar way to the 'short tube', yielding a high value of \( h \) which reduced rapidly with increasing distance.
FIGURE 2.1
THE EXPERIMENTS OF SIEBER MISTABE ON HEATING AND COOLING OILS

HIGH REYNOLDS NUMBER CORRELATION

\[
\frac{\text{Nu}_{\text{ave}}}{\text{Pr} \frac{d}{L}} \left( \frac{\text{Pr}}{\text{Pr}_b} \right)^{0.14} = \frac{\text{Nu}}{\text{Nu}_b} = \left( \frac{\text{Nu}_b}{\text{Pr}_b} \right) \left( \frac{\text{Pr}}{\text{Pr}_b} \right)^{0.14}
\]

\[ \frac{L}{D} = \text{OCCUPED TUBE LENGTH} \]

\[ L = \text{OVERALL TUBE LENGTH} \]

\[ D = \text{TUBE DIAMETER} \]

\[ \text{Nu}_{\text{ave}} = \text{NUSSLEIT NUMBER - MEAN VALUE FOR A LONG TUBE} \]

* THE AUTHORS RELIED MAINLY ON OTHER SOURCES FOR DATA USED IN HIGH REYNSOLDS NUMBER CORRELATION.

LOW REYNOLDS NUMBER CORRELATION

\[
\frac{\text{Nu}}{\text{Pr}^{\frac{1}{3}}} \left( \frac{\text{Pr}}{\text{Pr}_b} \right)^{0.14} = \frac{\text{Nu}_b}{\text{Pr}_b^{\frac{1}{3}}}
\]

\[ \text{Re} \]

\[ \text{Re} = \text{REYNOLDS NUMBER} \]

THE EFFECT OF VISCOSITY VARIATION ON Nu AT LOW VALUES OF Re.

\[
\frac{\text{Nu}_b}{\text{Pr}_b^{\frac{1}{3}}} \left( \frac{\text{Pr}}{\text{Pr}_b} \right)^{0.14} \text{Re}
\]

\[ \text{Pr} = \text{THE VISCOSITY RATIO (Pr \text{Pr}_b)} \]

\[ \text{Pr} \]

\[ \text{Pr}_b = \text{BASELINE VISCOSITY} \]

\[ \text{Re} \]

\[ \text{Re}_b = \text{BASELINE REYNOLDS NUMBER} \]
FIGURE 2.2.
RADIAL TEMPERATURE DISTRIBUTION IN TUBE ACCORDING TO MARTINELLI. Re = 10,000.

FIGURE 2.3.
ILLUSTRATION OF HOW THE AXIAL VELOCITY DISTRIBUTION IS MODIFIED BY TEMPERATURE DEPENDENT VISCOSITY AND BY FREE CONVECTION.
FIGURE 2.4.
HEAT TRANSFER TO AIR FROM CHOLETTE BELLMOUTH ENTRANCE.

![Graph showing local coefficient of heat transfer vs. axial distance for various Re values.]

TUBE DIAMETER = 0.45 cm.

FIGURE 2.5.
HEAT TRANSFER TO AIR FROM BOELTER BELLMOUTH AND CALMING LENGTH.

![Graph showing local coefficient of heat transfer vs. axial distance for different Re values.]

TUBE DIAMETER = 14.53 cm.

*UNITS FROM ORIGINAL SOURCE.*

\[
\frac{\text{Blu}}{h_{\text{eff}}} = 5.678 \left(\frac{W}{m^2 K}\right)
\]
FIGURE 2.6.

HEAT TRANSFER FOR DIFFERENT ENTRANCE CONFIGURATIONS FROM BOELTER.

TUBE DIAMETER = 4.53 cm.

Re = 27,000

ORIFICE AT ENTRANCE 1:1.78 DIVERGENCE.

ORIFICE 1:1.49 DIVERGENCE.

RIGHT ANGLE-EDGE TYPE ENTRANCE.

LOCAL COEFFICIENT OF HEAT TRANSFER $h_L$ (in W/ft$^2$-°F) or (5.672 W/m$^2$-°K)

AXIAL DISTANCE $x/D$ IN DIAMETERS.
3. LITERATURE SURVEY.

3.1. Introduction.

A survey of recent work on convective heat transfer in tubes was carried out, which encompassed numerous related topics. This part of the thesis is sub-divided into eight sections, each dealing with a particular aspect of the survey, and containing a discussion on those publications which have dealt specifically with a particular problem relevant in this research. It should be noted that in the following the term 'non-viscous' is used to indicate a low viscosity.

3.2. THE LONG STRAIGHT TUBE.

3.2(i) 'NON-VISCOUS' FLUIDS.

During the last twenty years a considerable volume of literature has been published on the experimental findings of researchers who measured $\text{Nu}_\infty$ for long tubes, with gaseous or aqueous heat-transfer media, in the fully turbulent flow regime. In references (A1, A2, A6, B1, B2, B3, B5, B6, B8, B9, B10, B12, E11) attempts have been made to obtain a correlation between $\text{Nu}_\infty$, $\text{Re}$ and $\text{Pr}$, although in some of the cases quoted this did not constitute the main objective of the work.

For moderate temperature differences, so that the properties of the fluid remained substantially constant, the experimental data in most of the aforesaid references was correlated in the following way.

$$\text{Nu}_\infty = B \cdot \text{Re}^m \cdot \text{Pr}^n.$$  

The usual practice was to evaluate the properties of the fluid at the local, bulk temperature and for the parameters $B$, $m$ and $n$ to be given suitable, constant values. Typical values of the parameters are 0.023, 0.8 and 0.4 respectively, these being the values most
often used in practice, and which were postulated by McAdams (Ref. K2.) as described in part 2.3(iii). Despite the confidence of each worker in his own experimental data, and this was expressed as a tolerance on \( \text{Nu}_\infty \) number of between 2.5 and 15%, the suggested values of the parameters \( B, m \) and \( n \) varied between sources as follows -

\[
B = 0.018\text{ to } 0.031 \\
m = 0.77\text{ to } 0.87 \\
n = 0.333\text{ to } 0.420
\]

Two recent (1964) papers by Allen and Eckert (Ref. B10.) and Malina and Sparrow (Ref. B11.) commented on the results of almost identical experiments with water in long, electrically heated tubes. The precision obtained was undoubtedly very good, the measurements of coefficients of heat transfer, \( h_\infty \), being repeatable within a 2% tolerance. The possibility of the variation in the physical properties of water with temperature affecting \( \text{Nu}_\infty \) was considered, even though such effects would have probably been small. For a given \( \text{Re} \) and \( \text{Pr} \) the ratio \( (\text{Nu}_\infty)/(0.023, \text{Re}^{0.8}, \text{Pr}^{0.4}) \) was plotted against the temperature difference employed in calculating \( h \), that is \( (t_w - t_b) \). Hence, departures of the experimental \( \text{Nu}_\infty \) from the McAdams equation were indicated, and the effects of property variations could be eliminated by repeating the measurements for different values of the heat flux, then extrapolating the data to the 'zero heat-flux' condition. Figure 3.1 illustrates this procedure with the results of both Allen and Malina. The way in which the ratio \( (\text{Nu}_\infty)/(0.023, \text{Re}^{0.8}, \text{Pr}^{0.4}) \) varies with \( \text{Re} \) is also shown; the work of Allen was based on water with \( \text{Pr} = 8 \), and that of Malina with \( \text{Pr} = 3 \). In Allen's results the equation of McAdams is shown to underestimate \( \text{Nu}_\infty \) by 11% at \( \text{Re} = 10,000 \) increasing to 20% at \( \text{Re} = 100,000 \). In Malina's results the underestimation increases from 2% to 11% in the same range of \( \text{Re} \).
In conclusion, the dependence of $N_u$ on $Re$, as given by 
the McAdams equation, must be inadequate, furthermore there is 
some indication that the dependence on $Pr$ is unreliable. The 
accuracy of the McAdams equation in the calculation of $N_u$ for 
water can be expressed as a 20\% tolerance.

In 1961 Ede (Ref. A.1.) recorded values of $N_u$ for both 
water and air. The data was derived whilst conducting various 
experiments on more complicated configurations and with tubes 
having a range of diameter. The tubes were heated electrically, 
and particular emphasis was placed on the accuracy of thermocouple, 
flowmeter, and heat flux measurements. Small temperature differences 
were used in order to minimise variations in the properties of the 
fluids. Similar observations were made regarding the reliability 
of the McAdams formula as were made later on by Allen and by Malina. 
For water, the ratio $(N_u) / (0.023 \, Re^{0.8} \, Pr^{0.4})$ increased with $Re$ 
from approximately 1.0 at $Re = 10,000$ to 1.1 at $Re = 100,000$. The 
Prandtl number of the water was nominally 8. The results for air 
were somewhat different, in that the McAdams formula overestimated 
$N_u$ by approximately 20\% to 25\% in the same range of $Re$. Clearly 
the $Pr$ dependence as indicated by McAdams is unsatisfactory where 
accurate values of $N_u$ are required. Ede showed that the theoretical 
formula of Martinelli (described in 2.3(iii)) gave a much better 
indication of the $Pr$ dependence of $N_u$, but even this was far 
from satisfactory. The results of Ede are shown in Figure 3.2.

It is concluded that any further research into forced 
convection in long tubes with 'non-viscous' fluids would not be 
particularly productive at this time.

3.2(ii) VISCOUS FLUIDS.

From the early researches, discussed in 2.3(iii), two
empirical relationships emerged for correlating data on turbulent Nu with viscous liquids. For a fluid having constant physical properties they both look very similar. The McAdams equation was

$$\text{Nu}_\infty = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4},$$

and the Sieder and Tate equation (which assumes constant viscosity in this discussion)

$$\text{Nu}_\infty = 0.027 \text{Re}^{0.8} \text{Pr}^{3}.$$

In 1955 Hartnett (Ref. B2.) used a long, electrically heated tube, to measure coefficients of heat transfer for an oil with Pr in the range 60 to 200. The maximum experimental error in measuring Nu was assessed at 10%. The values obtained for Nu were approximately 10% higher than the Sieder and Tate equation predicts at Re = 50,000, but the difference reduced to 0% at Re = 10,000. The McAdams equation overestimated Nu by up to 20% approximately, and was found unsuitable. The viscosity of the oil did vary considerably with temperature in these tests, but Nu was corrected to give the equivalent constant property value with the 'Sieder and Tate factor' ($\text{Nu}_{\text{WALL}}^{0.14}$ — as discussed in 2.3(iii). Since no attempt was made to justify this procedure, there is an inherent weakness in the arguments presented in this research, though the reliability of the results was probably not seriously impaired.

Friend and Metzner (1958) pointed out that simple equations of the form $$\text{Nu}_\infty = B \text{Re}^m \text{Pr}^n$$ may not be satisfactory for correlating experimental data when Pr varies over a wide range of values, and that no satisfactory experimental or theoretical analysis existed at that time to justify such a supposition. Experiments were conducted to measure Nu with a steam-heated tube, and a number of sugar-based fluids were used to provide a wide range of Pr (Ref. A4.) The Prandtl number of the sugar solutions was shown to be only a
weak function of temperature in these experiments. Although the reliability of the experimental results was not good (about 20% tolerance on Nu∞) the work was interesting because it showed how Nu∞ could vary with Pr whilst Re remained constant. With Re = 10,000, Pr was varied from approximately 50 to 600. The results are shown in Figure 3. B. Two equations were given to represent the experimental data, the first being a rather crude, semi-theoretical formula which showed no improvement over the second, more conventional form of equation: \( \text{Nu}_{∞} = 0.022 \text{Re}^{0.8} \text{Pr}^{0.42} \).

The exponent of Re was never justified experimentally, and since the reliability of the correlation, with respect to the Pr term, can be shown to be

\[ \frac{\Delta \text{Nu}}{\text{Nu}} = \Delta n \log \left( \frac{\text{Pr}_{\text{max}}}{\text{Pr}_{\text{min}}} \right), \]

where \( \Delta n \) is error in Pr exponent, \( \Delta \text{Nu} \) is error in Nu,

a reduction of 20% in the exponent 0.42 (or 0.33) would be equivalent to 21% variation in Nu∞, which is about the same as the reliability of the results. The insensitivity of Nu∞ to the Pr exponent is evident from this calculation.

In 1964 Malina and Sparrow (Ref. 21.) extended their experiments with water (See 3.2(i)) to include oils with Prandtl numbers 48 and 75. The ratio of \( \left( \text{Nu}_{∞}/0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \right) \) was determined for the 'zero heat-flux' condition over a range of Re. With Pr = 48, the ratio was found to vary from 0.94 at Re ≈ 14,000, to 1.11 at Re = 43,000. The values for Pr = 75 were 0.97 at Re ≈ 12,000, and 1.07 at Re ≈ 28,000. Hence the ratio increased with Re and the rate of increase was greater for higher Prandtl numbers. It appears that a suitable form of expression for calculating Nu would be

\[ \text{Nu}_{∞} = B \text{Re}^{\gamma(\text{Pr})} \text{Pr}^{\gamma(\text{Re})}. \]
Sufficient experimental data has been provided to enable the value of Nu_\infty to be estimated with viscous fluids, and little purpose would be served by pursuing this line of research. A possible exception to this statement would be in the transitional flow regime where experimental data is sparse.

As a final comment it is suggested that when equations of the McAdams kind are utilised, the exponent of Re should certainly be greater than the value 0.6 - which is encountered frequently in the heat-transfer literature - when viscous fluids are to be considered. Furthermore, a suitable value for the exponent of Pr has not been satisfactorily established; although typical values of 0.4 and 0.5 to appear equally reliable. The latter follows because few experimenters have utilised highly viscous fluids, and a simple exponential relationship does not necessarily provide an adequate means of correlating the experimental parameters.

3.3. THE SHORT STRAIGHT TUBE

3.3.1 'NON-VISCOUS' FLUIDS.

The measurement of local coefficients of heat transfer in the entrance region of tubes has been the objective of many experiments following the early works of Cholette (Ref. B14.) and Boelter (Ref. H15.). Particular attention has been given to gaseous and aqueous media, and the flow has nearly always been turbulent. References (B1, B2, B3, B4, B5, B6, B7, B8, B9, B10, B11, B12, L2, E1, E11) relate to such experiments.

For 'non-viscous' fluids in the laminar regime few data are reported on local coefficients of heat transfer. Apart from the experiments of Kays and of Kroll (Ref. L1.), most of the research effort has been concerned with 'secondary effects' such as the influence of free convection - references B7, B12, E1,
fall into this category.

In 1953 Kays (Ref: C1) carried out a theoretical analysis of heat transfer to air in short tubes, with undeveloped flow at the inlet. To support the theory a comparison was made with empirical data. These were derived with steam heated and electrically heated tubes. The results were presented as a graph of $\text{Nu versus } (\text{RePr})/(\pi/2r_w)$, as in Figure 3.4.

The local Nusselt number was greater for uniform heating than for uniform temperature at the tube wall, the extent being 20% to 40%.

At small axial distances $\text{Nu}$ for the 'undeveloped' case exceeded the 'fully developed' values by about 40%, but this difference diminished with increasing axial distance.

The results of Kays were applicable only to fluids with $Pr = 1$. In such fluids the temperature profile develops at a similar rate to the velocity profile, so one would expect the difference between the 'developed' and 'undeveloped' $\text{Nu}$ to become less than stated as the Prandtl number increases above unity.

In 1966 McComas and Eckart (Ref: E.1) reported the results of experiments on electrically heated horizontal tubes with a long calming length. Local coefficients of heat transfer were obtained for a range of $Re$ from 100 to 900, with air as the medium. Figure 3.5 shows $\text{Nu versus distance}$ from the onset of heating with $Re \approx 740$ and 220. With the lower Reynolds number, $\text{Nu}$ reduced rapidly with increasing distance to a minimum value, which was of the same order as the fully developed Nusselt number 4.36. This minimum was reached >60 diameters downstream with $Re \approx 740$, and 5 to 10 diameters with $Re \approx 220$. Further downstream with $Re \geq 220$, the value of $\text{Nu}$ increased with distance, the rate of increase depending on the heat-flux imposed (this is indicated by
the value of $Gr$ in figure 3.5). The latter phenomenon was attributed to the secondary effects of free convection (as subject discussed later in this section).

The considerable influence of free convection on the coefficient of heat transfer – for laminar forced flow of 'non-viscous' fluids – makes data for the case of pure forced convection of doubtful practical value, except where fine-bore tubes are encountered. For this reason, the objectives of researchers in this area have centred mainly on the 'mixed' convection process.

In turbulent flow, local measurements have been obtained with gases and water. References B.1, B.2, B.3, B.5, B.8, B.9, B.10, B.11, and B.12 report results of this kind.

Mills (Ref: B.12) measured local values of $h$ for air passing through a brass tube, heated with an electrical winding. Various entrance configurations were investigated, including a long calming length upstream, and a bellmouth entrance. Figure 3.6 summarises these results. The local coefficient of heat transfer, $h$, with the calming length, reduced from a high initial value to a constant limiting value in the first 6 diameters downstream, and this development length did not appear to vary significantly with $Re$ in the range 16,670 to 102,600. The value of $h$ at 1 diameter downstream was 1.8 times the limiting value $h_\infty$. With the bellmouth fitted the $h$-functions were found to be quite different. Initially, $h$ was found to reduce much more rapidly than with the calming length, until a minimum was reached which was less than $h_\infty$. This initial length reduced from $\frac{1}{2}$ diameters at $Re = 10,240$, to $\frac{1}{2}$ diameters at $Re = 107,800$. Further downstream, $h$ increased with distance until the limiting value $h_\infty$ was reached. The overall
effect of fitting the bellmouth was to increase the length of the development region, and reduce the 'average' value of $h$ in this region of the tube. The characteristic 'dip' in the latter functions was explained by Mills as being caused by the transition from laminar to turbulent flow at the tube-inlet, the onset of turbulence occurring at the minimum value of $h$.

Wolf (Ref: B.8) measured values of $h$ for air and CO$_2$ in 3 tubes which were heated electrically. A calming length was fitted upstream, and large temperature differences (tube-gas) were utilised ($> 200K$). The physical properties of the gases varied substantially with temperature. To facilitate the comparison of the results obtained, the values of Nu measured were corrected to give equivalent values for particular, nominal Reynolds numbers. A further correction was made to account for the differences in $(t_w - t_b)$ between measurements, which included an allowance for variations in the physical properties. For this purpose the parameter $\beta^*$ was derived, as a measure of the intensity of heat transfer, and Nu was corrected to nominal values of $\beta^*$. (The reader is referred to the original text for details. Variable properties are considered later in part 3.4). Figure 3.7 shows $Nu_b^{R\beta}$ (the corrected value of Nu) versus (axial distance) for air and CO$_2$. The general shape of these functions is similar to those measured by Mills, but the value of $Nu_b^{R\beta}$ continues to reduce gradually at large distances. This was attributed to the increase in thermal conductivity of the gas with increasing bulk temperature. A reduction in the rate of heat transfer (as indicated by a proportional reduction in $\beta^*$) caused the rate of change of $Nu_b^{R\beta}$ with distance to increase, although the magnitude at any position was greater (mainly because properties were evaluated at the local
bulk temperature).

With water as the entrained fluid, most of the works have considered moderate temperature differences \((t_w - t_b)\), and the secondary effects of free convection and property variations in some of these cases has been found negligible. References B.1, B.2, B.3 and B.5 fall into this category.

In 1955 Aladev (Ref: B.5) measured \(h\) for a horizontal tube enclosed in a steam jacket. The conditions at the tube-entrance were not well defined, being something between fully developed and undeveloped. The results of these experiments, although crude, have been quoted in several standard text books. After taking measurements for a range of \(Re\), unique functions of \((Nu/Pr^{0.4})\) versus \((Re)\) were plotted for particular axial locations and from these results graphs of \((h/h_{\infty})\) versus \((distance)\) were derived for selected Reynolds numbers - as shown in figure 3.8(i). The magnitude of \((h/h_{\infty})\), at distances of the order \(\frac{1}{2}\) diameters, can be seen to reduce appreciably as \(Re\) is increased, from a ratio of 2 at \(Re = 10,000\) to a ratio of \(\frac{1}{2}\) at \(Re = 100,000\).

In 1955 Hartnett (Ref: B.2) presented data for water, but in these experiments electrical heating was applied to a vertical tube having a long calming length. Figure 3.8(ii) shows a comparison with the results of Aladev. The temperature development length was much shorter than given by the latter, and the measured values of \((h/h_{\infty})\) in the entrance region were much lower. The experiments of Hartnett were not so crude as those of Aladev and are probably more reliable, but the effects of the different inlet conditions might well account for some of the disparity between the results.

A comparison can be made between the values of \(h\) - in a short tube with calming length - for gases and for water, by
referring to the results of Mills (Fig. 3.6) and Hartnett (Fig. 3.8(ii)). Although the thermal entrance length was 10 to 15 diameters in both cases, the ratio \( h/h_\infty \) at 0.5 diameters according to Mills was 1.8 to 1.6 with \( Re = 16,670 \) to 102,600, and with water the ratio was 1.5 to 1.2 with \( Re = 16,900 \) to 44,300. A substantial difference is indicated between the \( (h) \) versus (distance) functions for water and gas.

In 1965 Stone (Ref: B.3) conducted a detailed investigation of \( h \) in the entrance region of an electrically heated tube, with water as the fluid. Various complex methods of correlating the experimental data were attempted, but his general conclusions were that a unique function of \( (h/h_\infty) \) versus (distance) could be specified, independent of \( Re \) (but not \( Pr \)) representing his results with a maximum error of \( \pm 20\% \). The range of \( Re \) encompassed was 10,000 to 100,000. The shape of this function was shown to be highly dependent on the length of the unheated region upstream, and figure 3.9 illustrates these findings. Considering the substantial deviations permissible with the Stone \( (h/h_\infty) \) functions, the results of Hartnett (Fig. 3.8(ii)) are in reasonable agreement, the majority of the latter data falling within 0 and \(-20\%\). It seems probable that the correlations of Stone should have incorporated the effects of \( Re \) in a different way. It is clear from figure 3.9 that \( (h/h_\infty) \) could be increased appreciably by shortening the calming length, and a comparison with the results of Aladev (Fig. 3.8(i)) indicates the likely reason for the disparity between the latter data and that of Hartnett.

In 1964 Malina (Ref: B.1) produced experimental data of \( (h/h_\infty) \) for water in short tubes, heated electrically. The limited range of results was derived meticulously, and good repeatability was obtained. Figure 3.10 shows \( (h/h_\infty) \) versus (distance) for
Re = 14,500 to 101,300. The results are similar to those of Hartnett (Ref: B.2), the thermal entrance lengths were slightly shorter and the magnitude of \( (h/h_w) \) was in reasonable agreement at all values of Re.

3.3(ii) 'Viscous' Liquids.

It was clear from a survey of a wide number of sources that only a small amount of experimental data was available on local heat transfer in the entrance-region of a tube, for high Prandtl number fluids.

In the turbulent regime, Hartnett (Ref: B.2) used an electrically heated tube with a long calming length to determine the distribution of \( h \) for oil. The range of Prandtl number covered by the tests was 60 to 460, and the Reynolds number varied from 1,600 to 47,000. The maximum Re attainable with the apparatus was limited by the available pressure drop to 2,000 for \( Pr = 480 \), 10,000 for \( Pr = 206 \), 25,000 for \( Pr = 107 \) and 47,000 for \( Pr = 61 \).

The limited amount of experimental data derived by Hartnett is shown in figure 3.20, where \( (h/h_w) \) is plotted versus axial distance. These results may be compared to the data obtained with water by Hartnett (discussed earlier) which are given in figure 3.8(ii). It can be concluded that the effect of increasing \( Pr \) was to reduce the value of \( (h/h_w) \) in the entrance-region; although the distance downstream at which the fully developed value of 1.00 was reached was apparently independent of \( Pr \).

Another limited investigation of the short tube was carried out by Kalina (Ref: B.1) whose experiments have been described earlier (Parts 3.2(i) and 3.3(i)) as well defined and reliable. Prandtl numbers of 3 (water), 48 and 75 (oil) were
considered, and the different values of \( \frac{h}{h_\infty} \) measured at small axial distances are compared in figure 3.10. The values of \( \frac{h}{h_\infty} \) were slightly lower than those reported by Hartnett, but were in qualitative agreement. No comparison could be made below \( Re = 12,000 \) since Malina did not present data in this range. The length of the development region was approximately the same from both sources.

Malina pointed out that the values of \( h \) presented were up to \( \frac{3}{4} \% \) greater than would have been obtained with a fluid having constant physical properties. In general the results indicated that the value of \( h \) tended to the limiting \( h_\infty \) more rapidly as \( Re \) was increased for a given \( Pr \). It was further shown that the development was more rapid as \( Pr \) was increased for a given \( Re \).

In practice, it is probable that transitional and turbulent flows having low \( Re \), in the range \( Re = 2,500 \) to \( 15,000 \) (say), will occur frequently because of limitations on the available pressure drop when handling a viscous medium. This is also the region in which the thermal development at the entrance of a tube is slowest, and local measurements most beneficial to the designer. It can be concluded that no satisfactory experimental work has been carried out for such conditions. Undoubtedly, the reason for avoiding the transitional Reynolds numbers is the difficulty which might occur in conducting experiments with an unstable flow regime. However, it is not always possible to select a satisfactory design point in this respect, and experimental data would be valuable. A few results presented by Hartnett (figure 3.20) give some credence to this argument.

With laminar flows the experimental data available was limited, and such data as exists has usually been obtained in order to justify a theoretical analysis, or to provide a method of determining the way in which the secondary effects of free convection
and viscosity-temperature dependence would modify the value of
Nu estimated for a system in which no such secondary effects arise.
It was decided to classify the references consulted in terms of
the main objective of the work, e.g. the effects of free convection
on the forced convective process. These references will be
discussed in parts 3.4. to 3.6.

3.4. VARIABLE FLUID PROPERTIES.

The rate of heat transfer to a piped fluid is likely to
influence the shape of the axial velocity profile, and therefore
must have some effect on the Nusselt number. In a gaseous medium
the properties $\rho, K$ and $\mu$ could vary appreciably with the radial
temperature distribution at high heat fluxes. For forced convection
in a viscous liquid the most significant variations are concerned
with the radial changes in viscosity with temperature. In general
the analysis of problems of gaseous heat transfer with property-
variations has differed from the case of liquids, accordingly it has
been decided to review only those works concerned with liquid media.
References C.2, C.3, D.6, D.7, D.9 and J.6 discuss methods of
estimating Nu for gases with variable properties.

In the turbulent regime few authors have attempted to
determine the effect of viscosity-variations on the Nusselt number.
Hartnett (Ref: B.2) and Davies (Ref: B.6) are examples of authors
who have attempted to compensate for viscosity-variations (with
water and oil) by using a relationship of the kind:

$$Nu_{\text{variable}} = \left( \frac{\nu_{\text{W}}}{\nu_{\text{G}}} \right)^{0.14} Nu_{\text{constant}}$$

which was proposed by Sieder and Tate (Ref: D.1) as discussed
previously. This equation was not justified by the users since
the viscosity corrections made were small. Aladov attempted to derive a similar method for correlating experimental data obtained with water and arrived at the following expression:

\[
\frac{\text{Nu}_\infty \text{ (variable)}}{\text{Nu}_\infty \text{ (constant)}} = \left(\frac{\text{Pr}_b}{\text{Pr}_w}\right)^{\frac{1}{2}} \times \left(\frac{\mu_b}{\mu_w}\right)^{\frac{1}{2}}.
\]

The expression was used primarily to account for the apparent differences in Nu obtained with heating and cooling. The range of \(\left(\frac{\text{Pr}_b}{\text{Pr}_w}\right)^{\frac{1}{2}}\) was approximately 0.9 to 1.1.

Allen (Ref: B.10) carried out experiments heating water, and showed that the correction factor \(\left(\frac{\mu_b}{\mu_w}\right)^{0.14}\) was valid only with \(\text{Re} = 0 \left(10^5\right)\). The limited amount of data presented indicated that a more suitable correction factor would be \(\left(\frac{\mu_b}{\mu_w}\right)^{f(\text{Re})}\)

where \(0 \rightarrow f(\text{Re}) \rightarrow 0.14\) as \(10^4 \rightarrow \text{Re} \rightarrow 10^5\). The ratio \(\left(\text{Nu}_\infty \text{ (variable)} / \text{Nu}_\infty \text{ (constant)}\right)\) never exceeded 1.15, and a greater range of values might have demonstrated that part of the correction could have been absorbed by limitations on the experimental method.

The results of Malina (Ref:B.1), which were obtained with water and oil \(\text{Pr} = 48\) and 75), showed a similar trend to those of Allen over a comparable range of Re. A correction factor was proposed of the kind \(\left(\frac{\mu_b}{\mu_w}\right)^n\) to compensate for the effect of viscosity variations on Nu. The value of \(n\) was shown to be in the range 0.05 to 0.08 for most of the experimental data, and the value 0.05 was recommended. Once again the variations in viscosity were small, however, and the ratio \(\left(\text{Nu}_\infty \text{ (variable)} / \text{Nu}_\infty \text{ (constant)}\right)\) never exceeded 1.1.

No experimental evidence could be found which enabled the effects of viscosity-variations on Nu in the entrance-region to be determined.
With laminar flow the effects of viscosity-variations on $Nu$ are amenable to mathematical analysis. Yang (Ref: D.3) solved a simplified version of the boundary-layer equations for uniform heat-flux, with fully developed flow at the inlet of the tube, and viscosity dependent on temperature. The equations were solved analytically using an iterative integral technique. The complicated form of the results make it difficult to draw general conclusions, but the ratio of $(Nu$ with variable viscosity)/(Nu with constant viscosity) was of approximately the same magnitude as $(\mu / \mu_0)^{0.14}$. The value of $Nu$ tended to the constant property value at small axial distances but increased above the latter (for heating) with increasing distance. The value of $Nu$ was raised as the heat flux increased, or as the rate of change of $\mu$ with temperature increased. $(1/\mu)$ was assumed to be linear in temperature.

Rosenberg (Ref: D.8) solved the boundary-layer equations numerically for laminar flow in a tube, with the constant wall temperature condition. The inertial terms were retained in the solution because the flow conditions considered were undeveloped and developed at the inlet. The viscosity was related to temperature by one of two equations viz:

$$\mu = (C_1 + C_2 T)^{-1},$$

or

$$\log (\mu) = C_3 T + C_4.$$  

($C$'s are constant).

Results were calculated for a range of $Pr$ up to 1,000, and it was shown that the flow became 'fully developed' in a very short axial distance when viscous fluids were considered. The viscosity dependence of $Nu$ was shown to be mainly a function of $(\mu / \mu_0)$, but a weak function of $(x/x_w)/(RePr)$ — $\mu_0 = \mu$ at inlet $\sim \mu_b$.  

A comparison with the relationship:

$$\text{Nu}_m \text{ (variable viscosity)} = (\mu_L/\mu_w)^{0.14} \text{ Nu}_m \text{ (variable viscosity)}$$

(proposed by Sieder and Tate) demonstrated that the above expression enabled $\text{Nu}_m$ to be estimated for a short pipe within $2\%$ of the calculated result, in the range $1 < (\mu_L/\mu_w) < 100$. The magnitude of $\text{Nu}_m$ was consistently $15\%$ less than the empirical equation proposed by Sieder and Tate, but this has not been considered in assessing the effects of viscosity ratio.

Test (Ref: D.5.) analyzed the same problem as was discussed by Rosenberg using a similar numerical technique but the initial equations were more general. The effects of several terms in the energy and momentum equations on the local Nusselt number were evaluated before eliminating them as being small and obtaining the final solutions. The equation of state for viscosity was:

$$\log_{10} \mu_L \log_{10} (\mu_L/\mu_w + 0.8) = 9.1 - 3.16 \log_{10} T.$$  

Although mathematically rigorous, the results could not be generalized easily, and the increased complexity of the solution led to large truncation errors. The local Nusselt numbers calculated were compared with the results of some experimental data derived from a steam-heated tube with a calming length upstream. The entrained fluid was oil. The relationship (for local values) between $\text{Nu}$ and $\mu_L/\mu_w$ was ascertained and the Sieder and Tate correction was found to overestimate $\text{Nu}$ considerably. The following relationship was proposed:

$$\text{Nu (variable } \mu) = \text{Nu (constant } \mu) \cdot (\mu_L/\mu_w)^{0.05}.$$  

Insufficient data was presented to determine the magnitudes of the viscosity-ratio which were investigated.

Shannon (Ref: D.10) combined an experimental and analytical investigation of laminar heat transfer with variable viscosity. The flow was fully developed at the inlet, and a viscous liquid,
ethylene glycol, was used as the medium. In this analysis uniform heat-flux was imposed by electrical heating of the tube. The range of Re was 6 to 300, and Pr varied from 26 to 500. The experimental data was not presented, and only a vague description of the numerical method is discussed. The conclusions were that experiment agreed well with theory, and that the effects of viscosity variations could be estimated as follows:

For a liquid with viscosity $\mu = \mu_{\text{ref}} \exp(-\text{constant}(T-T_{\text{ref}}))$,

or for the case $\mu = \mu$ (ethylene glycol),

$$Nu (\text{variable } \mu) = Nu (\text{constant } \mu) (\frac{\mu}{\mu_{\text{ref}}})^m,$$

where $m = 0.14$ at large axial distances, but increases to 0.3 at small axial distances. The axial distance referred to was a dimensionless distance defined by $(x/d)/(RePr)$. In the numerical work $(\mu_0/\mu_w)$ had a maximum value of 100, whilst in the experiments the maximum value attained was $\text{3.}$

The magnitude of Nu obtained experimentally consistently underestimated the theoretical results by $0 - 15\%$.

Petukhov (Ref: D.2) investigated the case of undeveloped laminar flow of viscous liquid in a constant temperature tube. Experiments were carried out for heating oil which had a variable viscosity, and the main parameters were in the following ranges: $44 < Re < 2,100, \ 130 < Pr < 3,900, \ 1 < (\mu/\mu_{\text{inlet}}) < 12.5$ (where $\mu_0 =$ inlet value $\mu_0/\mu_a$). The dependence of $Nu$ on $(\mu/\mu_w)$ was compared with the equation of Sieder and Tate and the latter was found to be in reasonable agreement with the final proposal that

$$Nu (\text{variable}) = Nu (\text{constant}) (\frac{\mu}{\mu_w})^{1/6}.$$

The difference between the exponents 0.14 and 1/6 was small, and both were equally reliable for the range of the parameters stated above.
The local values of Nusselt number were correlated using the similar equation:

\[ \text{Nu (variable)} = \text{Nu (constant)} \left( \frac{U}{\nu_f / \mu} \right)^{1/6} \]

The experimental data used to arrive at the above expressions showed a maximum deviation from the estimated Nu of ±15% of Nusselt number.

The preceding discussion indicated that there is still considerable disagreement between sources as to the effects of the viscosity-temperature relationship on the local Nusselt number for both turbulent and laminar flow of liquid in heated tubes.

It should be pointed out here that quite different expressions to those discussed have been proposed for the related problem of cooling liquids in tubes, a subject not directly relevant to the present work.

3.5. FREE CONVECTONAL EFFECTS IN HORIZONTAL TUBES.

The effects of free convection on the heat-transfer coefficient in forced laminar flow can be appreciable whether the tube is horizontal, inclined or vertical. Several investigations have been carried out for the first and last cases, but the horizontal tube is probably the most common configuration in practice as well as being the least amenable to analysis, and therefore consideration has been given to this in these experiments.

The buoyancy forces in a horizontal tube are transverse to the direction of pumped flow primarily, but in a vertical tube the forced and free components are co-axial either aiding or opposing each other depending on the direction of the flow. In forming correlations for the value of Nusselt number, in horizontal flow, attempts have been made at drawing an analogy with expressions proposed for the simpler case of vertical flow (usually with no
sound basis.)

References E.4, E.5, E.13 and E.16 to E.22, deal with experimental and theoretical values for the Nusselt number in a vertical tube. No further comment will be made on these works since the problem was considered to be fundamentally different and discussion would tend to confuse rather than simplify the issue. An exception to the latter statement is the work of Martinelli (Ref. E.4) who determined values for $\text{Nu}_m$ using a combined theoretical and empirical approach.

The main result of Martinelli will simply be stated. An equation was derived for heated flow travelling upwards in a tube as follows:

$$\text{Nu}_m = 1.75 F_1 \left[ \frac{(\pi \text{Re}_m \text{Pr}_m D)}{4L} + 0.0722 \text{Pr}_m (\text{Gr}_w \text{Pr}_m D/L)^{\frac{2}{3}} \right]^{\frac{1}{3}}$$

where $F_1$ is a correction factor which allows arithmetic rather than logarithmic temperature differences to be used, and $F_2$ is a function which compensates for the reduction in buoyancy forces, in the axial direction, as the bulk temperature increases. The forced and buoyant terms are seen to be additive, the net result being that $\text{Nu}_m$ increases above the normal 'forced' value as the free convection parameter $\text{Gr}_w$ ($w$ = properties at wall) increases.

Aladev (Ref. E.11) attempted to find an equation for $\text{Nu}_m$, with $\text{Re} < 2,300$, which would satisfy a horizontal or vertical tube carrying water. The experimental analysis was fairly crude and applicable only to a tube 60 diameters long. The condition of uniform wall temperature was approximated. The form of the equation for the horizontal case reduced to:

$$\text{Nu}_m = 0.74 (\text{Re}_m^{0.2} \text{Gr}_m^{0.1} \text{Pr}_m^{0.3})$$

The form of the expression must be discounted because it demonstrates
that the contribution of free convection to \( \text{Nu}_m \) is the same for all values of \( \text{Re}_m \). (This criticism was made by Kern of the Colburn equation. Part 2.4.(ii)).

Other workers have used water as the medium. Martin (Ref: E.15) considered the local parameter \( \text{Nu} \) in the entrance-region of a constant-temperature tube with developed and undeveloped flow at the inlet. A slightly modified version of the Colburn equation was found to represent the experimental data adequately, viz: \( \text{Nu} = \text{Nu}_0 \left(1 + 0.127 \text{Gr}^{0.145}\right) \) for \( \text{Gr} > 10^7 \).

\( \text{Nu}_0 = \text{Nu} \) when \( \beta = 0 \).

Once again the equation is unlikely to be valid for conditions outside the range of the experiments, there being no consideration of tube length, \( \text{Pr} \) or \( \text{Re} \) in the free convective factor.

Jackson (Ref: E.14) experimented with air in a horizontal, steam heated tube. The correlation of results was based on the following expression:

\[
\text{Nu}_m = 2.67 \left[ \left(\pi \text{Re}_m \text{Pr}_m D/4L\right)^2 + (0.0087)^2 \left(\text{Gr}_w \text{Pr}_w\right)^{3/2} \right]^{1/6}
\]

for the laminar regime, which was based on the work of Martinelli and represented a vectorial addition of the forced and free convective components in such a way that the asymptotic cases of pure forced flow, and pure free flow, could be accommodated. Hence the omission of \( L \) in the last term. The weakness in this expression is that the thickness of the temperature layer increases with length when forced flow predominates, and therefore the free convective term (a function of the rate of secondary recirculation) must contain a length dependent parameter for such conditions. It is difficult to envisage any simple addition of forced and free components, such as the one postulated, which could satisfy both the asymptotic cases and also the intermediate states, when both
components are significant.

Ede (Ref: A.1) whose experiments, for water in long tubes with uniform heat-flux, have already been discussed, considered the case when \( \text{Nu} \rightarrow \text{Nu}_\infty \). The value of \( \text{Nu}_\infty \) in laminar forced flow is 4.36 and was therefore independent of length. For water with \( \text{Pr} \sim 8 \), the values of \( \text{Nu}_\infty \) with free convection were found to agree reasonably with

\[
\text{Nu}_\infty = 4.36 \left( 1 + 0.06 \, \text{Gr}_\infty^{0.3} \right).
\]

The expression was considered valid for the particular fluid, and was restricted to the case when forced flow predominated with \( \text{Gr}_\infty < 10^7 \). The data are presented in figure 3.2.

Brown (Ref: E.6) considered the case of water in a tube at approximately constant wall temperature. The significant groups of parameters used in correlating values of \( \text{Nu}_m \) were slightly different than had been hypothesized earlier. It was assumed that

\[
\text{Nu}_m = f \left( \text{Re}_m, \text{Pr}_m, \frac{D}{L}, \text{Gr}_m \right).
\]

The approach to the problem was influenced by the works discussed previously, but the analysis was largely empirical. The philosophy behind the final form of solution for the problem is not at all clear, but the best fit to the experimental data was found to be given by:

\[
\text{Nu}_m = 1.75 \left( \frac{\nu}{\nu_w} \right)^{0.14} \left[ \left( \text{Re}_m \, \text{Pr}_m \, \frac{D}{L} / 4 \right) + 0.012 \left( \text{Pr}_m \, \frac{D}{L} / 4 \right) \text{Gr}_m^{3/4} / \text{Pr}_m^{1/4} \right]^{3/4}.
\]

The introduction of \( \text{Re} \) into the free convective term was not adequately explained, and it is difficult to see how the free component of \( \text{Nu}_m \) could increase with \( \text{Re}_m \) for a fixed value of \( \text{Gr}_m \).

Oliver (Ref: E.3) carried out experiments on mixed convection with forced flow predominating, in a tube heated by a water jacket. The heat-transfer liquids were glycerol-water, ethyl alcohol, and water. A resulting equation for estimating
\( \text{Nu}_m \) was stated thus:

\[
\text{Nu}_m = 1.75 \left( \lambda_r / \lambda_w \right)^{0.14} \left[ \left( \text{Re}_m \text{Pr}_m D / 4L \right) + 0.00056 \left( \text{Gr}_m \text{Pr}_m L / D \right)^{0.7} \right]^{3/2}
\]

After surveying some of the works already discussed, Oliver showed that the experimental data was correlated more reliably, and agreed more closely with previous data, if the term containing \( \text{Gr}_m \) also included tube length. When the length was incorporated, as above, the intention was to demonstrate that the effects of free convection were more pronounced in long tubes than in short tubes. This had not been considered in any of the earlier equations, where the effect of increased length, on the free contribution to \( \text{Nu}_m \), was either not apparent or caused a reduction in magnitude.

A novel approach at correlating the experimental parameters in mixed convection was discussed by Shannon (Refs: E.7 and D.10). Tests were carried out with water at the ice-point, and with ethylene glycol. An electrically heated test section was used, and local measurements of Nu were made. When plotting the experimental values of Nu versus distance, Shannon noted that the effects of free convection on Nu became more pronounced at large axial distances and hence the free convective contribution to Nu increased as Nu reduced. It was further assumed that with low flows and high heat-fluxes, the value of Nu would tend to be proportional to \( (\text{GrPr})^{1/4} \). The latter is typical of laminar natural convective systems, but Shannon was influenced by reference J.10 in particular. The hypothesis was made that

\[
(\text{Nu} - \text{Nu}_0) = f \left( \frac{(\text{GrPr})^{1/4}}{\text{Nu}_0} \right)
\]

where \( \text{Nu}_0 \) = the value of Nu with \( \chi = 0 \).

The value of \( f \) was found to be zero when the independent variable was less than 2, but a very considerable amount of 'scatter' was
apparent when an attempt was made to correlate the experimental results graphically, as above.

It was found that widely different arguments had been presented in an attempt to define how Nu could be estimated for horizontal tubes, in the case of forced convection with some free convection superimposed. No single approach to the subject was shown to be reliable outside the experimental range, the logic behind the proposed equations was seldom acceptable, and very large errors, possibly of the order 100%, could be incurred when utilizing any of the equations hypothesized above. A detailed review, and analysis of the literature, would perhaps lead to a better understanding of the problem, but the latter was not a major objective of this study. It was decided to rationalize on the problem without being influenced by previous analyses, if and when the need arose during the work of this author.

Some theoretical investigations have been conducted which show a continuing development towards the state when all the important groups of parameters required to define Nu, with mixed convection, are known; even if a formal solution of the equations is not obtained. The theoretical approach is likely to prove extremely useful in determining the solutions to problems which have apparently eluded the empiricists.

Morton (Ref: E.8) sought a solution to the problem of mixed convection in a long tube with uniform heat-flux. Forced laminar flow predominated so that $Nu_\infty = 4.36$, with no secondary flow. The energy and momentum equations were solved using a perturbation series which was valid for small values of the group $(Re Ra)_\infty$. The result was considered accurate when $(Re Ra)_\infty < 3,000$, which meant $Nu_\infty$ was modified by 10% at the most. The parameter Ra
had a peculiar, impractical definition which can be reduced into a more recognisable form to give:

\[
\text{Re Ra}_\infty = \text{Gr Pr} \frac{r_w}{(dt_w/dx)} / (8 (t_w - t_b) \text{Nu}_\infty / 4).
\]

Hence the result:

\[
\text{Nu}_\infty = 4.36 \left(1 + f_1 \text{Pr} \right) \left(\text{Re}_\infty \text{Ra}_\infty \right)^2
\]

can be expressed

\[
\text{Nu}_\infty = 4.36 \left(1 + f_2 \text{Pr} \right) \left(\text{Nu}_\infty \text{Gr}_\infty \right)^2
\]

which leads to

\[
\text{Nu}_\infty = \left[1 - (1 - 7.44 \times 10^{-8} (\text{GrPr})^2) \right]^{1/2} / (8.54 \times 10^{-9} (\text{GrPr})^2)
\]

where \( f_2 = \text{Pr}^2 / 3 \) for \( \text{Pr} > 50 \)

Mori (Refs: E.9 and E.10) improved on the solution of Morton using a boundary-layer analysis, so that the results were valid for larger values of the complex \( \text{Re Ra}_\infty \) (i.e. values > 10^4). The final equation proposed was

\[
\text{Nu}_\infty = f_3 \left(\text{Pr} \right) \left(\text{Re Ra}_\infty \right)^{1/5}
\]

where \( f_3 = 0.842 \) for \( \text{Pr} = 1 \).

Values of \( \text{Nu}_\infty \) several times greater than 4.36 could be calculated. Some experiments were carried out with air in the region of \( \text{Re Ra} = 10^5 \), and good agreement with theory was demonstrated. The unusual parameters chosen to define the problem (Re Ra) tend to obscure the true meaning of the result. In order to measure these parameters experimentally the problem must be overspecified, as will be obvious from the following.

Substituting the more conventional parameters into the equation above, for \( \text{Pr} = 1 \),

\[
\text{Nu}_\infty = 0.842 \left(\text{Nu}_\infty \text{Gr}_\infty \right)^{1/5} / 4^{1/5},
\]

which is

\[
\text{Nu}_\infty = 0.596 \text{Gr}_\infty^{1/5}.
\]

It is now clear that the result obtained by Mori simply
represents the limiting case when free convection predominates. This is clear because the reduced equation is very similar to other well known relationships for free convection on the outside of a tube or vertical surface. (see for example Ref: K.2 where 0.59 Gr^{1/4} is proposed.)

Faris (Ref: J.11) carried out a similar boundary-layer solution, using the perturbation technique, for fully developed heat-transfer with uniform flux at the wall. The resulting equation given was \( \text{Nu}_{\infty} = 4.36 \left( 1 + f_4(Pr) \left( Gr^* Pr^2 \text{Re}^*_\infty \right) \right) \).

Once again the equations were formulated in terms of unconventional parameters, obscuring the meaning of the result. The limit of accuracy was specified in terms of Morton's parameters as being \( (\text{Re} \text{ Ra})_\infty < 3,000 \text{ Re}^{1/4} \). The starred variables above had definitions different to those commonly used. Rewriting the equation in terms of more conventional parameters -

\[ \text{Nu}_{\infty} = \left( 1 - (1 - 5.17 \times 10^{-8} (GrPr)^2/\text{Re}\_{\infty}^{1/2}) \right)/(0.593 \times 10^{-8} (GrPr)^2/\text{Re}\_{\infty}) \]

This indicates that initially \( \text{Nu}_{\infty} \) increases with \( (GrPr) \) and reduces with increasing \( \text{Re} \), which is as would be expected. The result must always be limited to those cases where free convection does not dominate in calculating \( \text{Nu}_{\infty} \), because the trend changes to one of \( \text{Nu}_{\infty} \) increasing with \( \text{Re} \) increasing, and \( \text{Nu}_{\infty} \) reducing with \( Gr \) increasing.

Cheng (Ref: E.24) produced a numerical solution of the problem posed by Morton and Mori, but in this case rectangular channels were considered. From the analysis the authors produced a graphical relationship between \( \text{Nu}_{\infty} \) and the parameter \( (\text{Re} \text{ Ra}) \) which was a unique relationship for a given channel. (The parameter was defined in a similar way to Morton and Mori.) The solution
became unstable for large values of \((Re \, Ra)\), but the method could be used to produce similar results in a circular tube for an intermediate range of \((Re \, Ra)\), between the ranges of Morton and Mori.

Siegwart (Ref: J.10) discussed the equations governing mixed convection in a horizontal tube. An approximate result was obtained for fully developed laminar flow, with uniform heat-flux, using an approximate integral procedure. The case considered was for \(Pr \rightarrow \infty\), and it was shown that

\[
Nu_{\infty} = 0.471 \left(Gr \, Pr\right)^{\frac{1}{4}}.
\]

The result would clearly be invalid for low values of \((Gr \, Pr) \sim 10^4\), since a lower limit of 4.36 exists. The result agrees within 22\% of that derived from the formula proposed by Mori, which was derived specifically for the case when free convection begins to dominate.

Some recent papers not reviewed are given in references J.9 and J.12.

3.6. THEORETICAL HEAT TRANSFER IN TUBES.

Theoretical analyses for combined convection, and for the types of separated flow encountered herein, are discussed in parts 3.5. and 3.7. The main purpose of this section is to illustrate the scope of the many papers published on laminar or turbulent heat-transfer in tubes, without secondary effects or a geometrical discontinuity. The result of each of these theoretical works can be described as being an approximate integral solution, an eigenvalue-series solution, a numerical solution or an asymptotic solution whose convergence depends on the magnitude of \(Pr\). The following table lists most of the publications acquired during the literature survey.
<table>
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<tr>
<th>Name</th>
<th>Ref</th>
<th>Date</th>
<th>Type Sol'n</th>
<th>Type Flow</th>
<th>Local/ P.D.</th>
<th>Turb. Model</th>
<th>Wall Bound</th>
<th>Fluid</th>
<th>Inlet Flow</th>
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<tr>
<td>Hubbard</td>
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<tr>
<td>Hunziker</td>
<td>C.9</td>
<td>1958</td>
<td>Series T</td>
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<td>Vel.</td>
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<td>Gas</td>
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<tr>
<td>van Driest</td>
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<tr>
<td>Reichardt</td>
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<td>1951</td>
<td>T</td>
<td>I10</td>
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<td>Loc.</td>
<td></td>
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<td>Gas/W</td>
<td>Dev.</td>
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<td>Asympt Series</td>
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<td>Gas/W</td>
<td>Dev.</td>
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<td>Num. L</td>
<td>Loc.</td>
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<td>Undev.</td>
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<td>Loc.</td>
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<td>CT/VT</td>
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<td>Dev./ Undev</td>
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<td></td>
<td>UHF</td>
<td>Gas/W</td>
<td>Dev.</td>
<td></td>
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</tbody>
</table>
Parts of references C.2, C.3, I.6 and C.13 were considered in conjunction with the theoretical work included in this thesis. Their relevance will be apparent later on (in part II).

In turbulent flow, the models for deriving the effective component of viscosity, defined by van Driest and Deissler, were considered for incorporation into this theoretical work. These can be stated thus –

\[ \mu_t' = (A^2 y^2 (du/dy)(1 - \exp(-B^2 y^2 (du/dy)/\mu)^{\frac{1}{3}})^2 \text{, from van Driest,} \]
\[ \mu_t' = (A^2 y^2 (u/y)(1 - \exp(-B^2 y^2 (u/y)/\mu')) \text{.} \]

A and B are constants.

Both expressions were arrived at by different reasoning, but appear remarkably similar. The values of \( \mu_t' \) in the above formulae were
intended to apply when $y$ was small i.e. in the region of a surface, where $(du/dy) \sim (u/y)$. The equations have the appearance of 'Prandtl's mixing-length' formula multiplied by a damping factor. The damping factor being an attempt at demonstrating that transverse eddies must be damped close to a surface, and as $u \to 0$ in the 'laminar' sub-layers the turbulence is suppressed by viscous action. Expanding the exponential, and assuming terms in $y^4$ become small near to the wall, both formulae reduce to

$$J'_t = C_\ast (2\frac{du}{dy})^2 y^4/\mu .$$

(0 constant)

As $Pr \to \infty$ this latter expression becomes sufficiently reliable for estimating the radial temperature distribution because the thermal layer is extremely small. The velocity distribution could not be determined in this way however, the complete expression for $J'_t$ would be required. The dependence of $J'_t$ on the exponent of $y$ becomes crucial, therefore, as $Pr \to \infty$. No adequate justification for the selection of the damping factor could be found, and no attempt has been made to find empirical proof at large values of $Pr$. For moderate values of $Pr$ of the order unity, the form of the damping factor was likely to be comparatively unimportant. In references 1.8, 1.9 and J.8, the authors proposed that the exponent of $y$ should be closer to 3 than 4, based on an analysis of $Nu_\omega$

(and $Sh_\omega$ for mass transfer experiments) - measurements derived with high Prandtl (or Schmidt - mass transfer) number fluids. There is clearly some doubt as to the limiting behaviour of $J'_t$ near to a heated surface, and this was recognised during the theoretical work carried out and described herein.

In the laminar regime, Sellears modified the result of Leveque (discussed in part 2.) to determine the distribution of $Nu$ for a fluid having a high $Pr$, where the boundary condition of
uniform heat-flux was imposed. Appendix B describes the procedure used by Sellars but for a different problem. The result can be stated - $\text{Nu} = 1.639 \left( \frac{x_w}{\text{Re Pr/x}} \right)^3$.

This equation can be considered valid provided $\text{Nu}$ is no less than $10$; but the solution is asymptotic for large values of the bracketed term which is the case most likely to occur with liquids having a high Prandtl number. For example, with $\text{Pr} = 100$ and $\text{Re} = 1,000$, the equation will be reliable for all axial distances less than 500 diameters from the onset of heating.
3.7. SEPAREATED FLOWS AND HEAT TRANSFER.

The sharp leading edge, at a sudden convergence in tube diameter, causes a separated region of flow to occur just downstream, sometimes known as the vena contracta. This distinguishes the configuration from developed or undeveloped flow at the entrance of a short tube, and a different distribution of Nu is likely to occur close to the discontinuity. Little experimentation has been carried out in this area. References B.11, B.12, H.1 and H.24 contain data for air and water, but the results most relevant to the present research are given by Grass (Ref: H.24) and Ede (Ref: H.1) using water as the medium.

Grass considered an electrically heated tube with a 2:1 convergence ratio, and the section upstream was unheated. Figure 3.11 shows the axial distribution of \( \frac{Nu}{Pr^{0.4}} \) downstream of the discontinuity. The shape of these functions, derived for different Reynolds numbers downstream, is qualitatively similar to measurements obtained with the short tube, as discussed earlier. However, the initial values of Nu close to the convergence were very much higher.

A more detailed study of the convergence system was carried out by Ede, in which three separate diameter ratios were investigated for a wide range of Reynolds number. The tubes were electrically heated so that uniform heat generation per unit length was imposed upstream and downstream of the discontinuity. The results are summarised in figure 3.17. The data presented for the 2:1 convergence compared favourably with the results of Grass. It was demonstrated that the Nusselt number close to the convergence reduced considerably as the diameter ratio was reduced to 1.25:1. An
increase in the diameter ratio to 3.33:1 also caused a reduction in Nu close to the discontinuity, but to a lesser extent, and the general trend was for the value of Nu to increase in the region 3 to 20 diameters downstream.

Several attempts have been made to generalise on the heat transfer process in regions of separated flow. Chapman (Ref: H.21) produced an analysis for a laminar boundary-layer of gas which separated from a surface, causing a separation 'bubble', then reattached further downstream. (The analysis was complex and the reader is referred to the original text.) It was concluded that the coefficient of heat transfer in the separated region was less than that for an equivalent attached region by a factor of approximately 2. Larson (Ref: H.20) attempted to obtain experimental corroboration of Chapman's theory, and extended his work into the turbulent region. Electrically heated models were investigated having various contours, both two-dimensional and axi-symmetric. It was concluded that for laminar separation of air the separated coefficient of heat transfer was 35% to 50% of the corresponding attached value. With turbulent flow, however, it was found that very high coefficients of heat transfer occurred in the reattached region of the boundary layer. It was demonstrated that for turbulent flow - $Nu \propto Re^{0.8}$ when attached, and $Nu \propto Re^{0.6}$ when separated.

Seban (Ref: H.11 and H.13) conducted some similar experiments with air-flow over two dimensional bodies containing a sudden step in the otherwise smooth surface. The flow was found to separate at the step, then reattach at a distance of 6 step-heights downstream. This was indicated by a sudden increase in the local coefficient of heat transfer, and by local measurements of velocity distribution.
In references J.2 and J.3 attempts were made at measuring the intensity of the turbulent velocity fluctuations in the region of a discontinuity; both a stepped plate and pipe-orifice were investigated. Whereas these works provide a deeper understanding of the mechanisms involved in separation, the work is of the preliminary nature, and is not of particular practical value. Future developments in this area could prove to be useful to the theoretician interested in separated heat transfer, a subject which is outside the scope of the present studies.

The theoretical investigations of Mills (Ref: H.16) and Macagno (Ref: H.17) provide some insight into the flow patterns for the sudden divergence in tube diameter. In both cases stable laminar motion was considered, and a numerical solution of the momentum equations was sought. The usual boundary-layer approximations were not invoked since large radial velocities were likely to occur downstream of the discontinuity. Mills investigated a pipe-orifice and Macagno a sudden increase in tube diameter. The range of flow rates considered was limited in both cases by the computational stability of the numerical method employed. Mills performed calculations up to $Re = 50$ downstream, and Macagno extended the range to $Re = 200$. In both cases the diameter ratio was $1:2$. Neither author attempted to solve the energy equations to yield the coefficients of heat transfer. The streamlines calculated indicated that a captive anvil eddy was formed downstream of the discontinuity, and the length of this separated region increased from $7$ diameter at $Re = 50$, to $4.5$ diameters at $Re = 200$ (downstream values). It could be postulated that the distance to the point of reattachment would correspond approximately to the position of the maximum local value of $Nu$, but this statement is speculative and no evidence was
found to support this conclusively. It should be stated here that the flow pattern is likely to be unstable, having a point of inflection in the axial velocity profile, and the critical Reynolds number for the onset of turbulent instability has not been considered in the above investigations.

Filetti (Ref: H.8) carried out a detailed investigation of heat transfer downstream of an abrupt enlargement in the cross section of a flat duct, and the heat transfer medium was air. Local heat-transfer rates were measured using heat-meters, and the channel walls were maintained at a lower temperature than the entrained air. The duct width increased in the ratio 1:2:1 and 1:3:1 for the two configurations studied. It was shown that a region of separation occurred on either side of the downstream section, but the length of each region differed since the flow pattern was asymmetrical. The Reynolds number was varied from 70,000 to 200,000 (based on hydraulic diameter). It was demonstrated that the average distance to the point of reattachment was independent of Re and was equal to 6 step heights. In the separated region Nu was comparatively small rising to a peak at the point of reattachment, then reducing with increasing distance downstream. The maximum value of Nu was proportional to Re\(^n\) where n = 0.689 for the short separated length and 0.593 for the long separated length.

Several workers have reported data for configurations similar to the divergence experiments in this thesis. Air has been used in some of these. Zemanick (Ref: J.1) investigated local heat transfer downstream of a sudden increase in tube diameter; the section upstream was unheated, and the section downstream was heated directly by applying a voltage to the tube. Three diameter ratios were
utilised, 0.43:1, 0.54:1 and 0.82:1. The results of the tests are recorded in figure 3.13. The axial distribution of the ratio—local Nusselt number to Nusselt number at large axial distances—is shown. A well defined peak occurred in the functions which has been discussed previously as being typical of separated flows at high values of Re. The maximum value of Nu was measured at 7 and 9 step heights downstream for each of the diameter ratios, and this was independent of Re except at the lower values of Re. The maximum value of Nu could be estimated approximately from

$$\text{Nu}_{\text{max}} = 0.2 \text{Re}^{0.3}_1,$$

where Re$_1$ is the value of Re upstream.

Emerson (Ref: H.7) measured local coefficients of heat transfer downstream of a 1:1.7 divergence ratio using air. The tubes were heated electrically so that uniform heat generation per unit length was achieved upstream and downstream of the discontinuity. In these experiments a most detailed investigation of the apparatus and instrumentation was carried out. It was shown that longitudinal conduction of heat in the tube material can have a profound effect on the accuracy of measured values of $h$ when small values of the latter are considered (such is the case with gases in convection.) A tube was manufactured with a thickness of only 0.05mm for use in the experiments to minimise the conduction problem. The results incorporated several geometries for causing separation, but only the divergence will be discussed here.

Figure 3.15 shows how $h$ varied downstream of the divergence, for a range of Re (downstream) of 14,500 to 105,000. Less than $\frac{1}{2}$ diameter downstream a minimum occurred in the h-distance function, and it was suggested that the 'high' values upstream of the minimum were 'artificial' values caused by conduction in the tube-wall.
The maximum value of \( h \) was measured 2\( \frac{2}{3} \) diameters downstream, (or 13 step heights). In an attempt to relate this to the distance at which reattachment occurred, flow visualization tests were carried out with a transparent 'mock up'. Droplets of oil were placed on the inner tube-wall and the position of zero shear stress located by observing the displacement of the drops. Figure 3.16 illustrates that the point of reattachment was slightly more than 2 diameters downstream (or 9\( \frac{1}{3} \) step heights). The maximum value of \( h \) was apparently downstream of the separated region. The effect of changing the heat transfer rate in the upstream leg (keeping the downstream value constant) is indicated in figure 3.16. An increase of 70% in the rate of heat transfer upstream caused a reduction of 15% in the value of \( h \) in the region of the maximum value. It was suggested by Emerson that the peak \( Nu \) would probably be located at the point of reattachment if the lateral temperature profile within the air upstream had been uniform (or zero heat-transfer upstream).

A few experiments have been carried out with water in a sudden divergence, at high Reynolds numbers. Grass (Ref: H.24) fitted orifices to the inlet of an electrically heated tube, and measured \( h \) downstream. The ratios of orifice to tube diameter employed were 1:1.25, 1:1.67 and 1:2.5. Figure 3.12 summarises these results. It is clear that the position of the peak \( Nu \) number moved downstream as the orifice size was reduced, and in the case of the 1:1.25 ratio, the peak value occurred too close to the discontinuity to permit the value to be measured. The position of the peak was apparently independent of \( Re \), and was located 2.6 diameters downstream (8\( \frac{1}{4} \) step heights) with 1:2.5 ratio, and 1.7
diameters (6\(\frac{1}{2}\) step heights) with 1:1.67 ratio.

A similar experimental configuration was investigated by Krall (Ref: H.9) using water. For Reynolds numbers above 10,000, the value of \(h\) was measured for 1:1.5, 1:2, 1:3 and 1:4 orifice to tube diameter ratios. The ratio of \(\frac{Nu}{x}\) to the fully developed value of \(\frac{Nu}{fd}\) at large distances (or \(Nu/Nu_w\)) is shown in figure 3.14. It is difficult to draw a comparison with the results of Grass, which were somewhat incomplete, however, small differences were observable. The position of maximum \(Nu\) was 1.5 to 2.5 diameters downstream for all the geometries considered. For the ratios 1:2 and 1:3 this distance was 1\(\frac{2}{3}\) to 2\(\frac{2}{3}\) diameters, which is rather less than the 2.6 diameters obtained by Grass with the 1:2.5 ratio. Although a direct comparison of the maximum \(Nu\) was not possible, the evidence suggests a reasonable agreement between the two sources. The maximum value of \(Nu\) for all the divergences could be expressed as 0.398 \(Re_1^2\), (where \(Re_1\) is the orifice Reynolds number).

The most detailed investigation of the divergence system with water has been carried out by Ede (Ref: H.1). The configurations included sudden increases in tube diameter, the divergence ratios studied being 1:1.25, 1:2 and 1:3.33. Uniform heat generation per unit length was imposed upstream and downstream of the discontinuity, and careful instrumentation of the apparatus allowed reliable measurements of \(h\) to be determined in the region of the divergence. A wide range of \(Re\) was covered extending to much lower values than were employed hitherto. The experimental findings are summarised in figures 3.18 and 3.19. Figure 3.18 gives the axial distribution of \((Nu/Pr^0.4)\) for the three divergences, the Reynolds number downstream ranged from 3,000 to 50,000. The
functions were derived by interpolating the extensive experimental data in such a way that the small axial variations in fluid properties (caused by the increasing bulk temperature) were eliminated.

The location of the peak \( \text{Nu} \) was 1 diameter downstream of the discontinuity for the 1:1.25 ratio, increasing to 2\( \frac{2}{3} \) diameters for the 1:2 ratio and 3 diameters for the 1:3.33 ratio (or 10, 9 and \( \frac{29}{3} \) step heights respectively.) The distance in diameters was approximately equal to the diameter ratio. The ratio (maximum \( \text{Nu}/\text{Nu}_w \)) increased with the diameter ratio, and could be expressed approximately by \( 15.3 \text{ Re}^{-0.22}(\text{dia. downstream}/\text{dia. upstream}) \) according to Ede.

Figure 3.19 shows the results derived for the lower Reynolds numbers. Reducing \( \text{Re} \) it was found that the temperature measurements became unstable and it proved impossible to obtain reliable data. A transition region occurred until \( \text{Re} \) downstream reduced to the order \( 10^2 \). As \( \text{Re} \) decreased, the trend was for the peak \( \text{Nu} \) to move downstream, and the ratio (maximum \( \text{Nu}/\text{Nu}_a \)) became much smaller. With \( \text{Re} \) of the order \( 10^2 \) the peak in the \( \text{Nu} \)-distance function vanished, for the lowest divergence ratio, and a sudden drop to a minimum was evident just downstream of the step. The value of this minimum was considerably less than \( \text{Nu}_a \).

Flow visualization tests carried out by Ede showed that at high values of \( \text{Re} \) the flow entered the downstream leg as a jet surrounded by a region of pronounced recirculation. The jet expanded to fill the tube. When the flow was laminar far downstream, the laminar distribution was very slow in developing, and the flow pattern close to the discontinuity was similar to the fully turbulent case to very low values of \( \text{Re} \) (\( \ll 2,000 \) downstream). With laminar
flow upstream, the flow through the divergence was stable for very low Reynolds numbers (100). However, an intermediate regime was evident in which the laminar flow broke down and became unstable several diameters downstream.

No experimental or theoretical work could be traced which dealt with viscous fluids in stepped tubes. It was apparent that little or no work had been carried out for heat transfer in the region of a sudden geometrical discontinuity, where the heat-transfer medium possessed a high Prandtl number. A comparison between the results of Emerson (Ref: H.7) and Ede (Ref: H.1) indicates the effect of increasing Pr from 0.7 to 6.0. Interpolating the data of Ede to give equivalent results for a 1:1.7 sudden divergence, as used by Emerson, significant differences are evident. For water, the maximum value of \( \frac{Nu}{Nu_\infty} \) was 4.3 to 2.5 reducing as Re increased from 5,000 to 50,000. This maximum was located 2.0 diameters downstream. For air, the value of \( \frac{Nu}{Nu_\infty} \) was maximum 2.9 diameters downstream and the magnitude was 2.5 to 2.8 in the range Re = 14,500 to 105,000. No definite relationship between \( \frac{(Nu/Nu_\infty)_{max}}{Re} \) and Reynolds number was established. Increasing Pr from 8 to 500 can, could lead to very significant departures from the 'non-viscous' results discussed in this section, and there is no reasonable justification for applying the data obtained with water or air when viscous fluids are encountered.
3.8. THE EFFECTS OF DISSIPATION.

The dissipation of mechanical energy into heat is likely to play an important part in the convection process when the shear stress is high in the region of the thermal boundary-layer, and the working temperature differences \((t_w - t_b)\) is small. In general, this is likely to be the case (for moderate Re, say \(10^3\) to \(10^5\)) when either the viscosity is small and velocity high, or the viscosity is high and the velocity moderate. The former is common with gases and the latter with oils.

The width of the thermal and momentum boundary-layers are comparable in magnitude with gases, but the thermal layer is much thinner with fluids having a high Prandtl number, so the theoretical treatment of the two cases must differ considerably. The analysis of dissipative heat-transfer for gas flowing over a plate is well known, and is treated in many standard text books (e.g., Ref. K.5). Few references were found which discussed dissipation with viscous fluids in tubes. References J.14 and C.21 provide an explanation of how Nu increases when dissipation becomes significant, and a limited amount of experimental work was done to demonstrate the phenomenon. In general, the effects of dissipation on Nu were pronounced when Pr \(>>\) 100. It was intended to attempt a theoretical assessment of such effects for the simplest configuration herein, i.e., the short tube.

Kudryashev (Refs: F.1 and F.2) analysed heat transfer with dissipation for the laminar flow of non-viscous fluids in the entrance region of a tube. The flow was fully developed at the entrance and the tube was at constant temperature. Two solutions of the energy equation were sought, the first was for the case when wall temperature was constant and equal to the inlet temperature of the fluid. The second solution assumed a constant wall temperature, which was higher than the inlet temperature of the fluid, and no dissipation was present. The general solution was assumed to be the sum of both particular solutions, and the local Nusselt
number was finally expressed:

\[ \text{Nu} = \frac{8 D + \sum_{n=0}^{\infty} \frac{H_n}{n!} \exp \left( -2 \frac{k_n^2}{r_w} \frac{x}{(r_w \text{RePr})} \right)}{5 D/6 + \sum_{n=0}^{\infty} G_n \exp \left( -2 \frac{k_n^2}{r_w} \frac{x}{(r_w \text{RePr})} \right)} \]

where the first five values of coefficients G, H and constants k in the above series are given in references F.1 and F.2.

\[ D = \text{Re}^2 \text{Pr} / \left( \left( \frac{8r_w^2}{\mu^2}(c_2 \frac{(t_w - t_o)}{2r_w}) \right) \right), \]

\[ t_o = \text{inlet temperature}. \]

The above result converges rapidly as \( \text{Pr} \) tends to zero, and would be found suitable for application with gases. It is interesting to note that the value of \( \text{Nu} \) tends to 9.6 at large axial distances, which can be compared with the non-dissipative limit of 3.65.

The analysis of fully developed heat-transfer, with dissipation, in long tubes is discussed in references F.3 and F.4. The procedures described enable the value of \( \text{Nu} \) to be determined for a tube of generalised cross-section, and can thus be extended to circular tubes. The boundary conditions considered (in Ref:F.3) appertain to a linear axial distribution of wall temperature, and internal heat generation is permitted within the fluid (such as might occur with nuclear heating). The result for a circular tube is expressed:

\[ \text{Nu} = 48 \frac{F}{11} \]

where \[ F = F \left( \frac{\mu^2}{q r_w^2} \right) \]

In the attempt at generalising on the problem, the function \( F \) was derived in a complicated form, and for fully developed heat transfer in laminar tube-flow, it is just as convenient to integrate the
energy equation from first principles. This is carried out in part 11. It can be concluded from the above analysis that the parameter \((\mu^2/qr_w)\) is a significant group in estimating the contribution of dissipation to \(\text{Nu}_w\).

In turbulent tube-flow, the effects of shearing rate on the value of \(\text{Nu}\) apparently have not been assessed for a viscous medium. The solution of this problem is likely to indicate that the main contribution of viscous dissipation occurs in the laminar sub-layers at the tube wall where high shear stresses can arise. In the mainstream, velocity and temperature gradients are small, and good thermodynamic mixing is inevitable. The model of turbulence proposed must have some effect on the dissipative energy, and some difficulty in justifying the solution is likely to occur.

No empirical data on dissipative heating could be found which would be of value in these investigations.

3.9. TUBE WALL CONDUCTION.

Although the temperature distribution in the heat-transfer medium is of prime interest in work of this nature by definition, it is recognised that a solid-liquid interface is present in practice, and that before reliance can be placed on the temperatures measured at the surface of the tube, the heat-transfer processes within the tube-wall itself should be probed. For instance, the relationship between the temperatures at the inner and outer surfaces of the tube must be influenced by the properties and thickness of the material, and the distribution of the heat generated electrically within the material. Further, the heat-flux at the solid-liquid interface can only be truly uniform for a linear axial temperature distribution inside and outside the tube.
In references G.1, G.2, and J.7, attempts have been made at solving the temperature distribution within a solid wall, and within the heat transfer medium entrained, simultaneously. In each case the mathematical techniques employed were sophisticated, and the distribution of velocity and temperature in the fluid was simple. This kind of formal approach to the problem was considered to be of little practical value in these experiments, and a more direct and elementary approach was sought, as described in a later section of the thesis.
FIGURE 3.1

Comparison between the results of Malina and Allen.
THE RESULTS OF EDE FOR EXPERIMENTS WITH LONG ELECTRICALLY HEATED TUBES.
THE VARIATION IN $\left( \frac{Nu}{\infty} \right)$ WITH $Pr$ WHEN $Re = 10^4$

FRIEND AND METZNER.

$\frac{Nu}{10^4} = \frac{Nu}{\infty}$ AT $Re = 10^4$. 
SUMMARY OF KAYS RESULTS.
LAMINAR HEAT-TRANSFER. DEVELOPED AND UNDEVELOPED FLOW OF LOW PRANDTL NUMBER FLUIDS WITH UNIFORM HEAT-FLUX AND CONSTANT TUBE TEMPERATURE.
LOCAL NUSSELT NUMBERS.
FIGURE 3.5.

The results of McComas and Eckart for local coefficients of heat transfer with laminar tube-flow (horizontal).
Heat-transfer medium—air.
Developed flow at entrance.

Diagram 1:

1. \( R_e = 214 \), \( Gr = 1000 \).
2. \( R_e = 218 \), \( Gr = 0.13 \).

Diagram 2:

1. \( R_e = 743 \), \( Gr = 492 \).
2. \( R_e = 757 \), \( Gr = 534 \).
Figure 3.6

The experimental results of mills for air in heated tubes.

Local heat transfer coefficients for calming section entry.

\[
\frac{1}{10^4 \text{Re}} \frac{d^2 h}{\text{ft} \cdot \text{h} \cdot \text{F}} = 5.478 \frac{W}{m^2 \cdot K}
\]

Local heat transfer coefficients for bellmouth entrance.

Local heat transfer coefficients for sharp 30° edge entrance.

\[
\chi = \text{distance from outset of heating}.
\]

\[
d = \text{tube diameter}.
\]
Figure 5.7.

The experimental results of Wolf for heat transfer to air and carbon dioxide, with calming length.

\[ N_u^{R_e, \beta} = \text{Nusselt number corrected to nominal values of } R_e \text{ and } \beta. \text{ (At local bulk temp.)} \]

\[ \beta = \left( \frac{T_w - T_w^c}{T_w - T_w^c} \right) \]
FIGURE 3.8:

2.8 (i).  

ALAD'EV (Ref. B.5).  
LOCAL COEFFICIENTS OF HEAT TRANSFER  
INTERPOLATED FROM MEASUREMENTS.  
CONSTANT TEMPERATURE TUBE (WATER).  
\( h_\infty = \frac{h}{h_\infty} \) AT 40 DIAMETERS.

\[ \begin{align*}  
\text{Distance Downstream (Diameters)} & \quad 0 & \quad 5 & \quad 10 & \quad 15 & \quad 30 \\
\frac{h}{h_\infty} & \quad 2.0 & \quad 1.5 & \quad 1.0 & \quad & \ 
\text{Re} & \quad 10,000 & \quad 50,000 & \quad 100,000 & \quad & 
\end{align*} \]

3.8 (ii).  

HARTNETT (Ref. B.2)  
LOCAL COEFFICIENTS OF HEAT TRANSFER  
SMOOTH EXPERIMENTAL DATA FOR WATER.  
UNIFORM HEAT FLUX, CALMING LENGTH.

\[ \begin{align*}  
\text{Distance Downstream (Diameters)} & \quad 0 & \quad 5 & \quad 10 & \quad 15 \\
\frac{h}{h_\infty} & \quad 1.5 & \quad 1.4 & \quad 1.3 & \quad 1.2 & \quad & \ 
\text{Re} & \quad 16,200 & \quad 33,600 & \quad 44,300 & \quad & 
\end{align*} \]
THE EXPERIMENTAL RESULTS OF STONE (Ref B.3.),
($R/R_{\infty}$) VERSUS (DISTANCE): SHOWING THE EFFECT OF
CALMING LENGTH.

SMOOTHED AND INTERPOLATED DATA, EXPERIMENTAL SCATTER ±20% MAX.

$Re > 8,000$

$\left( \frac{h}{h_{\infty}} \right)$

SHORT CALMING LENGTH = 0.8 DIAMETERS.

LONG CALMING LENGTH = 97 DIAMETERS.

DISTANCE FROM START OF HEATING (DIAMETERS).
FIGURE 3.10.

MEASURED VALUES OF \( \frac{h}{h_\infty} \) IN THE ENTRANCE REGION OF A TUBE WITH CALMING LENGTH. RESULTS FOR WATER AND TWO OILS.

MALINA (REF. B.I.).

TUBE HEATED ELECTRICALLY.

Thermal entrance region results, water, \( Pr = 3 \).

Thermal entrance region results, oil, \( Pr = 48 \).

Thermal entrance region results, oil, \( Pr = 75 \).

\[ h_\infty = \text{FULLY DEVELOPED VALUE.} \]

\[ h_l = \text{LOCAL VALUE.} \]

\( x = \text{DISTANCE FROM ONSET OF HEATING.} \)

\( D = \text{TUBE DIAMETER.} \)
HEAT TRANSFER: DOWNSTREAM OF A SUDDEN REDUCTION OF 2:1 IN TUBE DIAMETER ACCORDING TO GRASS. UPSTREAM SECTION UNHEATED.

CONVERGENCE RATIO 2:1.

\[
\left[ \frac{Nu}{Pr^{0.34}} \right]
\]

\[ Re = 20,000 \]

\[ Re = 10,000 \]

\[ Re = 5,000 \]

DISTANCE DOWNSTREAM OF CONVERGENCE (DIAMETERS).
FIGURE 3.12

HEAT TRANSFER DOWNSTREAM OF A SUDDEN DIVERGENCE IN TUBE DIAMETER. UPSTREAM SECTION UNHEATED. (GKASS).

\begin{align*}
&\text{(a)} = 1:2.5 \text{ RATIO OF DIAMETERS} \\
&\text{(b)} = 1:1.67 \\
&\text{(c)} = 1:1.25
\end{align*}

\begin{align*}
\text{Re} &= 20,000 \\
\text{DS} &= \text{DOWNSRAM}
\end{align*}

\text{DISTANCE DOWNSTREAM OF DIVERGENCE (DIAMETERS)}.

\begin{align*}
&\text{(a)} = 1:2.5 \text{ RATIO OF DIAMETERS} \\
&\text{(b)} = 1:1.67 \\
&\text{(c)} = 1:1.25
\end{align*}

\begin{align*}
\text{Re} &= 10,000
\end{align*}

\text{DISTANCE DOWNSTREAM OF DIVERGENCE (DIAMETERS)}.

\begin{align*}
&\text{(a)} = 1:2.5 \text{ RATIO OF DIAMETERS} \\
&\text{(b)} = 1:1.67 \\
&\text{(c)} = 1:1.25
\end{align*}

\begin{align*}
\text{Re} &= 5,000
\end{align*}

\text{DISTANCE DOWNSTREAM OF DIVERGENCE (DIAMETERS)}. 

\text{Re} = 20,000

\text{Re} = 10,000

\text{Re} = 5,000
**Figure 3.13**

$N_{U_{34}}$ = the value of $N_u$ at large axial distances.

**Symbols**
- $4,000$
- $7,000$
- $9,000$
- $10,000$
- $17,000$
- $20,000$
- $38,000$
- $47,000$

Axial distribution of local to fully developed Nusselt number ratio, $d/D = 0.43$

$d$ = Diameter upstream.

$D$ = Diameter downstream.

Axial distribution of local to fully developed Nusselt number ratio, $d/D = 0.54$

Axial distribution of local to fully developed Nusselt number ratio, $d/D = 0.32$

Heat transfer downstream of a sudden expansion, from Zel'dovich.
HEAT TRANSFER DOWNSTREAM OF AN ORIFICE IN A TUBE, FROM KRAYE.

\( d_0 = \text{orifice diameter} \quad D = \text{tube diameter} \quad \text{Nu}_{x_1} = \text{value of } \text{Nu} \text{ at large distances} \quad \text{Nu}_x = \text{Nu}. \)
LOCAL HEAT-TRANSFER COEFFICIENTS WITH AIR IN A 3-inch DIA PIPE DOWNSTREAM FROM A STEP CHANGE FROM 1\frac{1}{4} inch DIA (EMERSON).
HEAT TRANSFER AND OIL-DROP TESTS AT REYNOLDS NUMBER = 32000, \( \frac{3}{4} \) inch DIA PIPE ENLARGING TO 3-inch DIA (Ref. 9, 591).
EXTRA DIAMETERS EQUIVALENT TO EXTRA HEAT TRANSFER PRODUCED BY CHANGE OF SECTION SHOWN ABOVE CURVE
REYNOLDS NUMBER BEFORE AND AFTER CHANGE OF SECTION SHOWN BELOW CURVE

EFFECT OF AN ABRUPT CONVERGENCE INTERPOLATED FROM CROSS-PLOTS OF EXPERIMENTAL DATA (\(\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \)).
EXTRA DIAMETERS EQUIVALENT TO EXTRA HEAT TRANSFER PRODUCED BY CHANGE OF SECTION SHOWN ABOVE CURVE
REYNOLDS NUMBER BEFORE AND AFTER CHANGE OF SECTION SHOWN BELOW CURVE
EFFECT OF AN ABRupt DIVERGENCE INTERPOLATED FROM CROSS-PLOTS OF
EXPERIMENTAL DATA (E.D.).
REYNOLDS NUMBER BEFORE AND AFTER CHANGE OF SECTION SHOWN BESIDE EACH CURVE

EFFECT OF AN ABRUPT DIVERGENCE AT LOW REYNOLDS NUMBER, EXPERIMENTAL DATA (E.D.E.).
RESULTS OF HARKWITZ (Ref: B2), DISTRIBUTION OF HEAT TRANSFER COEFFICIENT IN THE ENTRANCE-REGION OF AN ELECTRICALLY HEATED TUBE WITH CALMING LENGTH FITTED UPSTREAM. DATA FOR OIL.

Thermal Entry Length Results for Oil Flow in the Transition Region

\[ \frac{h}{h_\infty} = \text{Local Coefficient of Heat Transfer} \]
\[ \frac{h_\infty}{\text{Value of } h \text{ at Large Axial Distances}} \]
\[ \gamma = \text{Distance from Onset of Heating} \]
\[ D = \text{Tube Diameter} \]
4. A RESTATEMENT OF OBJECTIVES.

4.1. This research was carried out under the terms of a Ministry of Technology (now Department of Trade and Industry) contract for the National Engineering Laboratory, East Kilbride. The project represents part of a wider project initiated at NEL to investigate those little explored areas of the heat-transfer field in which disturbed regions of flow arise, or where fluids of viscous or non-newtonian nature occur—particularly in piped flows.

At this stage of the thesis the objectives of the work can be stated in greater detail. The subjects of investigation were selected after a survey of previous research (3) and with consideration for the constraints of the contract. The following contains a summary of these objectives.

4.2. It was proposed that an investigation be carried out passing highly viscous fluids through horizontal tubes, the range of viscosities being 1 to 50 cP. Heat transfer by forced convection was to be studied for Reynolds numbers between 100 and 20,000, and the experimental configurations were to be as follows. First, a short, uniform diameter tube having either developed or undeveloped flow at the start of heating; second, tubes having a sudden increase in diameter, the proposed ratios of diameters being 1:3, 1:2 and 4:5; third, tubes having a sudden decrease in diameter, the proposed ratios being 3:1, 2:1 and 5:4.

The investigations were to include an assessment of the way in which the rate of heat transfer is affected by the variation in viscosity with temperature, the presence of natural convection, and the viscous dissipation of mechanical energy in the fluid.
5. DESCRIPTION OF THE APPARATUS.

5.1. THE CONFIGURATION.

The basic functions of the apparatus will be described in this paragraph. First, it was necessary to supply a steady flow of fluid to the section of tubing under investigation. The temperature of this fluid was to be constant with respect to time, and both the temperature and rate of flow were to be continuously variable between certain specified limits. Second, the temperature of the tube was to be controllable, so that the level could be set at some value above that of the fluid. Third, a means had to be provided for measuring the temperature at any required point on the inner surface of the tube, the bulk temperature of the fluid at any axial location, the rate of flow of the fluid, and the rate of heat transfer from the tube to the fluid.

The schematic diagram, figure 5.3, shows the apparatus which was employed in all the heat-transfer experiments described in this thesis. The test-section, or experimental tube, was heated electrically by a direct current passing longitudinally through the material, heat being generated within the wall of the tube. The entire test-section with its fittings was enclosed in a rectangular, wooden box which was lined with 4 cm. of polystyrene sheet. The box was firmly packed with polystyrene beads, about 5 mm. in diameter, so that at least 15 cm. of insulation was provided in all directions away from the test-section. Polystyrene was considered to be a good insulator, its conductivity being only 1/600 times that of the tube material, and 1/10 times that of the fluid in the tube. Calculations carried out in 5. 6. indicated that less than 5% of the heat generated was dispersed by conduction through the polystyrene. The condition of uniform heat-flux at
the inner surface of the tube was therefore well approximated, and the magnitude of the flux could be derived by measurement of the electrical power dissipated in the tube.

The electrical power supply was 415 volts - 3 phase a/c. A stabilised voltage controller permitted the selection of 0 to 415 volts and ensured that the output would not vary by more than ± 1% of the voltage setting. In order to match the voltage and current requirements at the test-section, the voltage was transformed to give a 0 to 30 v supply. A metal rectifier and smoothing unit was inserted to convert this into direct current, and the range of operating conditions established was 0 to 30 v, 0 to 1000A, or 0 to 30 kW. (All specifications for the components are given in Appendix (A)).

The general layout of the electrical equipment was such that no loops occurred in the alternating current circuitry, thereby avoiding the possibility of stray e.m.f's being induced into the instrumentation. The choice of direct current heating followed similar reasoning; if the tube were heated by an alternating current then it would be difficult to eliminate the possibility of stray voltages being induced in the thermocouples situated on the test-section.

The leads carrying the current to the test-section comprised 7 cm² of copper. An insignificant amount of heat was generated within the copper because of the high electrical conductance. The potential difference over the test-section was measured close to the points where the leads made contact with the tube, and the current was given by the volt drop across a calibrated shunt in series with the positive lead.

The heat-transfer fluid was drawn from the reservoir
by a centrifugal pump, circulated through the test-section, then returned to the reservoir (figure 5.3). The main filter was situated at the pump outlet, its primary purpose being the protection of the turbine-flowmeters. Downstream of the filter some of the flow was passed through a branch-pipe which was connected directly to the reservoir, hence bypassing the test-section. The quantity bled off was controlled by the 'bypass-valves', and these enabled a suitable operating point to be chosen for the pump independent of the test-section requirement. The temperature of the fluid discharged from the pump was dependent to some extent on the setting of these valves. The main flow, to the test-section, was controlled by the 'inlet-valves'. After this point, the flow was diverted into one of two parallel pipelines, each incorporating a turbine-type flowmeter, the capacity being different in each case, (details in Appendix (A)). A secondary filter having a finer mesh was included in the line with the smaller flowmeter.

After passing through a 180 degree bend, the fluid entered the test-section. Mixing boxes (see figure 5.3) were situated at the inlet and outlet ends of the tube, to enable the bulk temperature to be measured. The inlet-end box also served to baffle irregularities in the flow following the 180 degree bend upstream. (More details of the mixing boxes are given in 5.4.) A restriction was applied at the outlet end of the test-section (the back-pressure valve), its function being to maintain a system pressure higher than atmospheric throughout, hence eliminating the possibility of air-leakage into the heat-transfer fluid. All connections with the reservoir were submerged, and the tank was fitted with a rubber-edged lid to minimise the hygroscopic absorption of water from the atmosphere by the fluid.
The line diagram given in figure 5.4 shows the dimensions of the system and sizes of the connecting pipework.

To ensure that the temperature of the test-section never approached a dangerous level, a thermostatic control was affixed to the tube. The thermostat was connected via a relay to the circuit breaker in the tube’s power supply, and a suitable, maximum temperature level could be selected at which the power switched off.

A water-cooler was situated between the pump outlet and the main filter, and this facilitated the transfer of heat out of the system so that a suitable, steady temperature could be maintained in the heat-transfer fluid at the entrance of the test-section. The heat exchanger was a ’shell and tube’ type with the water contained in the tube side. The water supply was provided by a cooling-tower circuit. The tower was capable of dissipating 88 kW, whilst delivering \( \frac{6\times 10^3}{2} \) of water at 24°C, with an atmospheric wet-bulb temperature of 21°C. In practice, water temperatures of 10 to 15°C could be obtained, since the maximum dissipation required was approximately 30 kW and the atmospheric temperature typically 5 to 20°C.

5.2. DESIGN CONSIDERATIONS.

5.2. (i) INTRODUCTION.

To simplify the problem of designing the apparatus it was necessary to write a specification which would help to initiate the calculations involved in the selection of the components, e.g. the pump, heat exchanger, flowmeters, and so on. The following specification was in keeping with the general objectives of this research.
(1) The fluid used should be ten times as viscous as water (or have a nominal viscosity of 10 cP).

(2) The experimental tube should be nominally 2.5 cm. diameter and at least 200 diameters in length.

(3) With the maximum rate of flow, and with 100 diameters of the tube heated, the maximum temperature difference between the tube and fluid should be 40°C approximately.

(4) The maximum Re should be nominally 20,000.

(5) The electrical power supply should be limited to a maximum of 30 kW, or 30 V and 1000 A.

The statement (5) must be added to the specification since part of the apparatus (for the provision of a high current at a low voltage level) was already made available at the outset. Some elementary calculations showed these limitations to be no obstacle in the ensuing experiments, and this will be illustrated shortly, (part 5.2.(iii)).

Prior to discussing the design-calculations, the problems associated with the selection of the heat-transfer media will be outlined. Part 5.2.(ii) deals with this topic.

5.2.(ii) THE HEAT-TRANSFER FLUID.

In the selection of the heat-transfer medium the viscosity range to be considered was 1 to 50 cP (as discussed in part 4), and the fluids had to exhibit newtonian behaviour (that is, shear strain being directly proportional to shear stress). Light oils were considered first, but it eventually became apparent that glycols had certain advantages, and subsequently propylene glycol mixtures were selected.

The dependence of viscosity on temperature varies considerably from one fluid to another, so initially it was thought
the best approach would be to utilise at least three oils in the experiments, each having a particular kind of viscosity-temperature dependence. For purposes of comparison, one of the fluids was to have a viscosity which was almost independent of temperature, hence consideration was given to a silicone based fluid (silicone fluids exhibit less viscosity variation with temperature than any other known fluid).

The advantages of propylene glycol mixtures over light oils in the context of this research, becomes apparent from the following description. Propylene glycol is often used as a heat-transfer fluid, it is completely miscible with water, it is non-toxic, it has a boiling point much higher and a freezing point much lower than those of water, dry propylene glycol is non-corrosive, and the viscosity of water-glycol mixtures ranges from 1 cP to 56 cP at 20°C. In changing the composition of a water-glycol mixture, the viscosity changes, and also the temperature dependence of the viscosity changes. It would appear, therefore, that no advantages would accrue from having several oils rather than several glycol mixtures as far as the experimental objectives are concerned, and obvious operational advantages can be derived from handling liquids with the same base (ie. propylene glycol.)

Silicone oil was considered for use in experiments on 'constant viscosity', viscous liquids, but this aspect of the work was found to be unjustifiably expensive. The cost of a suitable silicone oil is about £4.8 per kg, and such an investment should only be made after examining the cost effectiveness of the research, which in this case was qualitatively estimable from the experimental data derived with propylene glycol. The rate of change of viscosity with temperature for a 10 cP fluid at 20°C is approximately 0.2 cP per degree K for silicone oil, and 0.4 cP per degree K for water-
glycol. Even for silicone oil the variations in viscosity with
temperature are therefore comparatively large. It was observed
in the 'glycol' experiments that quite substantial viscosity
variations were necessary before any measurable changes in the
coefficient of heat-transfer were detected. This implied that
under certain conditions glycol could be considered as having
a near constant viscosity. Such was the case when small heat
fluxes were applied and the viscosity of the glycol was not greater
than approximately 10 cP.

5.2(iii) CALCULATIONS.

The following calculations are included to exemplify
the design process, the selection of the components was of course
based on repetitious calculations and tempered by practical
limitations. The final scheme, containing detailed specifications
of the components, had to be analysed to give the performance
characteristics of the apparatus. The calculations herein show
how this analysis was carried out.
The first requirement is for an assessment of the pressure distribution in the system. The estimates are made assuming the heat-transfer fluid to be 61% (by weight) propylene glycol and the specification in 5.2(i) is to be observed. The following assumptions are made (see the diagram above) -

1. $p_{18} = 0 = p_{19}$
2. There is no 'bypass' flow.
3. All valves are open, except the bypass valves.
4. The properties of the fluid are constant with temperature and are estimated at 20°C reference temperature, i.e.

\[
\rho = 1.04 \times 10^3 \text{ kg/m}^3, \quad \mu = 10 \text{cp}, \quad c = 3360 \text{ J/kg K}, \quad K = 0.337 \text{ W/mK}
\]

5. The test-section is perfectly insulated.

The pressure drop in any length of tube or fitting is estimated on the basis of the Fanning equation, and all fittings are given an 'effective' length, $L$ diameters, to conform with this.

A detailed explanation can be found in McAdams book (p 145 Ref. K2). Hence,

\[
\frac{dp}{dL} = 2f\left(\bar{u}^2\right)
\]

with

\[
L = \text{tube, or effective length (diameters)}
\]

\[
(2r_w) = \text{local diameter},
\]

\[
\bar{u} = \text{mean velocity},
\]

\[
f = \text{friction factor},
\]

\[
Re = \text{local Reynolds number}.
\]

From McAdams -

for a 90 degree bend $L_{bend} \simeq 32$ diameters,

for an open valve $L_{valve} = 7$ diameters,

for a tee-junction $L_{tee} \simeq 90$ diameters.

For the test-section, $L_{12,13} = 200$, $(2r_w)_{12,13} = 2.5$ cm, and

\[
\frac{dp}{dL_{12,13}} = 0.096\frac{\mu^2}{\rho} \quad Re_{12,13}^{1.8}
\]

\[
\rho (2r_w)_{12,13}^2
\]
so that \[ (p_{12} - p_{13}) = 0.00295 \cdot \text{Re}^{1.8}_{12,13} \cdot \frac{N}{m^2} \]

For the valves, \((2r_w)_{6,7} = 4.5 \text{ cm}, (2r_w)_{8,9} = 3 \text{ cm}, \)
\[(2r_w)_{15,16} = 2.5 \text{ cm}, \]
so that \[ (p_6 - p_7) = 0.320 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{6,7} \cdot \frac{N}{m^2}, \]
\[ (p_8 - p_9) = 0.718 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{6,9} \cdot \frac{N}{m^2}, \]
and \[ (p_{15} - p_{16}) = 1.04 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{15,16} \cdot \frac{N}{m^2}. \]

The total pressure drop in the valves, referred to \(\text{Re}_{12,13}\) at the test-section, is \[ \Delta P_{\text{valves}} = 0.000167 \cdot \text{Re}^{1.8}_{12,13} \cdot \frac{N}{m^2}. \]

For the ancillary pipework including bends the appropriate dimensions are:

<table>
<thead>
<tr>
<th>Position</th>
<th>1 - 2</th>
<th>3 - 4</th>
<th>5 - 6</th>
<th>7 - 8</th>
<th>9 - 11</th>
<th>14 - 15</th>
<th>16 - 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2r_w)_{\text{cm}})</td>
<td>\</td>
<td>\</td>
<td>\</td>
<td>\</td>
<td>\</td>
<td>\</td>
<td>\</td>
</tr>
<tr>
<td>(L_{\text{pipe Dia}})</td>
<td>10</td>
<td>36</td>
<td>27</td>
<td>41</td>
<td>143</td>
<td>24</td>
<td>67</td>
</tr>
<tr>
<td>(L_{\text{bends Dia}})</td>
<td>32</td>
<td>32</td>
<td>122</td>
<td>0</td>
<td>64</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>(L_{\text{effective Dia}})</td>
<td>42</td>
<td>68</td>
<td>149</td>
<td>41</td>
<td>207</td>
<td>24</td>
<td>99</td>
</tr>
</tbody>
</table>

The pipe between 9 and 11 includes the flowmeter, but since the pressure drop in turbine-type flowmeters is usually very small its influence was neglected.

Application of Fanning's pressure-drop equation yields:
\[ (p_1 - p_2) = 1.08 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{1,2} \cdot \frac{N}{m^2}, \]
\[ (p_3 - p_4) = 1.75 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{3,4} \cdot \frac{N}{m^2}, \]
\[ (p_5 - p_6) = 6.80 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{5,6} \cdot \frac{N}{m^2}, \]
\[ (p_7 - p_8) = 1.83 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{7,8} \cdot \frac{N}{m^2}, \]
\[ (p_9 - p_{11}) = 21.3 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{9,11} \cdot \frac{N}{m^2}, \]
\[ (p_{14} - p_{15}) = 3.55 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{14,15} \cdot \frac{N}{m^2}, \]
\[ (p_{16} - p_{17}) = 14.7 \cdot 10^{-4} \cdot \text{Re}^{1.8}_{16,17} \cdot \frac{N}{m^2}. \]
The total pressure drop in the ancillary pipework, referred to the test-section Re is therefore:

\[ \Delta p_{\text{pipes}} = 0.00351 \, \text{Re}^{1.8} \, \text{Re}^{1.3} \, \frac{N}{m^2} \]

For the cooler, the pressure drop was estimated from the manufacturers' specification, and the latter may be stated in the following way:

\[ (p_2 - p_3)_{\text{spec.}} = 48 \, \text{kN/m}^2 \], when mass flow rate = 4.0 \, \text{kg/s},

viscosity \( \mu_{\text{spec.}} = 72 \, \text{cP} \), and density \( \rho_{\text{spec.}} = 0.05 \, 10^3 \, \text{kg/m}^3 \).

This data may be put into the same form as the Fanning equation and an effective length, \( L_{\text{cooler}} \), determined. It can be inferred that

\[ (p_2 - p_3) = C \, \frac{\mu^2}{\rho_{12,13}} \, \text{Re}^{1.8} \]

where \( C \) is a constant containing \( L_{\text{cooler}} \).

\( C \) is determined from the data in the specification just quoted, giving

\[ (p_2 - p_3) = 0.0565 \times 10^{-4} \, \frac{\mu^2}{\rho_{12,13}} \, \text{Re}^{1.8} \, \left( \frac{N/m^2}{(10^{-3} \, \text{kg} \cdot \text{m}^3)} \right) \]

hence

\[ (p_2 - p_3) = 0.000565 \, \text{Re}^{1.8} \, \frac{N}{m^2} \]

For the mixing boxes, it was difficult to obtain a reliable estimate of the pressure drop. It was assumed that the arrangement resembled a venturimeter with a contraction ratio of 3:1, this being a guess at the value which would be required for adequate mixing. The well known pressure-drop equation is given by Owezarak (p. 446 Ref: K 8) and states:

\[ (p_{11} - p_{12}) = (p_{13} - p_{14}) = \frac{1}{(2r_{w})_{13,14}} \, \text{Re}^2 \, \text{Re}^{1.3} \, \left( \frac{1}{(2r_{w})_{13,14}} \, \left( \frac{1}{C_d} - 1 \right) (s^2 - 1) \right) \]

Where \( S = 3 \), the contraction ratio,

and \( C_d = 0.95 \), the estimated coefficient of discharge.

Hence

\[ (p_{13} - p_{14}) = 0.000134 \, \text{Re}^2 \, \text{Re}^{1.3} \]
For convenience, this may be approximated in the region of
\[ \text{Re}_{12,13} = 20,000 \] by
\[ (p_{11} - p_{12}) = (p_{13} - p_{14}) \approx 0.000973 \text{ Re}_{12,13}^{1.8}. \]

The pressure drop through the filter \((p_4 - p_5)\) can be reasonably set at zero, but the outlet-end kinetic loss should be assessed, \((p_{17} - p_{18})\). The loss of kinetic energy gives
\[ (p_{17} - p_{18}) = \frac{\bar{u}_{17}^2}{2}, \] where \(\bar{u}_{17}\) is the outlet velocity.

Hence
\[ (p_{17} - p_{18}) = 0.0000768 \text{ Re}_{17}^{2}, \]
or approximately,
\[ (p_{17} - p_{18}) \approx 0.000558 \text{ Re}_{12,13}^{1.8}. \]

Finally the pressure drop requirement at the pump, and hence the maximum system pressure, is
\[ (p_1 - p_{19}) = p_1 = (p_{12} - p_{13}) + \Delta p_{\text{valves}} + \Delta p_{\text{pipes}} + (p_2 - p_3) + (p_{11} - p_{12}) + (p_{13} - p_{14}) + (p_{17} - p_{18}), \]
or
\[ p_1 = 0.00980 \text{ Re}_{12,13}^{1.8} \frac{H}{n^2}. \]

To determine the maximum \(\text{Re}_{12,13}\) which could be obtained with the apparatus, the system pressure drop \((p_1 - p_{19})\) must be matched to the pump's 'pressure-flow rate' characteristic. For a centrifugal pump running at a small fraction of its maximum flow rate the pressure is almost constant (falling slightly with increasing flow rate). For the pump utilised in the apparatus the 'pressure-flow rate' characteristic can be approximated by a constant 717 kN/m². The maximum \(\text{Re}_{12,13}\) which can be obtained with the apparatus is therefore
\[ \text{Re}_{12,13} = \left( \frac{717,000}{0.00980} \right)^{1/1.8} = 23,410. \]

The second requirement is for an assessment of the bulk-temperature distribution in the system (denoted t). The
following additional assumptions are made -

6. The entire apparatus is perfectly insulated.

7. The test-section is made up of 100 diameters of unheated tube followed by 100 diameters of heated tube.

8. The required temperature at the inlet of the tube is 25°C.

9. The maximum power dissipated in the tube is 30 kW.

For the maximum flow rate condition \( Re_{12,13} = 20,000 \), and noting that \( Pr = 100 \) (from the properties in assumption 4.), the approximate temperature difference between the heated half of the test section and the fluid, \( \Delta t_{\text{tube}} \), can be found.

\[
\Delta t_{\text{tube}} = \frac{VI}{\eta (2r_w)_{12,13} L_{12,13} K \text{ Nu}_m}
\]

where \( \text{Nu}_m \) is the mean Nu number for the test section given by the well known equation (discussed in 2.3(iii))

\[
\text{Nu}_m = 0.027 \ Re_{12,13}^{0.8} \ Pr^{0.37},
\]

and becomes

\[
\text{Nu}_m = 346
\]

Hence,

\[
\Delta t_{\text{tube}} = \frac{30,000}{0.025 \times 100 \times 0.337 \times 346} K
\]

or

\[
\Delta t_{\text{tube}} = 32.75 K
\]

The initial specification in 5.2(i) required 4°C approximately, but the value 32.75°C can be considered sufficiently high for experimental purposes.

The temperature of the fluid at the start of heating is given by

\[
t_{12'} = t_{12} + \frac{(p_{12} - p_{12'})}{\theta C}
\]

Hence

\[
t_{12'} = 25 + \frac{0.001475 \times (20,000)^{1.8}}{1.04 \times 10^3 \times 3360} \degree C
\]

or

\[
t_{12'} = 25.023 \degree C
\]
At the outlet end of the test-section, $t_{13}$ is given by

$$t_{13} = t_{12} + \frac{4V_{y_1}}{Re_{12,13} Pr \Pi K (2r_w)^{12,13}} + \frac{(p_{12} - p_{13})}{\rho C}$$

or $t_{13} = 25.023 + \frac{4 \times 30.000}{20,000 \times 100 \times 0.357 \times 0.023} + 0.023 \, ^\circ C$

$$t_{13} = 27.314 \, ^\circ C.$$  

The reservoir temperature, $t_{18}$, is obtained as follows

$$t_{18} = t_{13} \frac{(p_{13} - p_{18})}{\rho C}$$

where $(p_{13} - p_{18}) = 0.00346 \, Re_{12,13}^{1.8}$ from the pressure drop calculations, hence

$$t_{18} = 27.314 + \frac{0.00346 \times (20,000)^{1.8}}{1.04 \times 10^3 \times 3360} \, ^\circ C,$$

$$t_{18} = 27.369 \, ^\circ C.$$  

It is assumed that the temperature at the pump inlet is the same as the reservoir temperature

$$t_{19} = 27.369 \, ^\circ C.$$  

At the pump outlet the temperature is given by

$$t_1 = t_{19} + \frac{(p_1 - p_{19})}{\rho C} \left( \frac{1}{\eta} - 1 \right),$$

where $\eta$ = the hydraulic efficiency of the pump,

so that $t_1 = 27.369 + \frac{0.00980 \times (20,000)^{1.8}}{1.04 \times 10^3 \times 3360} \left( \frac{1}{0.30} - 1 \right) \, ^\circ C,$

$$t_1 = 27.731 \, ^\circ C.$$  

The estimate for pumping efficiency was 30%, which appears to be a low value because of the operating conditions; the pump-power was 1/5th the rated load. It is as well to underestimate the efficiency to ensure that the temperature rise is at least as great as would be achieved in practice.
The temperature at the inlet to the cooler, \( t_2 \), is derived in the same way as \( t_{12} \), hence

\[
t_2 = 27.732 \, ^\circ C.
\]

At the outlet end of the cooler the temperature, \( t_3 \), is obtained from \( t_{12} \) at the test-section as follows -

\[
t_3 = t_{12} - \frac{(p_3 - p_{12})}{\rho c}
\]

where \( (p_3 - p_{12}) \) is the sum of the pressure drops calculated between (3) and (12).

Therefore

\[
t_3 = 25 - 0.046 \, ^\circ C
\]

\[
t_3 = 24.954 \, ^\circ C.
\]

The heat which the cooler must dissipate, \( Q\text{\_cooler} \), is given by

\[
Q\text{\_cooler} = \frac{\pi R e \text{\_Pr} \cdot K \cdot (r_w)_{12,13} \cdot (t_2 - t_3)}{2}
\]

\[
Q\text{\_cooler} = 36.76 \, kW.
\]

As a check, the pumping power, \( Q\text{\_pump} \), and test-section power, \( Q\text{\_test} \), can be utilised.

\[
\bar{Q}\text{\_cooler} = Q\text{\_test} + Q\text{\_pump},
\]

therefore

\[
\bar{Q}\text{\_cooler} = 30 \, kW + \frac{(p_1 - p_{12})}{\rho} \cdot \frac{\mu}{\rho} \cdot (r_w)_{12,13} \cdot Re_{12,13},
\]

and

\[
\bar{Q}\text{\_cooler} = 36.80 \, kW.
\]

A small, 'rounding off' error is evident in \( Q\text{\_cooler} \).

The overall coefficient of heat transfer of the cooler is not easy to assess, therefore the size necessary to dissipate \( \bar{Q}\text{\_cooler} \) (that is the number of tubes, diameter of tubes, tube-spacing and so on) was best determined by the manufacturer. The following information was specified - the density, specific heat, viscosity, thermal conductivity and temperature at the inlet and outlet of the cooler on the glycol side, also the inlet temperature on the water side. The cooler in this apparatus was incorporated on the basis of the specification given in the appendix (A).
The figures given refer to a much more viscous fluid (72 cP) so that the temperature difference between oil and water, or the temperature rise on the water side, appear much greater than they would be for a less viscous fluid (say 10 cP).

Supposing a 2.5 K rise in the temperature of the cooling water, $\Delta t_{\text{water}}$, then the required rate of flow is given by a simple heat balance:

$$\text{water mass-flow rate} = \frac{\text{glycol mass flow rate} \times C_v (t_2 - t_3)}{C_{\text{water}} \Delta t_{\text{water}}}$$

$$= 3.93 \left( \frac{\text{kg}}{\text{s}} \right) \times \frac{3.36 \times 10^3}{4.18 \times 10^3} \frac{2.78}{2.5}$$

Water mass-flow rate = 3.51 kg/s.

To check whether the cooling flow rate can be attained, the maximum flow must be calculated for the water-supply system. The cooling water was supplied to the heat exchanger from a constant head tank with a fall of 4.6 m, through a 5 cm pipe, the outlet end being open to the drain. An estimate of the maximum flow rate possible under these conditions is as follows:

Say $z =$ the head of water,

$\Delta p_c =$ the pressure drop on the water side of the cooler,

$\dot{m} =$ mass-flow of water,

subscript $w'$ refers to the cooling flow.

Now

$$\dot{m} \left( \frac{z}{2 r_w} \right) \left( \frac{v^2}{2} \right)_{w'} + \Delta p_c$$

is the energy equation, and $\Delta p_c$ can be obtained with reference to the Fanning equation putting $\Delta p_c = \frac{N \dot{m}^{1.8}}{w'}$ where $N$ is a constant.

$N$ is determined from the manufacturers specification (appendix (A)) which states $\Delta p_c_{\text{spec.}} = 13.8 \text{ kN/m}^2$ when $\dot{m}_{\text{spec.}} = 3.79 \text{ kg/s}$ for water. Hence,

$$\Delta p_c = 4.22 \times 10^3 \frac{N}{w'}^{1.8} \frac{(\text{kg})^{1.8}}{(\text{m}^2)^{1.8}}$$
Inserting the value \( \dot{w} = 10^3 \text{ kg/m}^3 \) gives

\[
45,200 = 500 \ddot{w}^2 + 1,605 \ddot{w}^1 \cdot \ddot{w} + 4,220 \ddot{w}^1 \cdot \ddot{w},
\]

or \( \ddot{w} = 2.95 \text{ m/s} \),

so that \( \dot{w} = 5.80 \text{ kg/s} \).

This is nearly twice the estimated requirement.

The next step is to determine the true dimensions of the test-section, and these are readily obtained from consideration of the electrical resistance of the tube. The nominal tube size chosen in the initial specification, 5.2(i), was 2.5 cm. The actual sizes of the experimental tubes were to some extent limited by the ability of the manufacturer to supply them. Ideally, an infinitely thin wall thickness is desirable to eliminate thermal conduction. Unfortunately this implies negligible electrical conduction. A compromise (further discussed in part 5.3) was reached which led to the selection of stainless steel for the test-section material; this has one of the lowest thermal conductivities of the more common metals, i.e. nominally 15.5 \( \text{ W/mK} \). The electrical resistivity of stainless steel (type 321 AISI) is approximately \( \rho = 0.72 \times 10^6 \Omega \text{m} \). For a 2.5 cm. bore the wall thickness, \( \Delta \), is calculated as follows:

If the maximum dissipation of heat is required, then \( V = 30 \text{ volts and } I = 1,000 \text{ amp} \). The thickness necessary for a 100 diameter length is given by

\[
\frac{V}{I} = \frac{\rho L}{\pi (2r_w + \Delta) \Delta}
\]

this case becomes

\[
\frac{30}{1,000} = \frac{0.72 \times 10^6}{\pi (0.025 + \Delta) \Delta}
\]

or \( \Delta = 0.072 \text{ cm} \).

The value of \( \Delta \) stated is the optimum size, but small departures from the calculated value are acceptable in practice.
5.3. THE EXPERIMENTAL TUBES AND FITTINGS.

5.3.(i) SPECIFICATIONS.

Having designed the apparatus it was necessary to select the actual experimental tubes which would give a desireable range of operating conditions. The specification of these tubes is as follows:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Inner dia. (cm)</th>
<th>Wall thickness/Variation (cm)</th>
<th>Length (m)</th>
<th>Roundness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short tube</td>
<td>2.616</td>
<td>0.121/0.004</td>
<td>5.2</td>
<td>0.0056</td>
</tr>
<tr>
<td>Divergence &amp; Convergence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio 3.34:1</td>
<td>1.709</td>
<td>0.265/0.010</td>
<td>1.8</td>
<td>0.0020</td>
</tr>
<tr>
<td></td>
<td>5.715</td>
<td>0.0952/0.005</td>
<td>2.8</td>
<td>0.0300</td>
</tr>
<tr>
<td>Ratio 1.99:1</td>
<td>2.555</td>
<td>0.159/0.002</td>
<td>2.8</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td>5.088</td>
<td>0.0788/0.002</td>
<td>2.8</td>
<td>0.0074</td>
</tr>
<tr>
<td>Ratio 1.25:1</td>
<td>2.557</td>
<td>0.118/0.010</td>
<td>2.5</td>
<td>0.0040</td>
</tr>
<tr>
<td></td>
<td>3.189</td>
<td>0.103/0.004</td>
<td>3.1</td>
<td>0.0039</td>
</tr>
</tbody>
</table>

All tubes: Material - AISI Type 321 stainless steel

Straightness - 1 in 600

Surface finish of the bore - $3 \mu$m

Method of manufacture - cold drawn and seamless

The above measurements were taken on samples cut from both ends of each tube because in the manufacture of cold drawn seamless tubing the dimensional variations normally increase with the distance between the sampling points. Stainless steel was chosen because of its relatively low thermal and electrical conductivities in comparison with other common metals, also its high resistance to corrosion. The low conductivities imply that the tube wall need not be unreasonably thin in order to obtain a given heat flux, also the distortion of the temperature
distribution in the tube, due to the axial conduction of heat, is minimised because of the high thermal resistance.

The surface finish of the inner tube surfaces was such that they could be described as hydraulically smooth up to Reynolds numbers of the order $10^5$, that is the frictional force was independent of the roughness at the surface according to the findings of Moody (these are well known, e.g., Ref. K3.)

When two tubes were connected in series, the selection of the tube wall thicknesses was such that the heat generated per unit length was the same for both tubes. This was a matter of practical convenience, since the condition stated gave rise to a nearly linear rise in the bulk temperature of the fluid, a similar surface temperature for each tube, and a practical thickness to diameter ratio for each tube. No other criterion appeared to have any advantage over the one chosen, and by keeping the copper leads well away from the diameter discontinuity, the possibility of distorting the temperature distribution in this region was eliminated.

5.3. (ii) THE TUBE FITTINGS.

The fittings for connecting the test-sections to the mixing-boxes, and to each other (in the case of a diameter discontinuity), are shown in figures 5.5. and 5.6.

The brass fittings utilised to produce a sudden change in diameter were of high electrical conductivity to transfer the potential from the upstream to the downstream tube with a minimal volt drop. The brass was brazed to the stainless steel ensuring an even distribution of solder in the process, thereby achieving a good electrical contact. In between the two parts of each fitting a good electrical contact and hydraulic sealing were established by the insertion of a thin, brass shim which was compressed by eight,
small securing bolts, these being evenly tensioned around the periphery of the flanges. Functional tests indicated no unevenness of heating close to a fitting which would have indicated a poor electrical contact. Voltage drops measured across the 3.34:1 fitting were entirely negligible being of the order $5 \times 10^{-4} \text{v}$.

The copper leads were attached to the test-sections using the copper straps shown in figures 5,7. The contact area was machined to the tube outside diameter whilst the upper and lower portions were separated by 0.008cm., this amount being allowed for the compression strain when tightening the strap.

5.3(iii) END EFFECTS.

It has been said (5.3 ii) that a high electrical conductivity was desirable in certain fittings attached to the test-sections. This implied high thermal conductivity as well, which was clearly undesirable, since these fittings could act as a sink and distort the heat flux at the inner surface of the tube. In the case of the short-tube experiments, a practical solution was found by fixing a heater to the copper strap (to be discussed in 5.5). The brass fittings joining two tubes of different diameter could not be dealt with in this way, and it was necessary to try to analyse the system thermoelectrically to discover the extent of the problem, and the implication of ignoring this 'end effect' altogether.

In part 3.8, it is pointed out that there is no generalised procedure available for handling this kind of conduction problem, and an analysis must be carried out.

Assume that the system shown in figure 5.1, closely resembles the region of a diameter discontinuity, and say that -
(a) The following take on constant values:

- the fluid temperature throughout,
- the coefficient of heat transfer on all inside surfaces,
- all the physical properties of the fluid and tube,
- the tube and fitting thicknesses.

(b) The tube and fitting walls are thin.

(c) The exterior surfaces are ideally insulated.

\[ h = \text{coefficient of heat transfer at inner surface}. \]

**Figure 5.1.**

The problem is to determine how the temperature in the tube upstream \( t_i \), effects the temperature distribution in the downstream tube. The diagram below indicates this.

No conduction through fitting. \hspace{1cm} Conduction through fitting.

Considering conduction in the tube wall; thermal conductivity \( K_t \).

Put \( \phi = (t - t_L) = \phi(x) \), for a thin wall.

so that

\[
\left( \frac{1}{2\pi K_t \Delta} \frac{dv}{dx} \right) + \phi'' = \left( \frac{h}{K_t \Delta} \right) \phi, \hspace{1cm} (5.1)
\]
and since
\[ \phi_n = \left( \frac{1}{2 \pi n h} \right) \frac{d \phi}{dx} \]  
(5.2)
then
\[ \phi'' + \left( \frac{h}{k_1 \Delta} \right) (\phi_n - \phi) = 0 \]  
(5.3)
now
\[ \phi (0) = \phi_o, \quad \text{and} \quad \phi (\infty) = \phi_{\infty}, \quad \text{so that} \]
\[ (\phi - \phi_o) = (\phi_o - \phi_{\infty}) \exp \left( - \left( \frac{h}{k_1 \Delta} \right)^{\frac{1}{3}} x \right). \]  
(5.4)
\[ \phi_o \] may be derived in terms of \( \phi \), by considering conduction in the fitting conductivity \( k_1 \).

Now \[ \phi = \phi (y) \] for a thin wall,

hence
\[ \phi'' + \frac{\phi'}{y} - \frac{h}{k_1 d} \phi = 0 \]  
(5.5)
The solution of this equation is
\[ \phi = A \mathcal{I}_n \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} y + B \mathcal{K}_n \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} y \]  
(5.6)
with
\[ \frac{d \phi}{dy} = \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} \left( A \mathcal{I}_n - B \mathcal{K}_n \right). \]  
(5.7)
\( \mathcal{I}_n \) and \( \mathcal{K}_n \) are Modified Bessel's Functions of the first and second kind, and order \( n \).

By matching \( \phi(y) \) and \( \phi'(y) \) at \( y = R \) with \( \phi(x) \) and \( \phi'(x) \) at \( x = 0 \), the relationship
\[ \phi = f \left( \phi_1, \phi_o, x, \frac{h}{k_1 d}, \frac{h}{k_1 \Delta}, x, R \right) \]
may be found.

A simple result may be obtained as \( x \to 1 \), as follows:

In the fitting \( \phi'' \gg \frac{\phi'}{y} \),

hence
\[ \phi'' = \left( \frac{h}{k_1 d} \right) \phi \]

Now since \( \phi (R) = \phi_o \), \( \phi' (R) = \phi'_o \),

and
\[ \phi' = \mathcal{K} \exp \left( \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} y \right) + \mathcal{K} \exp \left( - \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} y \right) \],

(5.8)
then
\[ \phi_1 = \phi_o \cosh \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} (R - x) - \phi'_o \left( \frac{k_1 d}{h} \right)^{\frac{1}{3}} \sinh \left( \frac{h}{k_1 d} \right)^{\frac{1}{3}} (R - x). \]  
(5.9)
Now \( \rho'_0 (y) \) and \( \rho'_0 (y) \) may be matched with \( \rho'_0 (x) \) and \( \rho'_0 (x) \).

\[
\rho'_0 (y) = \rho'_0 (x),
\]

and
\[
\rho'_0 (y) = \frac{K_t \Delta}{k_f d} \rho'_0 (x).
\]

But \( \rho'_0 (x) \) is known to be
\[
\rho'_0 (x) = \pm \left( \rho_\infty - \rho_0 \right) \left( \frac{h}{K_t \Delta} \right)^{\frac{1}{3}}, \quad \text{ve. when } r > R,
\]

therefore all terms in \( \rho_0 \) and \( \rho'_0 \) may be replaced by \( \rho_\infty \) and \( \rho_1 \),
giving (after reverting to "t" notation):
\[
\frac{t - t_L}{t_\infty - t_L} = 1 + \exp \left\{ - \left[ \left( \frac{hR}{K_t \Delta} \right)^{\frac{1}{2}} \frac{x}{R} \right] \right\}
\]

(5.10)

where
\[
Z = \left( \frac{t_1 - t_L}{t_\infty - t_L} \right) - \cosh \left[ \left( \frac{hR}{k_f \delta} \right)^{\frac{1}{2}} \left( 1 - \frac{x}{R} \right) \right]
\]

\[
\cosh \left[ \left( \frac{hR}{k_f \delta} \right)^{\frac{1}{2}} \left( 1 - \frac{x}{R} \right) \right] - \frac{K_t \Delta}{k_f d} \sinh \left[ \left( \frac{hR}{k_f \delta} \right)^{\frac{1}{2}} \left( 1 - \frac{x}{R} \right) \right]
\]

Clearly, the fittings which would permit the greatest
distortion of the heat flux in the downstream-tube for this
apparatus would be those having 1.25:1 ratio, since \( \frac{r}{R} \) would be
a minimum. Also, the experiments most likely to be affected
would be those where small heat-transfer coefficients were en-
countered, such as when utilizing fluids of low viscosity at low
Reynolds numbers.

Before estimating the temperature distribution for a
typical case, it should be mentioned that the above equation is
valid for \( r > R \) with a difference sign in the denominator.

The following data applies in the case of the 1.25:1
convergence-ratio - \( R = 1.27 \text{ cm.}, r = 1.59 \text{ cm.}, \delta = 0.1 \text{ cm.}, \)
\( \Delta = 0.5 \text{ cm.}, K_t = 15.5 \frac{W}{mK}, k_f = 100 \frac{W}{mK} \).
Assuming a typically small value for the coefficient of heat transfer of 100 \( \frac{w}{m^2 K} \), the temperature difference required is given within 1\% by the limiting case as \( hR \frac{R}{K} \frac{1}{(1 - \frac{x}{R})} \to 0 \), that is

\[
\frac{t - t_L}{t_{\infty} - t_L} = 1 + \left[ \frac{(t_1 - t_L)}{(t_{\infty} - t_L)} - 1 \right] \exp \left\{-\left( \frac{hR}{K_t} \frac{R}{\Delta} \frac{x}{R} \right)^{\frac{1}{2}} \right\}, \quad (5.11)
\]

giving \( \left( \frac{t - t_{\infty}}{t_1 - t_{\infty}} \right) = \exp \left\{-1.02 \frac{x}{R} \right\} \), which becomes 0.1 as \( \frac{x}{R} \to 2.26 \)

Hence \( \frac{t}{R} = 2.26 - t_{\infty} = 0.1 \left( t_1 - t_{\infty} \right) \). \quad (5.12)

Now rewriting in terms of \( Nu_1 \) and \( Nu_2 \), the Nusselt numbers upstream and downstream away from the discontinuity, yields

\[
\frac{(t_X/R) = 2.26 - t_{\infty}}{(t_{\infty} - t_L)} = 0.1 \left[ \frac{Nu_2}{Nu_1} - 1 \right] = \xi_{2.26} \text{ say.}
\]

Making the further assumption that \( Nu \propto Re^{\frac{1}{2}} \) (which seems plausible in view of the previous discussion on laminar heat transfer in part 2.3.1), the equation becomes

\[
\xi_{2.26} = 0.1 \left[ \left( \frac{x}{R} \right)^{\frac{1}{3}} - 1 \right], \text{ where } \xi_{2.26} \text{ is the approximate error incurred by neglecting axial conduction in the determination of the local coefficient of heat transfer. The greatest error would occur when } \left( \frac{x}{R} \right) = 3.34, \text{ giving } \xi_{2.26} = 5\%.
\]

The result of this analysis can be stated simply; in this, the most severe case likely to be encountered during experimentation, the effects of 'end-conduction' on heat transfer in the downstream tube are probably confined to an initial length of one diameter. In most experiments this length is likely to be much less.

It is worth noting that the typical value for the coefficient of heat transfer, \( h \), was given approximately by the
numerical equation \( h = 10. \frac{\text{Nu}}{m^2 \ K} \) for propylene glycol, where a low value of 10 was allotted to \( \text{Nu} \) (the local value of \( \text{Nu} \) near to the diameter-discontinuity). A similar equation for gaseous media (where the thermal conductivity is approximately one tenth that of glycol) would be \( h_{\text{gas}} = 1. \frac{\text{Nu}}{m^2 \ K} \).

This would yield a value for \( \frac{\text{Nu}}{R} \) of 7.2, when \( t - t_c \) reaches 95\% of its limiting value \( (t_{\infty} - t_c) \). It would appear that the present experimental configuration is inadequate for investigating heat transfer with gases, since axial conduction could lead to erroneous results when determining \( h \) in the region of the first 3.6 tube diameters downstream of the diameter discontinuity.

5.4. THERMOCOUPLES AND VOLTAGE MEASUREMENTS.

The thermocouples were made from 3\( \mu \)m lengths of 0.0027 cm. diameter Nichrome and Constantan which yield a high voltage-temperature gradient whilst having a low thermal conductivity. The cold junctions were formed by soft soldering each wire to a copper lead then setting each pair in paraffin wax, contained in a 0.5 cm. diameter, glass tube. These glass tubes were immersed in Dewar flasks filled with water and ice chips. The copper leads were connected to a vernier potentiometer via a selector switch, and the thermocouple e.m.f.’s were measured by comparison with a standard cell, the voltage being given as the potentiometer setting corresponding to a null reading galvanometer. The hot junctions were formed by resistance-welding the two wires either in-parallel or in-line. The latter configuration was required for the couples positioned inside the mixing boxes, and the former was required in all other applications. The mixing-box junctions were bead-shaped, but the test-section
junctions were flattened into a spade-shape for fixing to the meter surfaces.

The diagram (figure 5.8) shows how a cruciform array of ten thermocouples was positioned in each mixing box so that local temperatures in the flow could be measured downstream of the venture-type mixing tube. The flow was considered to be adequately mixed when the ten local temperature measurements were the same. A range of mixing tubes was available with throat diameters from 0.3 to 1.5 cm., and a suitable size could be chosen to produce a high Reynolds number at the throat, thereby causing a leveling of the temperature in the fluid whilst minimising the pressure drop necessary for mixing to occur.

The greater part of each mixing box was manufactured from 'Tufnol' and 'Perspex' plastics, which are both good thermal and electrical insulators. The test-section was thus isolated so that practically no loss of heat or current could occur through the ends of the experimental tube. A transparent section was incorporated into the boxes to permit observation of the fluid during the experiments, thus enabling incipient fouling or air bubbling to be detected. Sealing was effected by the use of rubber O-rings, between the demountable parts, p.t.f.e. tape was compressed between the threaded joints, and silicone-rubber (which cured at room temperature) was used as a sealant around the thermocouple leads.

The method of affixing thermocouples to the outer surface of the test-section was as follows. At a given axial location the periphery of the tube was covered with a strip of adhesive, cellophane tape of thickness 0.005 cm. This served to insulate the thermocouple electrically whilst presenting negligible thermal
resistance, so that the temperature drop across the tape was immeasurably small (an estimate being 0.025% of the temperature difference used in calculating the coefficient of heat transfer). The thermocouple wire was bound firmly onto the surface of the tube with several layers of the adhesive tape, so that, at a given axial position a minimum of 5 cm. of the lead encircled the tube. Since the peripheral region described was almost isothermal, the possibility of heat-conduction in the leads affecting the readings was avoided.

The voltage supplied at the test-section was measured with the same potentiometer as the thermocouple e.m.f.'s. A voltage divider, ratio 50:1, was included into the circuit to reduce the potential to a manageable level. The volt drop in the calibrated shunt was measured directly with the vernier potentiometer, the resistance being only 5 μΩ.

5.5. FUNCTIONAL TESTS AND CALIBRATION OF THERMOCOUPLES.

Calibration of the thermocouples was carried out in strict adherence to the National Physical Laboratory recommendations (given in "Calibration of Temperature Measuring Instruments" H.M.S.O. 1955, code 48/120/12). Six sample thermocouples were taken from the test-section and the 'hot' junctions set in paraffin wax in glass tubes. These tubes were immersed in a uniform-temperature, recirculating water-bath (as described on page 28 of the publication) and the thermocouple readings were recorded for bath temperatures of 10 to 90°C, in approximately 4K increments. The bath temperature, which was continually rising at a slow rate, was measured with N.P.L. mercury-in-glass thermometers to an accuracy of ± 0.02 K. The calibration was checked at six monthly intervals.
The reliability of the calibration could be expressed as a tolerance of ± 2 μV, or ± 0.05 K. This differed from the accuracy of the standard because of non-uniformities in the thermocouple wire.

The method of fixing the thermocouples to the test-section was investigated by positioning three hot junctions in close proximity on the tube, using three separate techniques, as below -

with no applied voltage, fluid at 30°C was circulated around the system and the three thermocouple readings were found to be identical, thus supporting the selection of method (1) for affixing the hot junctions.

The 2.616 cm. bore tube was positioned so that 103 diameters of unheated length preceded 93 diameters of heated length. Thermocouples were distributed axially along the tube to investigate the possibility of the heavy copper lead, at the start of heating, distorting the local heat-flux. Figure 5.9, shows how during a test at zero applied voltage, with the heat-transfer fluid hotter than ambient, an axial temperature gradient occurred along the tube wall near to the start of heating. This problem was alleviated by fitting a 0 to 50 W electrical heater to the lead at the point of departure from the insulated box. The best operating condition was found to be when the heater was adjusted to give zero temperature
gradient in the copper lead, as recorded by two spaced thermocouples. Figure 5.9. shows the distinct improvement which ensued.

Further functional testing was carried out including the continuous reading of flow rate, which was found to vary with time within 1%. The frequency meter was calibrated against a signal generator over 0 to 1000 c/s, and the two turbine-type flowmeters were calibrated (by the manufacturer) with fluids having kinematic viscosities of 1, 25 and 50 cSt. This type of meter is very reliable, and can be used to measure flow rates with an accuracy of about \( \pm \frac{\%}{100} \) (for further details of turbine flow-meters, see the papers in Refs: (K9, K10, K11, K12, K13.)

The continuous reading of potential difference at the test-section was carried out with an 'ultra-violet recorder'. The voltage varied by approximately \( \pm 1\% \) due to fluctuations in the grid-supply, and at certain peak consumption periods the fluctuations became rapid, so it was considered inadvisable to conduct tests at such times.

A simple functional test was carried out on the thermocouple selector switch to reveal any spurious e.m.f.'s; though the probability of these occurring was small. The switch was actuated quickly and repeatedly, whilst a fan heated the contacts to approximately 40\(^\circ\)C. The thermocouple readings were found to be unaffected by the action taken.

As a further check on the reliability of the tube-thermocouple readings, a comparison was made between the mixing box and test-section temperatures during an 'isothermal' test. With 12 thermocouples distributed axially along the 5.715 cm. diameter tube, and with a 75% glycol solution recirculating at a high rate of flow, the mixing box and test-section temperatures
were recorded. Since zero voltage was applied to the tube, the inlet and outlet temperatures were equal (any difference was immeasurable). The temperature at the tube-exterior was slightly less than the fluid temperature, due in part to conduction through the wall, and in part to imperfect contact of the thermocouples or conduction in the leads. The former was readily assessed, but the latter was estimated from the stated readings. The depression of the thermocouple readings was found to increase with temperature in the following way:

<table>
<thead>
<tr>
<th>Fluid temperature (°C)</th>
<th>28</th>
<th>37</th>
<th>69</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error in thermocouple readings (K)</td>
<td>-0.02</td>
<td>-0.04</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

(Ambient temperature 23°C)

These deviations were tolerated because they represented a small limitation on the accuracy of the measured coefficients of heat transfer - typically <0.5%. The method of applying the thermocouples to the test-section was kept simple in view of the large number of locations which were found necessary; approximately 700 applications were used in the experiments.

5.6. THE HEAT BALANCE.

Generally, in heat-transfer experiments it is advisable to attempt an assessment of the distribution of energy around a boundary which encloses the system. The outcome may not be particularly useful in itself, but an indication of experimental reliability is usually given which helps to justify conclusions based on the results; furthermore it is often possible to assess limiting operational conditions for the apparatus. An example might be when the heat flux in an experiment is limited by the heat loss through the insulation. It is clearly undesirable to rely
heavily upon an estimate for conduction losses when determining
the forced-convection coefficient of heat transfer. The heat
input to the system should be sufficiently high to limit the
insulation losses to say 5%.

When aqueous propylene-glycol was used as the heat-
transfer fluid, it was found that the increase in temperature
between the inlet and outlet of the test-section was very small;
in most of the experiments carried out the magnitude was less
than 0.5 K. Such small temperature differences were difficult to
determine reliably, so it was decided that heat balances would be
carried out only when the temperature rise was greater than 0.5 K;
the probably accuracy of these temperature measurements being
0.02 K, or a maximum of 4% of the temperature rise.

The following calculations relate to the initial experi-
ments carried out where the rise in bulk temperature was greater
than 0.5 K. Out of the first 39 tests only 10 met this requirement.

with high Prandtl numbers the axial temperature gradient
in the fluid is small, even when the tube to fluid temperature
difference is comparatively large. This means that the local
bulk temperature of the fluid is well approximated by the inlet
temperature. It can be reasoned therefore that large errors in
the heat balance do not necessarily reflect limitations upon the
accuracy of the measured coefficients of heat transfer.

The distribution of heat around the system is as follows:

(A) Electrical energy input. +ve
(B) Heat conducted in copper leads.
(C) Frictional dissipation energy.
(D) Conduction at tube ends.
(E) Conduction through insulation.
(F) Energy in fluid at inlet.
(G) Energy in fluid at outlet.

\[ \sum (A),(B),(C),(D),(E),(F),(G) = 0 \] for equilibrium.
Overleaf (Figure 5.2.) is given a table of the estimated distribution of energy for the cases just mentioned. The experiments were concerned with the heat transfer in a short, uniform diameter tube, and the tests selected included Reynolds numbers in the range 268 to 9,076, with Prandtl numbers in the range 184 to 595. The average temperature difference from tube to fluid varied between 8 and 22 K.

A specimen calculation is now detailed.

Data for test 63. Fluid = 94.6% aqueous propylene-glycol.

Voltage $V = 10.015$ v. Current $I = 544.0$ A.

Heated length of tube $L_H = 93$ diameters. Unheated length, upstream $L_U = 103$ dia. Thermal conductivities: Insulation $K_{ins} = 0.033 \frac{W}{mK}$, copper $K_c = 384 \frac{W}{mK}$. Radii: Tube exterior $R_A = 1.429$ cm., interior $R_i = 1.308$ cm., outer insulation $R_B = 15$ cm.

Temperature gradient in copper leads: Inlet $\xi_i = +0.042 \frac{K}{cm}$, outlet $\xi_o = -0.180 \frac{K}{cm}$.

Temperatures: Average over heated length $T_p = 48.0$ °C, ambient $T_A = 21.8$ °C, fluid inlet $T_i = 24.54$ °C, fluid outlet $T_o = 25.50$ °C.

Volume flow rate $F = 2.67 \times 10^{-3} m^3$. Cross section of lead $A = 2.5 \times 2.5 cm^2$.

(A) Electrical power = $10.015 \times 544.0 = 5448$ w.

(B) Total heat loss by lead conduction $Q_{cond} = K_c A (\xi_i + \xi_o)$

$$Q_{cond} = 384 \times 0.025^2 (-0.042 + 0.180) 10^2$$

$$Q_{cond} = 3.31 \text{ w}$$

(C) Axial conduction from ends of tube $\approx 0$ w.

Axial conduction of heat must be considerably less than radial conduction.

(D) Conduction through insulation, $Q_{ins} = \frac{2\pi K_{ins} \Delta T}{\ln \left(\frac{R_B}{R_A}\right)}$
<table>
<thead>
<tr>
<th>Test No.</th>
<th>6s</th>
<th>10s</th>
<th>14s</th>
<th>17s</th>
<th>18s</th>
<th>22s</th>
<th>31s</th>
<th>32s</th>
<th>33s</th>
<th>37s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical (W)</td>
<td>5448</td>
<td>4375</td>
<td>7530</td>
<td>4155</td>
<td>7520</td>
<td>5185</td>
<td>422.7</td>
<td>231.2</td>
<td>821.0</td>
<td>959.0</td>
</tr>
<tr>
<td>(A) %</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Frictional (W)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D) %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total (W)</td>
<td>5448</td>
<td>4375</td>
<td>7530</td>
<td>4155</td>
<td>7520</td>
<td>5185</td>
<td>422.7</td>
<td>231.2</td>
<td>821.0</td>
<td>959.0</td>
</tr>
<tr>
<td>Copper Leads (W)</td>
<td>3.3</td>
<td>3.8</td>
<td>6.6</td>
<td>6.3</td>
<td>5.5</td>
<td>6.0</td>
<td>6.4</td>
<td>7.5</td>
<td>6.7</td>
<td>4.8</td>
</tr>
<tr>
<td>(B) %</td>
<td>0.06</td>
<td>0.07</td>
<td>0.09</td>
<td>0.15</td>
<td>0.07</td>
<td>0.12</td>
<td>1.51</td>
<td>3.26</td>
<td>0.80</td>
<td>0.48</td>
</tr>
<tr>
<td>Tube Ends (W)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C) %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insulation (W)</td>
<td>6.2</td>
<td>6.0</td>
<td>10.9</td>
<td>9.6</td>
<td>11.1</td>
<td>9.7</td>
<td>6.6</td>
<td>8.5</td>
<td>11.6</td>
<td>2.4</td>
</tr>
<tr>
<td>(D) %</td>
<td>0.11</td>
<td>0.13</td>
<td>0.14</td>
<td>0.22</td>
<td>0.14</td>
<td>0.19</td>
<td>1.55</td>
<td>3.69</td>
<td>1.38</td>
<td>0.24</td>
</tr>
<tr>
<td>Fluid (W)</td>
<td>5522</td>
<td>4375</td>
<td>7670</td>
<td>4270</td>
<td>7715</td>
<td>5140</td>
<td>411.5</td>
<td>214.2</td>
<td>821.0</td>
<td>990.0</td>
</tr>
<tr>
<td>(E)-(F) %</td>
<td>99.83</td>
<td>99.80</td>
<td>99.77</td>
<td>99.63</td>
<td>99.79</td>
<td>99.69</td>
<td>96.94</td>
<td>95.05</td>
<td>97.82</td>
<td>99.28</td>
</tr>
<tr>
<td>Total (W)</td>
<td>5531</td>
<td>4385</td>
<td>7688</td>
<td>4286</td>
<td>7732</td>
<td>5156</td>
<td>424.6</td>
<td>230.2</td>
<td>839.0</td>
<td>997.0</td>
</tr>
<tr>
<td>Balance (W)</td>
<td>+83</td>
<td>+10</td>
<td>+158</td>
<td>+131</td>
<td>+212</td>
<td>-29</td>
<td>+1.9</td>
<td>-1.0</td>
<td>+18</td>
<td>+38</td>
</tr>
<tr>
<td>%</td>
<td>1.5</td>
<td>0.2</td>
<td>2.1</td>
<td>3.0</td>
<td>2.8</td>
<td>-0.6</td>
<td>0.4</td>
<td>-0.4</td>
<td>2.2</td>
<td>3.9</td>
</tr>
</tbody>
</table>

The heat addition by viscous dissipation is assumed to be small.

The heat loss from the tube ends is assumed to be small.
\[
\Delta T = \left[ L_H (T_p - T_A) + L_u (T_i - T_A) \right]
\]

\[
Q_{\text{ins}} = \frac{2\pi \cdot 0.033}{\log (10.5)} \left[ 2.433 \cdot (26.2) + 2.694 \cdot (2.7) \right]
\]

\[
\frac{Q_{\text{ins}}}{6.2 \text{ w}}
\]

(E) and (F). Enthalpy gain of fluid, \( q_{\text{fluid}} = (\cdot F \cdot C \cdot (T_0 - T_i) \]

\[
C_\text{water} = 2.63 \cdot 10^3 \frac{\text{kJ}}{\text{kg \cdot ^\circ C}} \quad C_\text{glycol} = 1.035 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}
\]

\[
Q_{\text{fluid}} = 1.035 \cdot 2.67 \cdot 2.63 \cdot 10^3 \cdot (0.76)
\]

\[
Q_{\text{fluid}} = 5.522 \text{ w}
\]

(G). The frictional dissipation energy can be considered negligible for the present purposes. The design calculations in 5.2.(iii) indicate that this is not unreasonable.

In conclusion, a satisfactory heat balance was obtained under favourable operating conditions. Of the power input to the system, no more than 20 w was dispersed to the surroundings, so that care was taken when operating with less than 400 w heating rate to ensure that insulation losses were less than 5%. In only one test did they exceed 3%.

5.7. MEASUREMENT OF GLYCOL COMPOSITION, AND PROPERTIES.

The percentage by weight of water in the glycol solutions was measured by two methods. The first, direct procedure was to carry out a titration with Karl Fischer reagent. This technique is described in B.S.2511:1954. Whereas the results were accurate - the glycol content being determined within \( \frac{3}{3} \% \) - the process was a slow one since the apparatus had to be cleaned and dried before use. A day to day check was kept by an indirect method, which consisted of measuring the density of the solution and consulting a chart of density, composition and temperature. To ensure a high degree
of precision, the density of 10 known solutions was measured over a range of temperature, with B.S. 718 hydrometers. Comparative densities were used to estimate composition and excellent agreement with the titration results was obtained, the repeatability being expressible as $\pm 0.5\%$ of the glycol content.

The major source of property data was the book "Glycols" by Curme and Johnson (Ref. K15). Other references consulted were K16, K17 and K18.

The actual data utilised in this thesis is tabulated in figures 5.10a to 5.10d.

To substantiate the selection of the property values some random checks were made on 70% and 100% glycol solutions at temperatures in the range 20 to 30°C. The density was measured as described above for an extended range of dilutions and temperatures and the data of Curme agreed within 0.05% of these results. The specific heat capacity was measured by comparison with water using a Dewar flask as a calorimeter and determining the 'final' temperature of heated copper and glycol mixture. Values within 1.6% of the reference were recorded. The viscosity was determined with U-tube viscometers in a recirculating water bath; a comparison was made with the viscosity of water. Maximum variations from the reference values of 2% were measured. It was concluded that viscosity, density and specific heat capacity, as recorded in figure 5.10 represented the 'true' properties with a sufficient accuracy. The thermal conductivity could not be readily determined and the results of Bates (Ref. K18) were referred to, as recommended by Curme (Ref. K15) and Gallant (Ref. K16). An alternative source of data was the work of Niedol (Ref. K17) which showed a disparity with Bates'. For the range of compositions and
and temperatures considered herein, the Riedel data averaged $3\frac{1}{2}\%$ lower with a maximum of $9\%$. A discrepancy of this magnitude would affect correlations for the coefficient of heat transfer in this thesis by an average of $2\%$. In 1968 a critical survey by Jamieson (Ref. K17.) suggested that the work of Riedel was probably more reliable than that of Bates.
OVERALL DIMENSIONS OF
THE APPARATUS, AND SIZES
OF CONNECTING PIPEWORK.
THE FLANGES REQUIRED FOR JOINING THE THREE LARGER TUBES WITH THE MIXING BOXES.
THE FITTINGS FOR JOINING TUBES OF DIFFERENT DIAMETER.

MATERIAL - BRASS. DIMENSIONS - CENTIMETRES.
FIGURE 5.7.

THE FITTINGS FOR ATTACHING THE ELECTRIC-SUPPLY LEADS TO THE TEST SECTION.

3 HOLES MACHINED TO OUTSIDE DIAMETER OF THE SMALL TUBES.

AFFIX LEADS FROM RECTIFIER HERE.

3 HOLES MACHINED TO OUTSIDE DIAMETER OF THE LARGE TUBES.

INSERT SECURING BOLTS.

MATERIAL - COPPER.  DIMENSIONS - CENTIMETRES.
FIGURE - 5.9.

The temperature distribution along an unheated tube, showing how the conduction of heat in the inlet-end lead affects local temperature measurements, and the improvement after fitting a guard heater.

Ambient Temp. = 23°C.
(a) Thermal Conductivity of Propylene Glycol and its Aqueous Solutions.

<table>
<thead>
<tr>
<th>Temp oC</th>
<th>Propylene Glycol, % by wt.</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Thermal Conductivity, (cal)/(cm.s.°C)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.00139</td>
<td>0.00117</td>
<td>0.00100</td>
<td>0.00083</td>
<td>0.00068</td>
<td>0.00054</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.42</td>
<td>1.19</td>
<td>1.00</td>
<td>0.82</td>
<td>0.67</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.45</td>
<td>1.21</td>
<td>1.00</td>
<td>0.82</td>
<td>0.66</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.48</td>
<td>1.23</td>
<td>1.01</td>
<td>0.81</td>
<td>0.65</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.00151</td>
<td>0.00125</td>
<td>0.00101</td>
<td>0.00080</td>
<td>0.00064</td>
<td>0.00050</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.54</td>
<td>1.27</td>
<td>1.01</td>
<td>0.80</td>
<td>0.63</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.57</td>
<td>1.29</td>
<td>1.02</td>
<td>0.79</td>
<td>0.62</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>1.60</td>
<td>1.31</td>
<td>1.02</td>
<td>0.78</td>
<td>0.61</td>
<td>0.47</td>
<td></td>
</tr>
</tbody>
</table>

(b) Viscosity of Aqueous Solutions of Propylene Glycol.

| Temp oC | Propylene Glycol, % by wt. | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---------|---------------------------|---|----|----|----|----|----|----|----|----|----|----|----|
|         | Absolute Viscosity, centipoises |   |    |    |    |    |    |    |    |    |    |    |    |
| 0       | 1.79                      | 2.6 | 4.2 | 7.1 | 12.5 | 18.0 | 29.0 | 47.0 | 72.0 | 135.0 | 243.0 |
| 10      | 1.31                      | 1.8 | 2.9 | 4.0 | 7.2 | 9.3 | 16.0 | 22.0 | 34.0 | 59.0 | 111.0 |
| 20      | 1.01                      | 1.35 | 2.1 | 3.0 | 4.4 | 6.4 | 9.3 | 14.0 | 20.0 | 33.0 | 56.0 |
| 30      | 0.80                      | 1.10 | 1.6 | 2.9 | 4.0 | 5.6 | 7.8 | 12.5 | 19.0 | 30.3 |      |
| 40      | 0.65                      | 0.88 | 1.2 | 1.6 | 2.2 | 2.9 | 3.9 | 5.4 | 7.4 | 12.0 | 18.0 |
| 50      | 0.55                      | 0.73 | 0.98 | 1.3 | 1.7 | 2.5 | 2.9 | 3.9 | 5.3 | 7.9 | 11.3 |
| 60      | 0.47                      | 0.61 | 0.79 | 1.0 | 1.3 | 1.7 | 2.2 | 2.8 | 3.9 | 5.4 | 7.7 |
| 70      | 0.41                      | 0.53 | 0.68 | 0.84 | 1.1 | 1.3 | 1.7 | 2.2 | 2.9 | 3.9 | 5.5 |

Footnote: Figures a, b and d are as given by reference (K.15.). The units therefore do not conform to the S.I. system.
### (c) Density of Aqueous Solutions of Propylene Glycol

<table>
<thead>
<tr>
<th>Propylene Glycol, % by wt.</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature °C</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>0</td>
<td>0.9999</td>
<td>0.9997</td>
<td>0.9992</td>
<td>0.9957</td>
<td>0.9923</td>
<td>0.9881</td>
<td>0.9832</td>
<td>0.9778</td>
</tr>
<tr>
<td>10</td>
<td>1.0092</td>
<td>1.0082</td>
<td>1.0061</td>
<td>1.0030</td>
<td>0.9993</td>
<td>0.9946</td>
<td>0.9895</td>
<td>0.9837</td>
</tr>
<tr>
<td>20</td>
<td>1.0196</td>
<td>1.0178</td>
<td>1.0152</td>
<td>1.0111</td>
<td>1.0069</td>
<td>1.0015</td>
<td>0.9960</td>
<td>0.9898</td>
</tr>
<tr>
<td>30</td>
<td>1.0327</td>
<td>1.0282</td>
<td>1.0238</td>
<td>1.0187</td>
<td>1.0134</td>
<td>1.0076</td>
<td>1.0015</td>
<td>0.9950</td>
</tr>
<tr>
<td>40</td>
<td>1.0442</td>
<td>1.0376</td>
<td>1.0314</td>
<td>1.0256</td>
<td>1.0199</td>
<td>1.0133</td>
<td>1.0066</td>
<td>0.9997</td>
</tr>
<tr>
<td>50</td>
<td>1.0520</td>
<td>1.0445</td>
<td>1.0380</td>
<td>1.0316</td>
<td>1.0250</td>
<td>1.0178</td>
<td>1.0104</td>
<td>1.0028</td>
</tr>
<tr>
<td>60</td>
<td>1.0559</td>
<td>1.0490</td>
<td>1.0426</td>
<td>1.0355</td>
<td>1.0279</td>
<td>1.0208</td>
<td>1.0128</td>
<td>1.0046</td>
</tr>
<tr>
<td>70</td>
<td>1.0593</td>
<td>1.0517</td>
<td>1.0440</td>
<td>1.0364</td>
<td>1.0290</td>
<td>1.0216</td>
<td>1.0136</td>
<td>1.0055</td>
</tr>
<tr>
<td>80</td>
<td>1.0593</td>
<td>1.0517</td>
<td>1.0440</td>
<td>1.0364</td>
<td>1.0290</td>
<td>1.0216</td>
<td>1.0136</td>
<td>1.0055</td>
</tr>
<tr>
<td>90</td>
<td>1.0563</td>
<td>1.0488</td>
<td>1.0412</td>
<td>1.0337</td>
<td>1.0258</td>
<td>1.0183</td>
<td>1.0104</td>
<td>1.0024</td>
</tr>
<tr>
<td>100</td>
<td>1.0500</td>
<td>1.0435</td>
<td>1.0363</td>
<td>1.0283</td>
<td>1.0213</td>
<td>1.0137</td>
<td>1.0056</td>
<td>0.9977</td>
</tr>
</tbody>
</table>

### (d) Specific Heat of Aqueous Propylene Glycol

<table>
<thead>
<tr>
<th>Propylene Glycol, % by wt.</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature °C</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>0</td>
<td>1.009</td>
<td>1.002</td>
<td>0.999</td>
<td>0.997</td>
<td>0.998</td>
<td>0.998</td>
<td>0.999</td>
</tr>
<tr>
<td>10</td>
<td>0.994</td>
<td>0.990</td>
<td>0.989</td>
<td>0.989</td>
<td>0.989</td>
<td>0.990</td>
<td>0.992</td>
</tr>
<tr>
<td>20</td>
<td>0.968</td>
<td>0.965</td>
<td>0.963</td>
<td>0.963</td>
<td>0.965</td>
<td>0.968</td>
<td>0.973</td>
</tr>
<tr>
<td>30</td>
<td>0.934</td>
<td>0.934</td>
<td>0.935</td>
<td>0.937</td>
<td>0.940</td>
<td>0.944</td>
<td>0.949</td>
</tr>
<tr>
<td>40</td>
<td>0.890</td>
<td>0.893</td>
<td>0.897</td>
<td>0.902</td>
<td>0.907</td>
<td>0.912</td>
<td>0.918</td>
</tr>
<tr>
<td>50</td>
<td>0.838</td>
<td>0.845</td>
<td>0.852</td>
<td>0.860</td>
<td>0.868</td>
<td>0.877</td>
<td>0.885</td>
</tr>
<tr>
<td>60</td>
<td>0.792</td>
<td>0.799</td>
<td>0.807</td>
<td>0.817</td>
<td>0.827</td>
<td>0.837</td>
<td>0.847</td>
</tr>
<tr>
<td>70</td>
<td>0.735</td>
<td>0.745</td>
<td>0.754</td>
<td>0.765</td>
<td>0.777</td>
<td>0.788</td>
<td>0.801</td>
</tr>
<tr>
<td>80</td>
<td>0.680</td>
<td>0.692</td>
<td>0.703</td>
<td>0.716</td>
<td>0.728</td>
<td>0.741</td>
<td>0.753</td>
</tr>
<tr>
<td>90</td>
<td>0.623</td>
<td>0.636</td>
<td>0.648</td>
<td>0.662</td>
<td>0.675</td>
<td>0.688</td>
<td>0.702</td>
</tr>
<tr>
<td>100</td>
<td>0.565</td>
<td>0.579</td>
<td>0.593</td>
<td>0.607</td>
<td>0.622</td>
<td>0.635</td>
<td>0.649</td>
</tr>
</tbody>
</table>
6. EXPERIMENTAL PROCEDURE.

6.1. A TYPICAL TEST CARRIED OUT.

To illustrate the experimental procedure a chronological list of the operations involved in a typical test is now presented. An indication of the time scale is given

Select suitable mixing-box inserts... (0h 0min)

With the cooling water running, and the power supply switched on, start the pump, set the maximum flow rate, and bleed off air....

Take a sample of glycol. Check the ice mixtures... (0h 10min)

Reduce the flow rate to the required value, manipulating the valves so that the bypass and back pressure gates are not fully open. Check the flow for fouling and air bubbles at the mixing box windows... (0h 15min)

Set the required voltage approximately. Reduce the cooling water rate, permitting the fluid temperature to rise at the test-section inlet, approaching the required value. Calibrate the potentiometer. Measure the density of the glycol to find its composition... (0h 45min)

Reset the flow rate, voltage setting, and cooling water rate to steady the inlet temperature of the fluid and the tube temperature as they approach the required values. Set the input to the guard heater at the inlet end, copper lead (1h 0min)

Continuously reset the flow, voltage and cooling water until the inlet and tube temperatures are nearly steady at the required values. Check that the thermocouple positions on the tube are suitable for investigating the temperature distribution. Check that the mixing box thermocouples read the same. Reset the input to the guard heater... (1h 15min)
Record the thermocouple readings at the inlet mixing box and selected readings on the tube surface at intervals of 5 minutes, until conditions are steady. Finally trim the flow rate and voltage.

Recalibrate the potentiometer. Continuously read the tube temperature at several, selected locations to estimate the magnitude of any fluctuations which may occur. Finally check the inlet temperature for constancy.

Begin the test -

Calibrate the potentiometer. Check the ice mixtures.

Record the following data and readings:

The time, room temperature, flow rate, voltages and current, inlet and outlet mixing-box thermocouples (20 off), thermocouples on the tube surface (up to 74 off), thermocouples on the copper leads (4 off), the inlet and outlet mixing-box thermocouples, voltages and current, flow rate, the time.

Check the ice mixtures. Calibrate the potentiometer.

Record the data and readings once more in reverse sequence.

Record the time.

End the test.

Small variations in the inlet temperature of the fluid (of the order 0.05K) were eliminated from the results by taking the average of the two sets of readings, provided the mixing box reading recorded half way through the test was the same as the average of the readings at the beginning and end.

6.2. THE CONTROL OF TEST CONDITIONS.

Some difficulty was encountered in controlling the temperature level at the inlet-end of the test-section. This was a consequence of the considerable time required for the interchange
of energy between the apparatus and environment to settle to a steady rate. Ideally, it should have been possible to conduct a series of tests in which the Prandtl number of the glycol solution had a particular value. However, the inlet temperature could only be preset with 2% accuracy, so that Pr was found to vary by up to 10% of the value required (due to the temperature dependence of viscosity). This did not significantly impede the experimental analysis.

By increasing the cooling water rate it was possible to reduce the inlet temperature of the fluid, and by increasing the applied voltage the temperature difference between tube and fluid could be set at a convenient value. Although the boundary condition of uniform heat-flux was imposed, it was found useful to discuss the experimental conditions in terms of temperature differences. This follows because 'heat-flux' is a somewhat abstract concept to visualize, and the limitations on the instrumentation were expressed in 'temperature' terms.

There were two ways of obtaining the desired Prandtl number. First, the glycol could be diluted to reduce its viscosity, and hence Pr. Second, the inlet temperature of the glycol at the test-section could be raised, with a subsequent lowering of the viscosity. Both methods were employed, the choice being a matter of practical convenience.

The quantity of fluid returned through the bypass valves was found to affect the operating conditions. If the amount bled off was much greater than the main flow rate, and the desired inlet temperature of the fluid was close to the minimum attainable, then small changes in the bypass-flow caused large variations in the inlet temperature. Conversely, if the bypass-flow was small, then the cooling water rate was low, and the inlet temperature became unsteady. The optimal valve settings were determined by experience.
For a given test-section configuration and glycol composition, only three parameters could be varied in the experiments, apart from the points just mentioned. The aforesaid variables were the inlet temperature of the fluid, the heat-flux at the tube wall, and the rate of flow of fluid. Investigations, into the distribution on the tube surface of the coefficient of heat transfer, were performed by finding the functional relationship between the local coefficient and the local properties, heat-flux, fluid bulk-temperature, and flow rate.

6.3. EXPERIMENTAL ERRORS, AN ASSESSMENT.

The following assessment of experimental errors refers to the maximum possible inaccuracies in the measured parameters, excluding those fluctuations in temperature which could arise, for instance, in the transitional flow regime. Mean errors would normally be approximately half the maximum errors. The individual tolerances have all been previously discussed, and the accumulative effect is now to be estimated.

- Tube temperature \( \pm 0.05 \, \text{K} \)
- Fluid temperature \( \pm 0.02 \, \text{K} \)
- Flow rate \( \pm 0.5\% \)
- Voltage \( \pm 1\% \)
- Current \( \pm 1\% \)
- Circumferential wall-thickness deviations in tubes \( 2.5\% \)
- Axial wall-thickness deviations in tubes \( 1\% \)
- Variations in radius of tubes \( 0.2\% \)

The maximum accumulative error in the coefficient of heat transfer, \( h \), is given by the formula -
\[
\text{if } f = f(x_0, \ldots, x_n), \text{ then } \frac{df}{f} = \sum_{n=0}^{N} \frac{df}{dx_n} \frac{dx_n}{f}
\]

Hence, from Newton's Law of Cooling the error in the axial distribution of \( h \) is \( 5.5\% \), whereas the circumferential distribution could vary by \( 7.0\% \). These estimates are based on a tube to fluid temperature difference of \( 3 \text{ K} \). At higher differences the corresponding errors become \( 3.2\% \) and \( 4.7\% \).

In practice the probable error in measuring \( h \) was likely to be no more than \( \pm 5\% \), according to the above analysis, and most results should have been within a \( 3\% \) tolerance.
7. ACCOUNT OF THE TESTS CARRIED OUT.

7.1. THE SHORT STRAIGHT TUBE.

7.1(i) FULLY DEVELOPED FLOW.

The first investigations dealt with the short, straight tube, and in these experiments the 2.616 cm. bore tube was arranged so that 93 diameters were heated, and a 'calming length' of 103 diameters preceded the heated section. As a consequence, the flow was devoid of any inertial stresses, apart from the minor ones associated with the viscosity-temperature dependence, and the fluctuating components due to turbulence, which were independent of the axial location.

Thermocouples were located at 1, 2, 3, 4, 5, 6, 8, 11, 15, 20, 30, 50 and 70 diameters downstream from the start of heating (sometimes modifications were made to this list.) Four thermocouples were affixed at each axial position, to enable the mean circumferential temperature to be determined, these were equi-spaced at the top, bottom and sides of the tube. To verify that groups of four thermocouples were adequate, twice the usual number were located at 1, 20 and 70 diameters downstream.

In practice, it was only necessary to apply more than four thermocouples at low Reynolds numbers (100 to 500) when the axial distance was greater than approximately 20 diameters.

The initial testing was fairly extensive since the effects of varying each of the experimental parameters - inlet temperature, heat flux, and flow rate - had to be determined for a wide range of conditions. In these experiments the use of fluids having different Prandtl numbers adds an extra dimension to the problem.

With 94.6% (by weight) glycol, the flow rate was set at a high value, $2.7 \times 10^{-3} \text{m}^3/\text{s}$, and the inlet temperature was adjusted to approximately 23, 26, 30, 34 and 40 °C in turn. Four to six tests
were carried out at each temperature, the heat flux being different in each case. The fluxes were chosen so that near to the outlet end of the test-section the temperature difference between the tube and the glycol was a suitable value - typically 4, 8, 12 and 20 K. These tests are denoted 1s to 22s.

A similar procedure was followed with the flow rate set at a low value. Fewer tests were carried out this time, the requirements being more easily assessed following the analysis of the former experimental data. Inlet temperature levels in the glycol of 16, 24, 32, and 40 °C were selected, and tests were carried out at each of these for 'tube to fluid' temperature differences between 6 and 35 K, near to the outlet end. These tests are denoted 23s to 33s.

Some final tests, designed to determine the effect of varying the flow rate, were carried out at inlet temperatures between 16 and 40 °C. The difference in temperature between tube and fluid, near to the outlet end, was varied between 5 and 30 K, and the flow rate was varied between the high and low values chosen in the earlier tests, 1s to 33s. These tests were designated 34s to 57s.

At an axial distance of 70 diameters, the range of values for the main dimensionless groups during tests 1s to 58s were nominally -

- Reynolds, \( Re \) - 100 to 10,000
- Prandtl, \( Pr \) - 180 to 650
- Grashof, \( Gr \) - 300 to 16,000
- Viscosity ratio, \( \frac{\mu_{\text{wall}}}{\mu_{\text{glycol}}} \) - 1.01 to 6.0

The heat flux was varied from 600 to 38,000 \( \frac{W}{m^2} \), and the viscosity of the glycol was varied from 15 to 60 cP.

There was little point in extending the range of \( Re \) above 10,000, since the local coefficient of heat transfer, \( h \), reached 95% of its constant, limiting value, \( h_{\text{a}} \), within the first three diameters of tube.
Having investigated the short tube with a developing temperature profile and fully developed flow, the case was considered in which the flow and temperature development were simultaneous. It was expected that the temperature profile in the fluid would be a different function of axial distance in the latter case, since the gradient of velocity at the wall was very much higher near to the start of heating, thereby increasing the axial convection of heat.

A 'bellmouth' entrance was fitted to the tube to produce the desired axial velocity profile at the start of heating. Figure 7.1 shows the construction of the bellmouth. The material was Tufnol fibrous plastic, which is a good thermal and electrical insulator, and the design consisted of a 'slow' diffuser, containing 'flow-straighteners', followed by a smooth contraction to the tube diameter. This shape of entrance leads to minimal, radial velocity components, and a nearly uniform axial velocity at the tube entrance. The ratio of diameters, upstream and downstream of the entrance, was estimated in the following way:

\[
\bar{u} = \text{mean velocity}
\]

Assume a parabolic velocity occurs upstream in the tube when conditions are most difficult for producing a uniform velocity downstream.

\[
u_1 = 2 \bar{u}_1 \left(1 - \frac{r_1^2}{r_{w1}^2}\right). \tag{7.1}
\]

If the total pressure just inside the downstream section differs little from that in the upstream section, then one is led to assume that (vorticity/radius) is constant, i.e. if the boundary layer
equation is rewritten as follows -
\[
\frac{d}{dx} \left( p + \frac{2}{r^2} \int_0^r \left( \frac{u^2 r}{2} dr \right) \right) = 2 \frac{\mu}{r} \frac{du}{dr} \quad (7.2)
\]
where radial velocities and pressure gradients are considered to be of second order magnitude then
\[
\frac{1}{r} \frac{du}{dr} = \text{constant}
\]
thence,
\[
\frac{u_2}{r_2^2} \frac{dr_2}{r_1^2} = \frac{1}{r_1} \frac{du_1}{dr_1} = -4 \frac{\bar{u}_1}{r_{1w}^2}
\]
and integrating,
\[
u_2 = -4 \frac{\bar{u}_1}{r_{1w}^2} \left( \frac{r_2^2}{r_1^2} + C \right) \quad (7.3)
\]
C is obtained with the continuity equation
\[
\bar{u}_2 = \frac{2}{r_{2w}^2} \int_0^{r_{2w}} u_2 r_2 dr_2
\]
giving
\[
C = -r_{2w}^2 \left( 1 + \frac{r_{1w}}{r_{2w}} \right) \quad (7.4)
\]
The downstream velocity may now be written
\[
u_2 = \bar{u}_2 \left( 1 + \left( \frac{r_{2w}}{r_{1w}} \right)^4 - 2 \left( \frac{r_{2w}}{r_{1w}} \right)^2 \left( \frac{r_2}{r_{2w}} \right)^2 \right) \quad (7.5)
\]
The velocity \(u_2\) differs from a constant value by the amount
\[
\left( \frac{r_{2w}}{r_{1w}} \right)^4
\]
at the greatest. This represents 1.2\% of the mean velocity for a contraction of 3:1. Although the analysis was crude, some idea of the bellmouth's performance was given.

In the tests with undeveloped flow, 187 diameters of the 2.616 cm. tube were heated, there being no calming length. It was considered unnecessary to conduct experiments with such extensive operating conditions as in the previous tests with the calming length, unless the results obtained differed considerably. Glycol with 94.6\% composition was used as in the previous experiments, and the thermocouples were located at similar axial distances from the onset of heating, 1, 2, 3, 4, 6, 8, 11, 15, 20, -30 and 70 diameters. These tests were designated 50 \(B\) to 72 \(B\).
The ranges of the main dimensionless groups, at an axial
distance of 70 diameters, were nominally:

- Re = 100 to 10,000,
- Pr = 170 to 570,
- Gr = 300 to 14,000,
- \( \frac{h_{wall}}{\lambda} \) = 1.2 to 4.2.

The heat flux varied from 550 to 28,000 \( \frac{W}{m^2} \), and the
viscosity from 15 to 53 cP. Temperature differences between the tube and
fluid, at 70 diameters from the inlet, were in the range 4 to 26 K,
and the temperature of the glycol at the inlet varied from 16 to
40 °C.

To permit a direct comparison between the coefficient of
heat transfer with undeveloped and developed flow, under similar
experimental conditions, i.e. with similar temperatures and flow
rates, the tests 74aB to 79cB and 80a to 85a were carried out.

Since the difference between the heat-transfer coefficients obtained,
in either case, was found to be more pronounced at lower Prandtl
numbers, the glycol content was reduced to 69.2%, and the inlet
temperature raised to 27 °C, thereby producing a low Prandtl number,
91, and a low viscosity, 9 cP. The range of Re was approximately
500 to 15,000.

The values of some important test-parameters are given
in part 7.5. for all the experiments carried out with the short tube,
in 1a to 85a.

7.2. THE SUDDEN CONVERGENCE.

Three ratios of upstream to downstream diameter were
investigated in the 'sudden convergence' experiments, these being
3.34:1, 2:1 and 1.25:1. The tube dimensions are given in part
5.3.(1). Long heated lengths of tube were utilized upstream
and downstream of the discontinuity in diameter, as follows:

<table>
<thead>
<tr>
<th>Diameter ratio</th>
<th>Heated length upstream (dia.)</th>
<th>Heated length downstream (dia.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.34:1</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>2 :1</td>
<td>51</td>
<td>90</td>
</tr>
<tr>
<td>1.25:1</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

From experience gained with the short, straight tube, it was decided to have four discrete values of Pr for the fluid entering the test-section. This was achieved with two different dilutions of propylene glycol, 97% and 73%, at two different inlet temperature levels. Some practical difficulty was encountered in establishing these conditions, but a reasonable approximation to the objective was attained.

Again, from previous experience, it was found unnecessary to repeat tests for a range of temperature difference from tube to fluid. The differences selected were between 5 and 12 K near to the outlet end of the tube, but only one value was used at a given Reynolds number.

The thermocouple arrangement, downstream of the discontinuity, was similar to the short, straight tube configuration for each of the diameter ratios. Groups of four thermocouples were located at 0.5, 1, 2, 3, 4, 5.5, 7, 8.5, 10, 12, 15, 20, 25, and 40 diameters downstream of the change in section. It was never found necessary to use groups of more than four thermocouples, probably as a consequence of the relatively small tube diameters downstream.

On the larger tube, the measured variation in the local coefficient of heat transfer was found to be slight in the region of the discontinuity. Because results in this region were uninteresting, few thermocouples were applied, the main locations being at the top and bottom of the tube at 0, 1, and 2 diameters upstream.
of the discontinuity.

The range of Re was different for each of the four selected
Prandtl numbers because of limitations on the available pressure drop.
Approximate values of these parameters are now given.

<table>
<thead>
<tr>
<th>Pr</th>
<th>Maximum Re downstream tube</th>
<th>Minimum Re downstream tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>27,000</td>
<td>500</td>
</tr>
<tr>
<td>150</td>
<td>10,000</td>
<td>550</td>
</tr>
<tr>
<td>200</td>
<td>8,500</td>
<td>450</td>
</tr>
<tr>
<td>350</td>
<td>4,700</td>
<td>450</td>
</tr>
</tbody>
</table>

The above applies to all three diameter ratios.

Heat fluxes in the downstream-tube, for each configuration,
varied from approximately 1,000 to 25,000 \( \frac{W}{m^2} \), the fluxes upstream
being less by a factor \( x \frac{(\text{downstream dia.})}{(\text{upstream dia.})} \). The viscosity ranged
from 5 to 31 cP.

The values of some important test-parameters are listed
in part 7.5, which includes data on all the 'convergence' tests.
These tests are designated 1c1 to 18c1 (2:1 ratio), 1c2 to 18c2
(1.25:1 ratio), and 1c3 to 18c3 (3:3:1 ratio).

Having avoided the repetition of tests for a range of heat
fluxes, it was considered that some justification for this step should
be shown. Tests 1c3, 4c3 and 6c3 were duplicated but with approximately
double the heat flux. These tests were numbered 19c3, 20c3, 21c3
respectively.

To evaluate the effects of the heat flux in the upstream
tube on the local heat-transfer coefficient in the downstream tube,
several tests were carried out with zero voltage applied to the
section upstream. This was achieved by moving one copper lead
close to the diameter discontinuity. The 'tube to fluid' temperature
difference near the outlet, the flow rate, and the inlet temperature
were set as close as possible to the values recorded in tests 1c3,
4c3 and 6c3, which included high, medium and low Reynolds numbers. These tests were limited to a Prandtl number of 60, and the 3:1 ratio. The corresponding references are 22c3, 23c3, 24c3.

7.3. THE SUDDEN DIVERGENCE.

The three pairs of tubes in the 'sudden convergence' experiments were reversed to give three divergences in diameter - 1:3.34, 1:2, and 1:1.25. Again, long heated lengths of tube were utilised upstream and downstream of the discontinuity in diameter, as follows:

<table>
<thead>
<tr>
<th>Diameter ratio</th>
<th>Heated length upstream</th>
<th>Heated length downstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:3.34</td>
<td>100</td>
<td>48</td>
</tr>
<tr>
<td>1:2</td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>1:1.25</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

Four discrete Prandtl numbers were selected, as in the 'convergence tests,' and once more it was found that the only significant variations in the local coefficient of heat transfer were those occurring in the downstream tubes. Thermocouples were situated top and bottom at 0, 2, 3 (or 5) diameters upstream of the discontinuity (others situated further upstream were found to give uninteresting information) but the most suitable axial locations on the downstream tube were found to vary with Re. Most of the results were obtained with thermocouples at 0, 0.5, 1, 2, 3, 4, 5.5, 7, 8.5, 10, 11.5, 13, 15, 20, 25 and 40 diameters downstream of the discontinuity. Groups of four thermocouples were positioned, with groups of eight at 1, 15 and 40 diameters to provide a check on the reliability of the mean circumferential temperature, as mentioned in 7.1.1(i).

A different range of Re was possible with each fluid and each diameter ratio. The approximate values were -
<table>
<thead>
<tr>
<th>Diameter Pr Ratio</th>
<th>Maximum Re Upstream/Downstream</th>
<th>Minimum Re Upstream/Downstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:3.34 55</td>
<td>33,000 / 11,000</td>
<td>750 / 250</td>
</tr>
<tr>
<td>1:3.34 150</td>
<td>10,500 / 3,500</td>
<td>750 / 250</td>
</tr>
<tr>
<td>1:3.34 250</td>
<td>6,000 / 2,000</td>
<td>420 / 140</td>
</tr>
<tr>
<td>1:3.34 500</td>
<td>3,000 / 1,000</td>
<td>210 / 70</td>
</tr>
<tr>
<td>1:2 55</td>
<td>28,000 / 14,000</td>
<td>500 / 250</td>
</tr>
<tr>
<td>1:2 150</td>
<td>10,000 / 5,000</td>
<td>600 / 300</td>
</tr>
<tr>
<td>1:2 275</td>
<td>7,000 / 3,500</td>
<td>400 / 200</td>
</tr>
<tr>
<td>1:2 400</td>
<td>3,000 / 1,500</td>
<td>400 / 200</td>
</tr>
<tr>
<td>1:1.25 60</td>
<td>27,500 / 22,000</td>
<td>500 / 400</td>
</tr>
<tr>
<td>1:1.25 150</td>
<td>10,000 / 8,000</td>
<td>570 / 450</td>
</tr>
<tr>
<td>1:1.25 225</td>
<td>7,500 / 6,000</td>
<td>375 / 300</td>
</tr>
<tr>
<td>1:1.25 400</td>
<td>4,375 / 3,500</td>
<td>500 / 400</td>
</tr>
</tbody>
</table>

The heat fluxes in the downstream-tubes were 200 to 6,000 W/m², 500 to 8,500 W/m², and 700 to 18,000 W/m², for the ratios 1:3.3, 1:2, and 1:1.25 respectively.

The initial 'sudden divergence' tests concerned the 1:3.3 ratio. First, a series of fifteen tests, 1d1 to 15d1, was carried out with Pr = 55 at the inlet. The flow rate was changed to give a range of Re, but there was no attempt to determine the effects of heat flux on the coefficient of heat transfer. The temperature difference from tube to fluid, at 40 diameters downstream, was in the range 3 to 7 K, the heat fluxes being low in order to minimise 'secondary' effects: such as the dependence of the local coefficient of heat transfer on the functionality between the viscosity and temperature. The next stage was to investigate specifically the effects of varying the heat-flux, and this was done by repeating some of the earlier tests with higher fluxes, as follows -

Test Numbers |
--- | --- |
16d1 to 18d1  | 2d1  |
19d1 to 21d1  | 3d1  |
22d1 to 23d1  | 5d1  |
24d1          | 15d1 |
25d1          | 12d1 |
26d1          | 15d1 |
27d1          | 14d1 |
28d1          | 2d1  |

Data on some test-parameters is given in part 7.5.
Further testing, with different glycol temperatures and compositions, gave evidence as to the effects of Pr on the local heat-transfer coefficient. These were labeled:

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Pr</th>
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</thead>
<tbody>
<tr>
<td>29d1 to 34d1</td>
<td>150</td>
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<tr>
<td>35d1 to 43d1</td>
<td>500</td>
</tr>
<tr>
<td>44d1 to 50d1</td>
<td>250</td>
</tr>
</tbody>
</table>

The 'tube to fluid' temperature differences in the above tests were approximately 10 K at 40 diameters downstream of the discontinuity.

Finally, a few tests were carried out to determine the influence on the downstream-section of the heat flux in the smaller tube by applying zero voltage to that tube in a similar manner to the 'convergence-experiments'. The copper lead upstream was adjusted to a position close to the discontinuity in diameter, thus achieving the desired condition. The four tests carried out, 51d1 to 54d1, resembled previous tests in that the flow rate, tube and fluid temperatures were similar to those selected for tests 12d1, 20d1, 9d1 and 7d1. Thereby, a range of Re was covered with a particular Pr (≈).

In the case of the 1:2 and 1:1.25 divergences far fewer tests were necessary. No investigation into the effects of heat flux values was required, since the main objective was to find the relationship between the diameter ratio and the local coefficients of heat transfer. Moderate heat fluxes were applied giving 'tube to fluid' temperature differences of 5 to 10 K at 40 diameters downstream. Four different Prandtl numbers were used, these were nominally 60, 150, 250 and 400. Further data on these tests is given in part 7.5, where tests 1d2 to 22d2 refer to the 1:2 diameter ratio, and tests 1d3 to 22d3 to the 1:1.25 ratio.
7.4. FURTHER TESTING

7.4(i) FLOW VISUALIZATION.

In the course of these experiments some limited investigations were made to help explain certain results obtained with the sudden divergencies. The manner in which the local coefficients of heat transfer varied with the axial distance at low values of Re was found to be somewhat erratic and often the measurements were unrepeatable without any obvious cause. An insight into the fluid flow patterns was desirable, and by carrying out the aforesaid investigations, a knowledge of the heat transfer mechanisms was derived, which helped to explain the considerable irregularities in the experimental results. Part of the analysis in 9.7. alludes to the tests described here.

An analagous apparatus was constructed from transparent materials (acrylic plastics) as shown in figure: 7.2. The discontinuous tube was immersed in a water bath to permit some degree of control of the surrounding temperature, and to minimise refractive parallax. Dye was injected into the flow in the smaller tube upstream, and photographs recorded the traces on entering the larger tube. The fluid passing through the tubes was supplied from a constant head tank and controlled by a gate valve at the exit of the tank. The temperatures were measured with a thermometer. To obtain a sufficiently high Re the viscosity of the fluid had to be fairly low, consequently water was used in most of the tests and a weak dilution of propylene glycol in the remainder.

A series of isothermal tests was tried with a 1:3 'divergence' ratio, the range of Re being 200 to 2,400 in the large tube. The tests were then repeated with the temperature of the water bath raised above the temperature of the entrained fluid. Whereas the conditions do not even approximate to the 'uniform heat-flux' imposed in the manual experiments, it was supposed that a qualitative impression of the flow...
patterns would be obtained for the case of 'flow with heat transfer'.

The tests just described were carried out once more, but this time with a 1:2 divergence ratio. When the 1:1.25 ratio was investigated, it was found difficult to obtain photographic evidence of the results due to refraction in the tube wall.

With the same apparatus an investigation was carried out into the effects of heating on the flow pattern in a long, uniform tube. This was a brief examination because the observations, whilst providing a description of the flow pattern, did not contribute significantly to the quantitative analysis of the main experimental results. Photographs were obtained for different values of Re and at different bath temperatures.

Part 9.7. contains the continuing discussion of the flow visualization tests, when the significance of these results becomes apparent from the elucidation of the measured coefficients of heat transfer.

7.4.(ii) LARGER DIVERGENCES.

The local coefficient of heat transfer in the larger tube, \( h \), during the 'sudden divergence' experiments, was in general much greater than the corresponding value with no discontinuity present, \( h_\infty \). The ratio \( (h/h_\infty) \) was found to increase with the ratio (larger diameter/small diameter). The question arises, what happens as the diameter ratio tends to a very large value? The ratio \( (h/h_\infty) \) may or may not increase ad infinitum. Because of this rather speculative argument, and also to support the empirically based hypothesis that the heat-transfer coefficient near to the discontinuity is a function of Re in the smaller rather than the larger tube, some further tests were carried out.

The fittings on the following tubes were modified,

(as in figure: 7.3. )
so that short lengths of small bore-tube could be inserted at the end of the test-section upstream. This had the effect of producing the 'apparent' diameter ratios of 1:14.4, 1:9, and 1:6. This fairly crude expedient enabled evidence to be obtained in support of the hypothesis just stated. The heat generation per unit length was different in each half of the test-section, but the arrangement was such that 92 diameters of the smaller tube were heated and 46 diameters of the larger tube. The sizes of the inserts were as follows: 0.397, 0.635 and 0.953 cm. all being of length 4 cm.

Three tests were carried out in which Re in the smaller tube was approximately 13,000 and Pr at the inlet was 55. These extra tests are denoted 1dx, 2dx, 3dx.

7.5. THE RESULTS.

In the 246 tests completed the reduction of the raw experimental data into usable form yielded approximately 50,000 pieces of information, and this was prior to the calculations which were required for the correlation of the results. In view of the large volume of data handled, it was decided to present a detailed account of a few tests, then to list some important test parameters for all cases. The most significant variables not recorded here appear graphically in part 9.

The results referred to are now given.
The definition of symbols used in the computer output is as follows:

R.E. Local value of Reynolds number
N.U. " " Nusselt number
P.H. " " Prandtl number
G.R. " " Grashof number
M. " " (Bulk viscosity)/(Viscosity at wall)
X. " " Distance from start of heating, or discontinuity (diameters)

V. Applied voltage to tube (v)
I. Current in tube (A)
R. Radius inside tube (cm)
VISO. Nominal viscosity (cP)
CL. Calming length (dia)
HL. Heated length (dia)
TFO. Outside tube temperature (°C)
TPI. Inside tube temperature (°C)
TX. Bulk temperature (°C)
TIN. Temp. at tube inlet (°C)

In the tests without diameter change, the values of TFO, TPI and TX are listed directly below R.E, PR and I.

NOTE: The data following the specimen test results give an indication of the range of the main test parameters.
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<thead>
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<th>V</th>
<th>9.8050000</th>
<th>I</th>
<th>529.00000</th>
<th>TIN</th>
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<th>R</th>
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</thead>
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\[ X = 2.000 \quad NU = 42.90 \]
\[ RE = 384.0 \quad PR = 174.4 \quad M = 1.611 \quad GR = 7296.79 \]
\[ TPO = 51.11 \quad TPI = 50.95 \quad TX = 39.99 \bigdeg \text{C} \]

\[ X = 3.000 \quad NU = 37.45 \]
\[ RE = 384.3 \quad PR = 174.3 \quad M = 1.707 \quad GR = 8357.19 \]
\[ TPO = 52.70 \quad TPI = 52.55 \quad TX = 40.01 \bigdeg \text{C} \]

\[ X = 4.000 \quad NU = 34.97 \]
\[ RE = 384.5 \quad PR = 174.2 \quad M = 1.768 \quad GR = 8998.07 \]
\[ TPO = 53.66 \quad TPI = 53.50 \quad TX = 40.03 \bigdeg \text{C} \]

\[ X = 6.000 \quad NU = 30.80 \]
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\[ TPO = 55.50 \quad TPI = 55.34 \quad TX = 40.06 \bigdeg \text{C} \]

\[ X = 8.000 \quad NU = 28.39 \]
\[ RE = 385.6 \quad PR = 173.8 \quad M = 1.992 \quad GR = 11137.7 \]
\[ TPO = 56.84 \quad TPI = 56.68 \quad TX = 40.10 \bigdeg \text{C} \]

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\[ TPO = 64.04 \quad TPI = 63.89 \quad TX = 41.12 \bigdeg \text{C} \]

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\[ R = 1.308 \quad CL = 103.0 \quad HL = 93.00 \]

\[ \text{VISC} = 14.44 \]
TEST NO. 58.58.

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TPO = 43.61  TP1 = 43.39  TX = 36.93DEGC

X = 2.000  NU = 33.25
RE = 1615  PR = 199.8  M = 1.437  GR = 3934.46
TPO = 45.13  TP1 = 44.91  TX = 36.94DEGC

X = 3.000  NU = 75.49
RE = 1616  PR = 199.7  M = 1.491  GR = 4353.49
TPO = 45.98  TP1 = 45.76  TX = 36.94DEGC

X = 4.000  NU = 69.46
RE = 1616  PR = 199.7  M = 1.543  GR = 4733.70
TPO = 46.75  TP1 = 46.53  TX = 36.95DEGC

X = 6.000  NU = 60.68
RE = 1617  PR = 199.6  M = 1.643  GR = 5421.05
TPO = 48.14  TP1 = 47.92  TX = 36.96DEGC

X = 8.000  NU = 55.67
RE = 1618  PR = 199.5  M = 1.722  GR = 5916.59
TPO = 49.14  TP1 = 48.92  TX = 36.97DEGC

X = 11.00  NU = 49.83
RE = 1619  PR = 199.4  M = 1.837  GR = 6617.00
TPO = 50.55  TP1 = 50.33  TX = 36.98DEGC

X = 15.00  NU = 44.60
RE = 1620  PR = 199.2  M = 1.944  GR = 7405.01
TPO = 52.13  TP1 = 51.91  TX = 37.00DEGC

X = 20.00  NU = 39.68
RE = 1622  PR = 199.0  M = 2.082  GR = 8342.00
TPO = 54.00  TP1 = 53.78  TX = 37.03DEGC

X = 30.00  NU = 33.85
RE = 1626  PR = 198.6  M = 2.323  GR = 9877.71
TPO = 56.92  TP1 = 56.70  TX = 37.07DEGC

X = 50.00  NU = 23.37
RE = 1633  PR = 197.8  M = 2.694  GR = 11815.1
TPO = 60.79  TP1 = 60.57  TX = 37.16DEGC

X = 70.00  NU = 25.25
RE = 1642  PR = 196.8  M = 2.952  GR = 13413.6
TPO = 63.79  TP1 = 63.57  TX = 37.26DEGC

V = 9.375  I = 250.0
R = 1.303  CL = 0.0000  HL = 187.20

VISC = 16.741
TEST NO. 6 & SB.

\[ x = 1.000 \quad \text{NU} = 202.4 \]
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\[ \text{TP0} = 44.13 \quad \text{TP1} = 43.74 \quad \text{TX} = 37.93 \text{DEGC} \]

\[ x = 2.000 \quad \text{NU} = 170.5 \]
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\[ \text{TP0} = 45.22 \quad \text{TP1} = 44.93 \quad \text{TX} = 37.93 \text{DEGC} \]

\[ x = 3.000 \quad \text{NU} = 150.8 \]
\[ \text{RE} = 6256 \quad \text{PR} = 190.8 \quad \text{M} = 1.388 \quad \text{GR} = 3995.01 \]
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\[ x = 4.000 \quad \text{NU} = 154.4 \]
\[ \text{RE} = 6257 \quad \text{PR} = 190.8 \quad \text{M} = 1.407 \quad \text{GR} = 4163.80 \]
\[ \text{TP0} = 45.95 \quad \text{TP1} = 45.57 \quad \text{TX} = 37.94 \text{DEGC} \]

\[ x = 6.000 \quad \text{NU} = 144.0 \]
\[ \text{RE} = 6258 \quad \text{PR} = 190.7 \quad \text{M} = 1.441 \quad \text{GR} = 4453.58 \]
\[ \text{TP0} = 46.51 \quad \text{TP1} = 46.12 \quad \text{TX} = 37.94 \text{DEGC} \]

\[ x = 8.000 \quad \text{NU} = 139.4 \]
\[ \text{RE} = 6260 \quad \text{PR} = 190.7 \quad \text{M} = 1.459 \quad \text{GR} = 4616.02 \]
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\[ x = 11.00 \quad \text{NU} = 137.0 \]
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X = 2.0000  NU = 324.78
RE = 15016  PR = 92.375  M = 1.2394  GR = 6683.28
TPO = 32.19  TPI = 31.62  TX = 27.77DEGC

X = 3.0000  NU = 319.31
RE = 15018  PR = 92.360  M = 1.2424  GR = 6802.00
TPO = 32.26  TPI = 31.69  TX = 27.77DEGC

X = 4.0000  NU = 317.93
RE = 15021  PR = 92.346  M = 1.2431  GR = 6834.29
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X = 8.0000  NU = 313.19
RE = 15031  PR = 92.290  M = 1.2455  GR = 6946.77
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X = 11.0000  NU = 313.81
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X = 15.0000  NU = 314.63
RE = 15047  PR = 92.191  M = 1.2441  GR = 6931.12
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X = 50.0000  NU = 317.04
RE = 15132  PR = 91.701  M = 1.2398  GR = 6957.64
TPO = 32.41  TPI = 31.84  TX = 27.89DEGC

X = 70.0000  NU = 317.24
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X = 2.0000  NU = 323.52
RE = 14888  PR = 93.137  M = 1.2422  GR = 6562.46
TPO = 32.01  TPI = 31.45  TX = 27.63DEGC

X = 3.0000  NU = 316.29
RE = 14891  PR = 93.123  M = 1.2462  GR = 6653.43
TPO = 32.10  TPI = 31.54  TX = 27.63DEGC

X = 4.0000  NU = 312.52
RE = 14893  PR = 93.109  M = 1.2482  GR = 6736.33
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X = 6.0000  NU = 308.23
RE = 14898  PR = 93.081  M = 1.2506  GR = 6834.27
TPO = 32.21  TPI = 31.65  TX = 27.64DEGC

X = 8.0000  NU = 310.76
RE = 14902  PR = 93.053  M = 1.2493  GR = 6795.70
TPO = 32.19  TPI = 31.63  TX = 27.65DEGC

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X = 15.0000  NU = 306.19
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X = 20.0000  NU = 307.17
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X = 30.0000  NU = 306.06
RE = 14954  PR = 92.748  M = 1.2500  GR = 6936.28
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X = 50.0000  NU = 306.13
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TPO = 32.35  TPI = 31.79  TX = 27.75DEGC

X = 70.0000  NU = 307.03
RE = 15049  PR = 92.190  M = 1.2462  GR = 7004.63
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**Notes:**
- RE: Reaction Number
- X: Temperature (°C)
- P: Pressure (atm)
- T: Time (min)
- DE: Degree of Efficiency
- NUM: Numerical Value
- QL: Quality Level
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**Figure 7.2**

**Flow Visualisation Apparatus**

- Fluid from Header Tank
- Approx. 60 diameters
- Approx. 25 to 35 diameters
- Dye
- Hyperbolic Injector
- Thermometers
- Heated Water Bath
- Outlet to Weighing Tank

**Side View**

**Figure 7.3**

The inserts used to produce large, sudden divergences in diameter.

**Insert**

- 4 screws
- Fixing plate

**Diameter A**

- Machined to give desired divergence ratio.
3. THE REDUCTION OF RAW EXPERIMENTAL DATA, AND
ASSOCIATED CALCULATIONS

3.1. INTRODUCTION.

The reduction of experimental results into a useful form
was initially carried out on a desk calculator, with a number of
graphical aids and much tabulated data. This procedure was tedious
and time consuming, but it did provide a logical sequence of calculations
which could be easily arranged as a computer programme.

The numerical data which could be extracted from each test
may be generalised as follows: ambient temperature; flowmeter
reading; voltages and current in experimental tubes; thermocouple
readings in the heat-transfer fluid at entry and exit of the tubes;
the composition of the fluid; the dimensions of the tubes; and
finally the thermocouple readings at various locations on the outer
surface of the tubes with their corresponding positions. These
were supplemented by a flowmeter calibration, thermocouple calibration
and physical property data for the fluid and other materials utilised.

From the data above the following variables had to be deter-
mined - the circumferential, mean coefficient of heat transfer at
each axial position on the tubes, the bulk temperature of the fluid
and the temperature at the inner surface of the tube for each location,
the heat flux, and various dimensionless groups dependent on the local
properties of the fluid, such as the local values of the Nusselt,
Reynolds, Prandtl and Grashof numbers. Further information was
added to this list in some cases, but the essential components are
as stated.

The calculations which were carried out in the reduction
of the experimental data are described in the next section, and a
typical computer programme incorporating these will be given. For
simplicity, the explanation of the calculations follows a similar
sequence to the 'programme'.

At the end of this section a discussion concerning the
heat balance will be presented.

8.2. THE CALCULATIONS.

8.2.(a) Flowmeter and thermocouple calibrations.

The frequency reading from the flowmeter was converted
into volume flow-rate by using Figure 8.2 which is referred to in
part 5. This step was not included in the computer programme.
The thermocouple readings could be converted into temperature
simply using a graph of the calibration data (Figure: 8.3.). This
technique was used when the peripheral temperature distribution
was being investigated, but a numerical method was found to be more
satisfactory for the majority of the calculations. The most useful
results, in general, were those appertaining to axial, but not
peripheral, tube locations. Hence the data input for the computer
programme included mean thermocouple readings for given axial
positions, and these were converted into temperatures with an inter-
polation 'procedure'.

The numerical interpolation was based on discrete temperature
increments and the corresponding thermocouple voltages. These values
were determined graphically from basic calibration results, and the
temperature increment was chosen at 4K so that linear interpolation
sufficed.

8.2.(b) The interpolation of property data.

The propylene glycol-water mixtures had to be specified
in terms of composition and temperature for the purpose of deter-
mining by reference the density, thermal conductivity, specific heat
capacity, and viscosity. The values referred to are tabulated in
figures 5.10a to 5.10d.
A unified approach was required for the interpolation of intermediate property values for use in the programme (the properties being stored in the computer at discrete 'temperature' and 'percentage glycol in water by weight' values, as shown in the tables). Upon inspection of the property data it was noted that density, conductivity, and specific heat capacity could be interpolated linearly over a large 'temperature' or 'per cent' range (i.e. less than 1% difference was estimated between interpolated property values and graphically presented reference data for 10K or 10% increments). The viscosity, however, could not be interpolated linearly unless very small increments were chosen (i.e. approximately 1% maximum departure from reference values occurred for 3K or 3% increments).

Rather than increase the computer data required to store the viscosity functions by a factor of about 10 times, it was decided to improve the interpolation formula. Instead of assuming linearity between any two adjacent, tabulated values, an exponential relation was proposed to fit any three adjacent values, as follows:

Interpolated viscosity \( f = A + B \cdot x^{n} \),

where, at a given composition \( x = (\text{temperature} - \text{reference temperature}) \),
or, at constant temperature \( x = (\text{per cent glycol} - \text{reference per cent}) \).

\( A, B \) and \( n \) may be determined from the three known conditions,

Hence,

\[
\begin{align*}
f &= f_1 + (f_3 - f_1) \left( t - t_1 \right) \frac{\log \left( \frac{t_2 - t_1}{t_3 - t_1} \right)}{\log \left( \frac{t_2 - t_1}{t_3 - t_1} \right)}
\end{align*}
\]

where \( t = \text{temperature or per cent glycol in water} \).

Since viscosity was known to reduce exponentially with increasing temperature or water content, a considerable improvement was expected over the linear analysis. Further, this exponential form could be utilised for the other properties since linearity was permissible as a special case.
Using the above technique, it was found possible to choose increments in temperature and per cent glycol of 1°C and 10%. The interpolation error was assessed by plotting graphs of viscosity versus temperature for given mixtures, the most severe test on the method being the case of 100% glycol at 'middle of increment' temperature values. A maximum interpolation error of 2% could occur at 25°C (graphical estimates are shown in Figure 8.4) but the mean error over the full temperature range (0 to 70°C) was approximately 0.7%. With 90% glycol the maximum possible errors were much reduced, the corresponding figures being 0.9% and 0.4%. For lower glycol content the error estimates were small and unrealistic. The estimated errors were very small for the other properties over the entire range of tabulated values.

In applying the interpolation procedure for a given composition and temperature, nine values of viscosity (the closest to the region of interest) were utilised in four applications of the formula.

It could be argued that a more reliable interpolation would have been a polynomial curve fit using as many reference points as were available, however, the present method was adopted as being sufficiently reliable, whilst combining mathematical simplicity with ease of application. Two dimensional arrays of different size and grid spacing could be handled without difficulty.

6.2.(c) Conduction in the tube wall.

In part 5, some calculations were carried out on the effects of axial conduction in the tube wall very close to fittings which could distort the temperature distribution. It was concluded that this effect was likely to be very localised for the test conditions anticipated. Assuming this to be so (though not at this stage entirely dismissing it as a possible source of discrepancy) an
investigation of conduction in a tube of infinite length may be carried out advantageously; the purpose of this analysis being to produce formulae for the local temperature difference across the tube wall and the local heat flux.

Figure: 8.1

Consider figure 8.1. The diagram shows the coordinates used to describe an elemental ring of the tube having axially symmetric temperature.

At the inside surface (i) heat is transferred to the fluid.

At the outside surface (o) a small quantity of heat is conducted through an imperfect insulator.

Assume: (i) Thermal and electrical properties vary little with temperature. (For stainless steel, as used in the experiments, electrical and thermal conductivities vary by a nominal 0.02 per cent per Kelvin).

(ii) The tube wall is thin compared with its radius. This cannot be defined any more specifically, but constitutes a necessary condition if the experimental method of measuring outside wall temperatures is considered valid, and this is likely to be the case when the overall temperature correction
is much smaller than the temperature difference used in
calculating heat transfer. (i.e. temperature drop across
the tube wall is much less than the temperature difference
between the inner tube surface and the entrained fluid.)

The conduction equation is

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2} + \frac{1}{k_e} \left( \frac{dT}{dx} \right)^2 = 0 \quad (8.1)
\]

From assumption (i) the fourth term becomes a constant (λ say).

A solution may be obtained by separation of the variables, but the
result would be in a fairly complex form and would be difficult to
apply in practice, particularly since the two axial boundary conditions
are not clearly defined.

An integral form of the equation was used in conjunction
with certain assumptions based on (ii) (above) to give a more promising
approach to the problem. This method, though not mathematically
rigorous, yields an approximate solution containing a first order
estimate for the effects of axial conduction in the tube wall (the
third term in the equation).

Integrating the equation with 'x' constant,

\[
r \frac{\partial T}{\partial r} + \int r \frac{\partial^2 T}{\partial x^2} dr + \frac{\lambda r^2}{2} + \int f(x) = 0 \quad (8.2)
\]

the first boundary condition is \( \frac{\partial T}{\partial r} = \frac{\partial T}{\partial r_o} \left( x, r_o \right), \ r = r_o \),

giving

\[
\frac{\partial T}{\partial r} + \frac{1}{r} \int r \frac{\partial^2 T}{\partial x^2} dr + \frac{\lambda r}{2} - \frac{1}{r} \left( \frac{\lambda r^2}{2} + \frac{\partial T}{\partial r_o} \right) = 0 \quad (8.3)
\]

A second integration with the condition \( t = t_o \left( x, r_o \right), \ r = r_o \),
is written -

\[
t - t_o + \int \left[ \frac{\partial T}{\partial r} \left( r \right) \right] dr + \frac{\lambda}{4} \left( r^2 - r_o^2 \right) - \left( \frac{\lambda r^2}{2} + \frac{\partial T}{\partial r_o} \right) \log \left( \frac{r}{r_o} \right) = 0 \quad (8.4)
\]

Now the second term may be evaluated approximately by making a
plausible assumption regarding the magnitude of \( \frac{\partial T}{\partial x^2} \).

From (ii) it is implied that the form of \( t_o \left( x \right) \) is similar to \( t_i \left( x \right) \)
so that
\[ \frac{\partial^2 T}{\partial x^2} \bigg|_0 \approx \frac{\partial^2 T}{\partial x^2} \bigg|_1 \]
It seems reasonable therefore to make \( \frac{\partial^2 T}{\partial x^2} \bigg|_0 \approx \frac{\partial^2 T}{\partial x^2} \bigg|_1 \)
at least this will indicate how the second term affects the resulting
temperature difference \((t_o - t_1)\), and a measure of reliability is
achieved in practice when applying the resulting formula to par-
ticular experimental conditions.

Now the equation is
\[(t - t_o) = (\lambda + \frac{\partial^2 T}{\partial x^2}) \left( \frac{r_i^2 - r_o^2}{4} \right) + \left( \frac{\partial^2 T}{\partial r^2} + \frac{r_o^2}{2} \left( \lambda + \frac{\partial^2 T}{\partial x^2} \right) \right) \left( \log \left( \frac{r}{r_o} \right) \right) \quad (8.5)\]
Putting
\[ \phi \equiv \left( \frac{\partial^2 T}{\partial r^2} \right) \quad \text{and} \quad s \equiv \left( \frac{r_i}{r_o} \right) \]
and replacing \( \left( \lambda + \frac{\partial^2 T}{\partial x^2} \right) \) by \( \left( \frac{s \phi_i - \phi_o}{1 - s^2} \right) \frac{1}{r_o} \) for con-
venience, the overall temperature difference is
\[(t_i - t_o) = \left( \frac{\phi_i - s \phi_o}{1 - s^2} \right) \left[ r_i \log(s) + s(1 - s) \frac{r_o}{2} \right] - \left( 1 - s^2 \right) \frac{r_i}{2} \phi_o \quad (8.6)\]
where the ratio of axial to radial conduction of heat out of an element
is approximately
\[ \frac{\text{Ratio}}{k_r e} = \frac{\partial^2 T}{\partial x^2} \left( \frac{dV}{dx} \right)^2 \quad (8.7) \]
when radial conduction predominates.

The integration of the axial derivative could be improved
by using a better approximation for the temperature profile, but this
was considered unnecessary since it was required only to estimate
the possible effects of axial conduction on the experimental results.

Equation (8.6) is the main result of the analysis, now the
gradients \( \phi_i \) and \( \phi_o \) must be stated. Figure 8.1. shows how the heat
is distributed in a short length of tube. The assumptions made
already in deriving (8.6) apply here also. The equation describing
this situation is:
\[ \frac{dQ_i}{dx} = I \frac{dV}{dx} + \pi K_p (r_o^2 - r_i^2) \frac{\partial^2 T}{\partial x^2} - 2 \pi K_{\text{ins}} (t_o - t_A) \left( \log \left( \frac{r_A}{r_o} \right) \right) \quad (8.8) \]
where \( t_A = \text{ambient temp,} \), \( r_A = \text{mean outer radius insulation,} \)
also \( Q_i = V \cdot I / L + \pi K_p r_o^2 (1 - s^2) \frac{\partial^2 T}{\partial x^2} - u_i \text{ins} \quad \text{and} \quad u_{\text{ins}} = 2 \pi K_{\text{ins}} (t_o - t_n) / \log \left( \frac{r_A}{r_o} \right) \quad (8.9) \)
with \( q' \) - heating rate per unit length

Subscript \( \text{ins} \) - insulation.

\( V, L \) - Total applied voltage and length over which applied.

Note that \( \varphi_0 \) and \( \varphi_i \) may be easily obtained using (8.9) and (8.10), as follows:

\[
\varphi_i = \frac{q'_i}{(2\pi k_f r_i)} \quad (8.11)
\]

and \( \varphi_0 = \frac{q'_{\text{ins}}}{(2\pi k_f r_0)} \quad (8.12)\]

The preceding analysis results in equations (8.6), (8.9) and (8.10) which permit a first order approximation for axial conduction effects. If these are set at zero, the result is similar to that obtained by Ede (Ref.H.2). Although no great accuracy is claimed by using this form of solution, a clarification of assumptions has been made, and an order of magnitude for axial conduction terms is permitted; this will be discussed quantitatively in part 9.

The term \( \frac{\partial^2 t}{\partial x^2} \) can be found in practice by using three adjacent, local temperatures, the accuracy improves as the gap between thermocouples reduces. The result is like a parabolic interpolation on the axial temperature profile,

\[
\frac{\partial^2 t}{\partial x^2} = \frac{(t_3 - t_2) - (t_2 - t_1)}{(x_3 - x_2) - (x_2 - x_1)} \cdot \frac{2}{(x_3 - x_1)} \quad (8.13)
\]

The values for the thermal conductivities of type 321 AISI stainless steel, and moulded polystyrene insulation are taken to be 15.5 W \( /mK \) and 0.033 W \( /mK \). The values are assumed to be constant.

8.2.(d) The bulk fluid temperature.

To calculate the bulk fluid temperature at each axial location, a summation must be made of the heat addition to the fluid between the tube entrance and the section of interest. In general, the configuration consisted of an unheated length of tube with conduction through the insulation, which preceded an upstream-tube.
with a given power input and heat conduction through the insulation, then a downstream-tube having a different power input. The calculation for temperature rise in the fluid is a simple one which can be applied over short lengths of the tubes, the total temperature rise being the summation of all such lengths.

\[ Q_1 = \text{local heating rate per unit length} \]

\[ H = \text{specific enthalpy of fluid} \]

If \((x_2 - x_1) \bar{\delta}_1 = (H_2 - H_1), \quad \Leftrightarrow \quad F \]

then \( (t_2 - t_1) = \frac{\bar{\delta}_1 (x_2 - x_1)}{\rho \cdot \bar{C} \cdot F} \) \hspace{1cm} (8.14)

where \( \rho \) = Density at inlet to tube,

\( F \) = Volume flow rate at inlet,

\( \bar{C} \) = Mean specific heat capacity.

Note that \( \bar{C} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} C \, dt \triangleq \frac{C_1 + C_2}{2} \)

and \( \bar{\delta}_1 = \frac{1}{(x_2 - x_1)} \int_{x_1}^{x_2} \bar{\delta}_1 \, dx \triangleq \frac{\bar{\delta}_{11} + \bar{\delta}_{12}}{2} \).

Since \( \bar{\delta} \) is a function of temperature, the fluid temperature rise is calculated using two iterations of the formula stated, updating \( \bar{\delta} \) at each stage.

8.2(c) Dimensionless groups and heat-transfer coefficients.

The local dimensionless groups Reynolds, Prandtl and Grashof numbers together with the viscosity ratio \( \left( \mu_{\text{bulk}} / \mu_{\text{wall}} \right) \) were calculated at each axial location in the tube. Little explanation is required for these straightforward computations, it should be mentioned however that all properties were obtained by a simple call on the interpolation 'procedure', and that the reference temperature utilised was the bulk temperature of the fluid. (This was the most convenient reference, and conforms with the generally established practice in tube flow problems, as discussed in part 2.3.(iii)). One obvious
exception is the viscosity ratio ($\mu_{\text{bulk}} / \mu_{\text{wall}}$) which requires viscosity at the local wall temperature. Some further notes follow which give details of formulae used in the programme:

(i) \[ \text{Re} = \left( \frac{2 \rho_0 P_o}{\pi \mu_b r_i^2} \right) \quad \text{Pr} = \left( \frac{\mu_b}{\kappa_b} \right) \]

Viscosity Ratio, $\mu_{\text{wall}} = \left( \frac{\mu_b}{\mu_w} \right)$

\[ \text{Gr} = \left( \frac{(2r_i)^3 (\ell_b)^2 \beta (t_w - t_b)}{(\mu_b)^2} \right) \]

Where
- $\rho_0$, $P_o$ - Density, Volume flow rate at inlet.
- $\mu_b$ - Local viscosity at bulk temperature.
- $\mu_w$ - Local viscosity at wall temperature.
- $r_i$ - Local, inner tube radius
- $\ell_b$ - Local, bulk specific heat capacity.
- $\kappa_b$ - Local, bulk thermal conductivity.
- $P$ - Per cent glycol in water by weight.
- $h$ - Local heat transfer coefficient.
- $t_w$ - Local, inner wall temperature.
- $t_b$ - Local bulk fluid temperature.
- $\beta$ - Local bulk coefficient of thermal expansion.

(ii) In deriving the Grashof group, it was necessary to know the local coefficient of thermal expansion at the bulk temperature. This property was obtained from the density reference data, and a graph of the values was plotted (Figure: 8.5). These coefficients were based on density difference over a 20% temperature variation in order to establish reasonable accuracy (density differences being small). The results were found to be linear in temperature, and hence expressible in a single formula for a given glycol mixture. The numerical constants, however, were highly dependent on the comp-
osition of the liquid, and were expressed as second order polynomials in 'per cent glycol by weight (P)'. The following numerical formulae were derived:

for \(30 < P < 70\), \(\beta \left( \frac{10^{-6}}{K} \right) = (6.965 - .0615.P) \cdot t(\degree C) - 7.050 + 15.27.P - 0.07675.P^2\)

and \(70 < P < 100\), \(\beta \left( \frac{10^{-6}}{K} \right) = (0.6880 + 0.0016.t(\degree C)) \cdot 10^3\).

The formulae well represented the experimentally derived \(\beta\) values, the average scatter being approximately 1.5\% with a maximum of 2.0\% in the range 20\degree C to 70\degree C. (Figure: 8.5).

6.3. A DATA REDUCTION COMPUTER PROGRAMME FOR TUBES HAVING A DIAMETER DISCONTINUITY.

The computer programme given here (see Figure: 8.6) was based on the calculations shown in 6.2. It was used to reduce the data obtained from the experiments on two straight tubes connected in line, having different heat generations and with different diameters. It will be obvious after inspecting it that few modifications would be necessary to permit its application to other similar experimental configurations. The cases of no diameter change, and of heating only one tube were dealt with using nearly identical programmes. A flow chart (Figure: 8.6.) shows the simplicity of the programme construction.

The programming language Algol 60 was used, and the computer was an ICL 1905. Early calculations were carried out on an Elliot 805 with the Elliot Algol language, but the programmes were substantially the same. A typical computing time for one set of test data would be about 20 to 30 seconds, but this depended on the total volume of data input. It should be noted that a large section of the programme deals with the output of results, and the requirements here could vary considerably. In the case shown various combinations
of dimensionless groups were computed, the details are unimportant since they illustrate merely the kind of output which was found to be useful in a particular analysis of experimental results.

A list of programming symbols (and units) follows:—

Subscript $H = 1$ for upstream tube, 2 for downstream tube.

Subscript $G$ = Number of axial thermocouple locations from start of heating.

$XX[H]$ Number of thermocouple locations (axial) less one.

$T_{IN}, T_{OUT}$ Bulk fluid temperature at inlet and outlet ($^\circ C$).

$Q_{LO}$ Volume flow rate at inlet ($10^{-3} \text{ m}^3/\text{s}$).

$V[H]$ Voltage applied to tube (Volts).

$I$ Current through tube walls (Amperes).

$L[H]$ Length of heated section (m).

$R_{I}, R_{O[H]}$ Inner and outer tube radii (cm).

$L_U$ Unheated length of tube upstream of heated section (m).

$T_{AMB}$ Ambient temperature ($^\circ C$).

$K$ Tube thermal conductivity ($W/K$).

$K_{INS}$ Insulation thermal conductivity ($W/K$).

$R_A$ Outer radius of insulation (cm).

$S[H]$ Ratio inner/outer tube radii.

$C_{COMP}$ Percentage of glycol in water by weight.

$K$ Thermal conductivity ($\text{cal/cm.s}^\circ C$). Tabulated properties of propylene glycol-water mixtures.

$V_G$ Viscosity (centipoise).

$C_P$ Specific heat capacity ($\text{cal/g.}^\circ C$).

$D_S$ Density ($10^2 \text{kg/m}^3$).

$X[H]$ Axial distance from start of heating (tube diameters).

$T_{IN[H,2]}, T_{OUT[H,2]}$ Local inner and outer surface tube temperatures ($^\circ C$, or calibration $V$).

$Q_X[H,2]$ Local heating rate per unit length, to fluid ($W/m$).
QLX, [H, C] Local heating rate per unit length, through insulation (W/m)
TX[H, C] Local bulk fluid temperature (°C)
KX, D, KX, CPX, VCX, KX [H, C]
Interpolated values of DJ, CP, VC and K for given mixture
and temperature.
RE[H, C] Local tube Reynolds number
GRX[H, C] Local tube Grashof number
PRX[H, C] Local, bulk fluid Prandtl number
NUX[H, C] Nusselt number at given axial location
M[H, C] Viscosity ratio, bulk fluid/tube surface
B[H, C] Local, bulk coefficient of thermal expansion of fluid (K⁻¹)
GALT, CALV Thermocouple calibration temperatures (°C) and corresponding
potentials (μV).
TI, T1 The size of temperature and per cent increments in property
arrays (K, %).
II, JJ The number of temperature and per cent increments stored
for a given property.

All other symbols used are dummy variables used to facilitate programme
construction. The following titles are given to function-procedures:
INT Interpolation procedure used to derive intermediate properties
from the reference data supplied
CAL Conversion of thermocouple readings (μV) to temperature (°C)

* Note that the use of 'caloric' does not conform with the SI system
of units (BSI publication PD5605), and is included here simply because
the relevant property data was obtained by direct reference to Currie
(Ref: K.A) who preferred this unit.

8.4. THE HEAT BALANCE.

In part 5.6, there is a description of the calculations
carried out to determine the distribution of energy over a closed
boundary which surrounds the system. It was found that the temperature
rise obtained in propylene glycol solutions passing through the test-
section was very small; usually less than 1 K, and often less than 0.5 K. This is a consequence of the high Prandtl numbers encountered in the experiments, because for a given temperature difference between tube and fluid, and for a given Re the rise in the bulk temperature of the fluid, $\Delta t_{\text{bulk}}$, is related to Pr in the following way:

$$\Delta t_{\text{bulk}} \propto \text{Pr}^{-\frac{5}{3}}$$

Initially, a heat balance was attempted for each experiment carried out, and the calculations were incorporated into the computer programme (8.3.) This practice was abandoned because of limitations on the reliability of the results; for example, for a rise in bulk temperature of 0.35 K, the confidence limits on the thermocouple readings gave rise to at least $\pm 20\%$ uncertainty in the calculated enthalpy rise of the fluid.

Heat balances were obtained for several tests on the uniform diameter tube where favourable conditions prevailed (a bulk temperature rise greater than 0.5 K). These were discussed earlier in part 5.6.
Figure 8.2.
Flowmeter calibration.

<table>
<thead>
<tr>
<th>Frequency (c/s)</th>
<th>Flowrate Small motor ($10^{-3}$ m$^3$/S)</th>
<th>Flowrate Large motor ($10^{-3}$ m$^3$/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kinematic Viscosity 1 c St</td>
<td>Kinematic Viscosity 1 c St</td>
</tr>
<tr>
<td>150</td>
<td>0.15608, 0.16888, 0.17563</td>
<td>1.6054, 1.5712, 1.5865</td>
</tr>
<tr>
<td>250</td>
<td>0.26018, 0.27572, 0.20277</td>
<td>2.6890, 2.6091, 2.6278</td>
</tr>
<tr>
<td>500</td>
<td>0.51866, 0.54356, 0.55212</td>
<td>5.3780, 5.2229, 5.2217</td>
</tr>
<tr>
<td>600</td>
<td>-</td>
<td>8.6253, 6.2965, 6.3009</td>
</tr>
<tr>
<td>800</td>
<td>0.82541, 0.86716, 0.88095</td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>1.03060, 1.07673, 1.09363</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8.3.
Thermocouple calibration.

<table>
<thead>
<tr>
<th>Temperature ($^\circ$C)</th>
<th>Microvolts</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>431</td>
</tr>
<tr>
<td>14</td>
<td>599</td>
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<tr>
<td>18</td>
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<td>1126</td>
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</tr>
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<td>1485</td>
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<td>1668</td>
</tr>
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<td>42</td>
<td>1851</td>
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</tr>
<tr>
<td>86</td>
<td>3961</td>
</tr>
<tr>
<td>90</td>
<td>4161</td>
</tr>
<tr>
<td>95</td>
<td>4360</td>
</tr>
</tbody>
</table>
FIGURE 8.4.

VISCOITY OF PROPYLENE GLYCOL MIXTURES—INTERPOLATION.

@ VISCOSITY ACCORDING TO INTERPOLATION FORMULA.

100% GLYCOL.

90% GLYCOL.

DYNAMIC VISCOSITY (CENTIPOISE).

TEMPERATURE (°C).
THE COEFFICIENT OF THERMAL EXPANSION FOR AQUEOUS PROPYLENE GLYCOL MIXTURES.

![Graph showing the coefficient of thermal expansion for aqueous propylene glycol mixtures. The graph plots temperature (°C) on the x-axis and coefficient of thermal expansion (10^(-3) K⁻¹) on the y-axis. Several curves represent different compositions of glycol: 20%, 40%, 60%, 80%, and 100%. Each curve is labeled with its respective composition percentage. The graph also includes a note indicating a composition of 30% glycol.](image-url)
Figure 8.6

Computer Programme Flow-Chart for Reduction of Raw Data

1. Start
2. Declare Variables
3. Read Input Data
4. Store Calculation Data (Temperature, Viscosity)
5. Read Inlet Properties
6. Call Procedure to Convert Into Temperature
7. Call Procedure to Convert Into...
8. Call Local Thermocouple Voltages on Tube Ends, Also Position Inlet...
'BEGIN' 'COMMENT' MEST11A SECTCHANGE

'COMMENT' READ IN NO. THERMO. LOCATIONS;

'INTEGER' 'ARRAY' XX[1:2]; XX[1]=READ; XX[2]=READ;

'COMMENT' DECLARE VARIABLES;

'BEGIN' 'REAL' TIN,TOUT,F(O,1),TAMB,KP,KINS,RA,COMP,LU;
'INTEGER' G,E,H,W; 'REAL' 'ARRAY' K[0;7],0;5,VC,CP,DS[0;7],0;10,TPO,
TPJ,X,GINS,Q,T,DX,CPX,VCX,XX,RE,PRX,H,NUX,B,GRX[112,0;XX[2]],CALV,
CALT[0;21],V,1,RO,RI,S[1:2];

'COMMENT' DECLARE SUB ROUTINES;

'COMMENT' INTERPOLATION SUB ROUTINE;

'REAL' 'PROCEDURE' INT(TW,PX,PI,TT,II,JJ,A);
'VALUE' TW,PX,PI,TT,II,JJ; 'REAL' TW,PX; 'REAL' 'ARRAY' A;
'INTEGER' PI,TT,II,JJ;
'BEGIN' 'REAL' 'ARRAY' AA[1:3],T[0:7],P[0:10]; 'INTEGER' Y,N,H;
'FOR' H=0 'STEP' 1 'UNTIL' II 'DO' T[N]=N*TT;
'FOR' M=0 'STEP' 1 'UNTIL' JJ 'DO' P[M]=M*PI;
Y=1; N=0; H=0;
FF: 'IF' TW>T[N] 'THEN' 'BEGIN' N=N+1; 'GOTO' FF; 'END';
DB1: 'IF' PX>P[N] 'THEN' 'BEGIN' H=H+1; 'GOTO' DB2; 'END';
Y=1; N=0; H=0;
'BEGIN' A[N]=A[N,M]; 'GOTO' DB2; 'END';
1; DD: 'IF' Y<3 'THEN' 'BEGIN' Y=Y+1; N=N+1; 'GOTO' CC; 'END';
N=N-1;
'BEGIN' INT:=AA[2]; 'GOTO' EE; 'END';
AA[1]));
/AA[3]-AA[1])/LN(T[N]-T[N-1])/(T[N+1]-T[N-1]));
EE: 'END' INT;

'COMMENT' THERMO. CALIBRATION SUB ROUTINE;

'REAL' 'PROCEDURE' CAL(AV,CV,CT); 'VALUE' AV; 'REAL' AV; 'REAL' 'ARRAY' CV,CT;
'BEGIN' 'INTEGER' U; U=0;
LL: 'IF' AV>CV(U) 'THEN',
'BEGIN' U=U+1; 'GOTO' LL; 'END';
CAL=CAL(U)+AV(CV(U-1))+(CV(U+1)-CV(U-1));
'END' CAL;

'COMMENT' CALIB. DATA;

CALC[0]=10; 'FOR' H=1 'STEP' 1 'UNTIL' 21 'DO' CALT[H]=CALT[H-1]+4;
CALT[21]=4360;
COMMENT: READ IN PROPERTY DATA

FOR H=0 TO 1 'UNTIL 5 'DO' FOR W=0 TO 7 'STEP' 1 'UNTIL' 7 'DO'
K[W,H]:=READ
FOR H=0 TO 9 'STEP' 1 'UNTIL' 10 'DO' FOR W=0 TO 6 'STEP' 1 'UNTIL' 7 'DO'
V[C,W,H]:=READ
FOR H=0 TO 9 'STEP' 1 'UNTIL' 10 'DO' FOR W=0 TO 6 'STEP' 1 'UNTIL' 7 'DO'
C[P,W,H]:=READ

COMMENT: START CALCS.

COMMENT: READ IN EXPERIMENTAL DATA:
AGAIN: TIN:=READ; TOUT:=READ; FLO:=READ; H:=READ; TAM:=READ; K:=READ
KINS:=READ; RA:=READ; COMPL:=READ; LU:=READ; 'FOR' G=1.2 'DO' 'BEGIN'
V[C]:=READ; L[C]:=READ; R[C]:=READ; R[C]:=READ
TIN:=CAL(TIN, 'CALV', 'CALT') 'IF' TOUT>100 'THEN' TOUT:=CAL(TOUT, 'CALV', 'CALT')

COMMENT: READ TUBE TEMPS AND LOCATIONS:

FOR H=1 TO 6 'DO' 'BEGIN' S[H]:=RI[H]/P[H];
FOR G=0 TO 9 'STEP' 1 'UNTIL' XX[H] 'DO' 'BEGIN' X[H,G]:=READ
TPO[H,G]:=READ; TPO[H,G]:=CAL(TPO[H,G], 'CALV', 'CALT') 'END'

COMMENT: CALC. LOSS THRO. INSULATION THEN LOCAL HEAT FLUX:

FOR G=1 TO 6 'STEP' 1 'UNTIL' XX[H]-1 'DO' 'BEGIN'
QINS[H,G]:=3.1416*KINS*(TPO[H,G]-TAM)/LN(RA/RO[H])
QITH[H,G]:=V[H]+1/L[H]+3.1416*KP*(RI[H]*1.2)/(1+RI[H]*1.2)*(1+RI[H]*1.2)
TPO[H,G]/(TPO[H,G]-V[H]-1)/((TPO[H,G]-V[H]-1)+0.5*(2*RI[H]*1.2)-QINS[H,G]) 'END'
QISH[H,G]:=QINS[H,G]+QISH[H,G]+
QISH[H,G]*QISH[H,G]*QISH[H,G]*QISH[H,G]*QISH[H,G] 'END'

COMMENT: CALC. LOCAL FLUID TEMPS:

DSX[1,1]:=INT(TIN, 'COMP', 10, 10, 7, 10, DS); CPX[1,0]:=INT(TIN, 'COMP', 10, 10, 6, 10, CP)
TX[1,0]:=INT(23.1416*KINS*LU/(TIN-TAM)/LN(RA/RO[1]))/(DSX[1,1])
FCX:=CPX[1,0]+4.18585; CPX[1,0]:=INT(TX[1,0], 'COMP', 10, 10, 6, 10, CP)
FOR H=1 TO 6 'DO' 'BEGIN' G=1 'STEP' 1 'UNTIL' XX[H] 'DO'
'BEGIN'
'IIF' H=2 'AND' G=1 'THEN' 'BEGIN' TX[2,0]:=TX[1,XX[H]]; CPX[2,0]:=CPX[1,XX[H]] 'END'
'BEGIN' TX[H,G]:=(X[H,G]-X[H,G])*(TX[H,G]-G)/(TX[H,G]-G)+(TX[H,G]-G)/(TX[H,G]-G)
E=0
'CYCLE': CPX[H,G]:=INT(TX[H,G], 'COMP', 10, 10, 6, 10, CP)
TX[H,G]:=(X[H,G]-X[H,G]+G/0.5+CPX[H,G]+FCX*DSX[1,1])/(4.18585)+TX[H,G]-G)
'IIF' E<1 'THEN' 'BEGIN' E=1; 'GOTO' 'CYCLE' 'END'; 'END'

COMMENT: OBTAIN LOCAL PROPERTIES AND CALC. INSIDE TUBE TEMPS.

FOR H=1 TO 6 'DO' 'BEGIN' 'FOR' G=0 TO 9 'STEP' 1 'UNTIL' XX[H] 'DO' 'BEGIN'
DSX[H,G]:=INT(TX[H,G], 'COMP', 10, 10, 7, 10, DS); VX[H,G]:=INT(TX[H,G], 'COMP', 10, 10, 7, 10, VX)
9. ANALYSIS OF THE RESULTS.

9.1. THE RELIABILITY AND SIGNIFICANCE OF TEMPERATURE MEASUREMENTS.

The local coefficients of heat transfer, which are referred to throughout this thesis, are measured or calculated from the mixed, mean temperature of the fluid, and the circumferential mean temperature at the inner surface of the tube. This is not to say that the temperature distribution within the fluid was axi-symmetrical, indeed vertical symmetry and asymmetry were both encountered. Although the relevance of such phenomena will be considered in part 9, the greater portion of this discussion concerns the axial distributions of Nu.

The local temperature and heat flux were susceptible to errors induced through inaccurate measurements, variations in the wall thickness, the conduction of heat in the tube-wall and changes with time in the fluid flow pattern. A critical appraisal of the thermocouple measurements has already been carried out, (5.3, 5.4, 6.3) so the first source of error mentioned may be disregarded in this discussion. The local heat flux was proportional to the wall thickness and must have had a similar fractional variation from the mean value. Since the cross-sectional area of a cold drawn, seamless tube can be considered constant, the circumferential mean temperature at any axial position must have been independent of small, dimensional variations. The second source of error can therefore be disregarded for practical reasons. Parts 9.2 and 9.3 include an appreciation of the 'conduction' and 'time dependent' aspects of tube temperature measurement. These possible sources of error bear closer examination since the magnitudes involved were difficult to determine.
9.2. THE EFFECTS OF TUBE WALL CONDUCTION.

In calculating the local temperature and heat flux at the inner surface of the tube an estimate was made for the amount of heat conducted axially, radially and circumferentially in the wall. The temperature difference across the wall and the heat flux were assessed for the case of no circumferential conduction and a small amount of axial conduction.

It was discovered during the experimental work however that at low values of Re (< 2000) and for large axial distances (> 20 diameters) the top of the tube was considerably hotter than the bottom. This gave rise to a distortion of the circumferential temperature distribution, and the corresponding 'point' value for the coefficient of heat transfer, \( h_s \) (this parameter is defined below, and has relevance only within the present discussion.) The extent to which circumferential conduction affected the measured values of \( h_s \) was determined as follows:

Consider the coordinate system below and the assumptions stated. The model proposed takes a rather crude view of the convection process, since conduction in the tube is the main concern.

Assume -

(i) The wall is thin and ideally insulated.

(ii) Axial temperature gradients are small at large axial distances.

(iii) The circumferential variation in \( h_1 \) (not \( h_s \)) is not great when \( h_\parallel \) is defined \( h_1 = q/(t_w - t_\infty) \)
Compare this with \( h_g = \frac{q}{(t_w - t_{cm})} \).

(iv) In the central core the fluid is stratified so that \( t_{\infty} \) varies linearly in the vertical direction, to a first approximation.

(v) An axial voltage gradient is applied to the tube.

The governing equation is

\[
\frac{I}{2mR_k \Delta} \frac{dV}{dx} = \frac{h_1}{K_p \Delta} (t_w - t_{cm}) - \frac{\partial^2 t_w}{\partial x^2}, \quad (9.1)
\]

where \( t_{cm} = t_{cm} - \frac{\Delta t}{2} \cos \left( \frac{\theta}{R} \right) \) \quad (9.2)

Putting \( \rho = (t_w - t_{cm}) \),

gives \( \rho'' - \left( \frac{h_1}{K_p \Delta} \right) \rho + \left( \frac{I}{2mR_k \Delta} \right) \left( \frac{h_1 \Delta t}{2K_p \Delta} \right) \cos \left( \frac{\theta}{R} \right) \) \quad (9.3)

Since the boundary conditions are \( \rho'(0) = 0 = \rho'(\pi R) \),

then \( \rho = \frac{I}{2mR_k \Delta} \left( \frac{h_1 \Delta t}{2} \right) \frac{\cos \left( \frac{\theta}{R} \right)}{h_1 R^2} \) \quad (9.4)

In the absence of circumferential conduction

\( \rho = \rho_o = \frac{I}{2mR_k \Delta} - \frac{\Delta t}{2} \cos \left( \frac{\theta}{R} \right) \) \quad (9.5)

so that \( (\rho - \rho_o) = \left[ \frac{\Delta t}{2} \cos \left( \frac{\theta}{R} \right) \right] \left( \frac{K_p \Delta}{h_1 R^2} \right) \) \quad (9.6)

The maximum error which would be incurred through neglecting circumferential conduction in the measurement of \( h_g \) is given approximately by -

\[
\text{error} \quad \varepsilon \approx 0.5 \left( \frac{K_p \Delta}{h_s R^2} \right) \frac{\left( 1 + \frac{K_p \Delta}{h_s R^2} \right)}{1 + \frac{K_p \Delta}{h_s R^2}} \quad (9.7)
\]
where \( \gamma = \frac{\text{temperature difference from top to bottom}}{\text{temperature difference used to calculate } h_s} \)

The greatest degree of temperature stratification (\( \Delta t \)) occurred in tubes having a large diameter, so for a typical case put

\[ k_P = 15.5 \frac{W}{m \cdot K}, \quad \Delta = 0.075 \text{ cm}, \quad R = 2.8 \text{ cm}, \quad h_s = 100 \frac{W}{m^2 \cdot K} \]

Now \[ \frac{k_P \Delta}{h_s R^2} = 0.148, \]

Therefore \[ \varepsilon = 0.0645 \gamma \]

As the degree of stratification reaches the same order of magnitude as the circumferential mean temperature difference used in calculating \( \text{Nu} \), it is evident that \( \varepsilon = 6.45\% \) maximum.

The measured value of \( h_s \) was likely to be much less than 6\% inaccurate during the most severe experimental conditions, and it was probable that the accuracy of measured values of \( \text{Nu} \) - a function of axial position only - was independent of the degree of temperature stratification. From the preceding argument it was proposed that any further investigation into the effects of circumferential conduction would have a small relevance to this work.

The dependence of the local heat flux and temperature drop across the tube wall on the amount of axial conduction was evaluated in the analyses 5.3(iii) and 8.2(c). These effects were found to be most severe at small axial distances were rapid changes occurred in the axial temperature distribution. The local heat flux was calculated and compared with the mean value for the entire tube, at each thermocouple station. This comparison yielded the percentage of heat generated which was dispersed by axial conduction in an elemental length.

In the experiments on the short, straight tube, measurements showed this percentage to be always \(< 1.1\%\). The greatest
estimate for axial conduction occurred at one diameter from the start of heating, with the lowest value of the (Re x Pr) product - 43,000 (as in test 79aB) - and with the bellmouth fitted. The amount of axial conduction calculated would probably have increased considerably at smaller distances had these been investigated.

To substantiate the order of magnitude obtained it was assumed that the axial distribution of Nu was similar to the theoretical solution of Sellars for laminar flow (Ref: C.13) viz - \[ Nu = 1.30 \left( \frac{RePr}{X_1} \right)^{\frac{3}{2}} \]

The ratio of axial to radial conduction from a small length of tube is given (part 8.2(c)) by

\[ \text{Ratio} = \frac{K_P}{K_L} \frac{\delta}{D} \cdot \frac{\partial^2 t}{\partial x^2} \left( \frac{\partial y}{\partial x} \right)^2 \]

or rewriting another way

\[ \text{Ratio} = \left( \frac{K_P}{K_L} \frac{\delta}{D} \frac{\partial^2}{\partial x^2} \left( \frac{\partial y}{\partial x} \right)^2 \left( Nu^{-1} \right) \right), \text{ where} P = \text{tube subscript} \]

\[ \text{L} = \text{liquid subscript} \]

\[ \delta = \text{wall thickness} \]

\[ D = \text{tube diameter} \]

\[ X_1 = \frac{x}{D} \]

hence \[ \text{Ratio} = 0.171 \left( \frac{K_P}{K_L} \frac{\delta}{D} \left( \frac{RePrX_1^5}{X_1} \right) \right)^{-\frac{3}{2}} \]

Now \[ \frac{\delta}{D} = 0.0466, \frac{K_P}{K_L} \approx 64, (RePr)_{\text{minimum}} = 43,000, \]

So that \[ \text{Ratio} = 0.015 X_1^{-\frac{5}{3}} \text{ maximum}. \]

In the turbulent flow regime the effects of axial conduction on measured values of Nu were less important than in laminar flow. This result can be justified by substituting a different expression for Nu in the above analysis. A theoretical investigation of turbulent heat-transfer (in part 11.2.(iv)) showed that for very small axial distances
\[ \text{Nu} = 0.1855 \left( \frac{Pr^{1.5} \, Re^{0.6}}{x_1^{5/3}} \right) \]

For a low \( Re = 4,000 \) and a low \( Pr = 100 \), the following equation applies - Ratio = 0.0053 \( x_1^{-5/3} \). It is clear that radial conduction exceeds axial conduction considerably when \( x_1 > 1 \).

In the experiments with the sudden convergence in tube diameter, the axial distribution of \text{Nu} was similar to that in the 'bellmouth' tests, the configurations being qualitatively similar. But since the section of tube downstream generally had a greater thickness to diameter ratio, since equally low (RePr) products were recorded, and because smaller axial distances were considered - it was probable that axial conduction would have a more significant effect on the measured coefficients of heat transfer in the 'convergence' experiments. The local heat flux was corrected for axial conduction by a maximum of 9\% in test 6c3, where the (RePr) product was small at 44,300; the convergence ratio was greatest at 3.341, and the axial distance was 0.5 to 1.0 diameters from the discontinuity. The correction reduced to 2\% at 2 diameters. With turbulent flow and low values of \( Re (\sim 4,000) \), it was found that a large correction to the heat flux of 10\% was necessary. This was the case in tests 13c3 and 14c3, where Pr was greatest at 340, and the axial distance was 0.5 to 1.0 diameters. The correction reduced rapidly to \( \leq 1\% \) at 2 diameters. At higher values of \( Re \) the conduction effects became negligible.

In the experiments with the 'sudden divergences' the second derivative of temperature, \( \frac{\partial^2 t}{\partial x^2} \), was greatest near to the positions of maximum and minimum tube temperature which occurred a short distance downstream of the discontinuity. Once again it was found that axial conduction had little effect.
on the local heat flux, except where low values of Re (200-400) were encountered and Pr was low (50). The maximum correction to the local heat flux was 10% (occurring in tests 18d3 and 16d1) and the magnitude of the corrections was similar for the three diameter ratios utilised.

9.3. MEASURED VARIATIONS IN TIME FOR TUBE TEMPERATURES.

In part 6.1, it was described how continuous readings of the tube temperature were carried out. The only experimental configuration for which such an investigation was not implemented was the short tube with a calming length. Variations in the instantaneous temperatures occurred, under certain experimental conditions, which could only be attributed to changes in the flow pattern. These were generally slow, cyclic fluctuations which were continuous but irregular, having a typical periodicity of 10 to 100 sec. More rapid fluctuations were undetectable because of transient conduction in the tube wall.

The changing electrical supply may well have contributed slightly to the measured variations, but such changes were easily determined and were found to be small.

For the short tube with a bellmouth-entrance, there was some evidence that the tube temperature was unstable during the transition from laminar to turbulent flow, that is in the range Re = 2,000 to 10,000. In figure 9.5 the points plotted represent measured variations in the tube temperature, over a period of 1 to 2 min., expressed as a percentage of the local temperature difference from tube to fluid (t_w - t_b). This criterion occurs frequently throughout the following discussion. The measurements were taken at axial distances of 1, 3, 4, 11, 20 and 70 diameters, but the magnitude of the variations did not depend
simply on the axial location. Maximum variations of approximately 7% occurred at \( \text{Re} \approx 3000 \), and for very large or very small \( \text{Re} \) the variations were less than 1%. Ede (Ref: A.1.) made similar observations in experiments with water and air, but variations of 20% at \( \text{Re} = 3,000 \) were recorded which reduced to zero at \( \text{Re} = 9,000 \).

The experiments on the convergences were in general agreement with the bellmouth tests, with respect to the temperature fluctuations. Typical data for the 1.25:1 and 3.34:1 convergences are plotted (Fig. 9.6) for distances of 1, 5.5, 15 and 40 diameters, in a similar manner to that already described. The magnitudes of the variations measured at one diameter downstream of the discontinuity were very large for the 3.34:1 convergence: a maximum of 35% of \((t_w - t_b)\) occurred at \( \text{Re} = 5,400 \). The measurements otherwise were comparable in magnitude and distribution with the bellmouth results.

In the case of the 'sudden divergence' experiments a more complicated description of the unsteadiness in the tube temperature is necessary. Considering the 1:3.34 divergence, the fluctuations were greatest in the approximate range \( \text{Re} \approx 200/668 \) (Downstream/Upstream) to 800/2,672 and the maximum variations measured were in the range 0% to 30%, with the exception of a few results taken when the flow was highly unstable. Figure 9.7 shows how the magnitude of the fluctuations varied with axial distance in a typical test. It is apparent that the maximum fluctuations occurred near to the position of minimum tube temperature (or maximum \( \text{Nu} \)). The probable reason for this became apparent after measuring the maximum and minimum tube temperatures for a particular flow rate and heat flux, then comparing these results with the corresponding 'instantaneous'
temperatures measured during a short time interval. This procedure was followed in a test where the fluctuations were particularly severe. The comparison of the axial temperature distributions thus obtained is given in figure 9.8.

It is clear that small axial movements in the disposition of flow could cause severe variations in the local, instantaneous temperature by virtue of the high axial temperature gradients near to the location of the minimum tube temperature. The minimum temperature recorded was found to be almost identical for the readings taken with and without a time dwell. The way in which the fluctuations varied with Re is shown in figure 9.9, which includes typical experimental data measured at 5 and 40 diameters downstream, and a diagram giving an idealised picture of the distribution of the maximum variations. Below Re = 200/668 the fluctuations were negligibly small. Between Re = 200/668 and 800/2,672 the fluctuations were either negligible (0%) or large (mostly 8% to 30%). Between Re = 800/2,672 and 2,500/8,350 the maximum fluctuations were typically 4% to 15%. From Re = 2,500/8,350 to 10,000/33,400 the fluctuations were approximately 4% to 7%.

The alternative condition proposed in the range Re = 200/668 to 800/2,672 was indicative of a bi-stable flow system, which could be described loosely as being either 'laminar' or 'turbulent' (the latter term implies a disturbed, laminar flow and not a sustained turbulence as in the usual sense of the term). The existence of the two flow conditions was established beyond doubt in a particular test (4B41) when the tube temperature oscillated between characteristically high and low values near to the diameter discontinuity. The three tests with extreme fluctuations in figure 9.9 had flows of this nature. By manipulating the flow-control valves it was possible to sustain
the lower temperature, which corresponded to turbulent flow. When the higher tube temperature prevailed, the fluctuations were always immeasurably small.

Figure 9.9 also illustrates the magnitude of the maximum fluctuations at a considerable distance (40 diam.) from the divergence. These variations became large in the Re range 2,000/6,680 to 10,000/33,400 when their magnitude decreased from 20% to 4% with increasing Re.

Further discussion on the relationship between heat transfer and the flow regime will be contained in part 9.7, but a recapitulation of the foregoing argument will be stated here.

Transitional flow in a divergence is more complex than in a uniform tube. Near to the discontinuity the laminar regime persists with increasing flow rate until a lower, critical Reynolds number is reached, which is considerably less than the 2,500 usually quoted for a uniform tube. At higher Re values, laminar flow may persist or turbulence may develop, large fluctuations in the tube temperature being associated with the latter. When Re > 2,500 upstream, laminar flow can no longer exist downstream, except at considerable distances from the discontinuity. Smaller temperature fluctuations persist in this region, of the same order of magnitude as were obtained with transitional flow in the 'bellmouth' tests. The fluctuations diminished and became negligible as Re tended to 10,000 downstream, when turbulence pervaded throughout.

Up to now just the 1:3.34 divergence has been considered and the results appertaining to the 1:1.25 and 1:2 ratios should be mentioned. As the ratio of diameters tended to unity the abovementioned 'lower critical Re' increased, reducing the size
of the first part of the transition range, \((Re < 2,500\) upstream). Further, the tube temperatures associated with laminar and turbulent flow approached each other, thereby making it more difficult to demonstrate the distinction between the different flow regimes. The magnitudes of the fluctuations recorded during transitional flow were generally similar for each of the three divergence ratios investigated; most of the variations being of the order \(5\%\) to \(30\%\) of \((t_w - t_b)\) the maximum being close to the position of minimum temperature.

9.4. **THE SHORT STRAIGHT TUBE; LOCAL TUBE TEMPERATURES.**

A brief description will now be given of the kind of temperature distributions obtained with the short, straight tube having a calming length. Because the results obtained with the bellmouth entrance were qualitatively similar, these results are omitted from the discussion.

Figure 9.1 shows the axial temperature profile for the top and bottom of the tube as measured in test 32s; the circumferential, mean tube temperature is also indicated, together with the bulk temperature of the fluid. In this test \(Re\) was low, increasing from 366 at the inlet to 372 at the outlet due to the temperature dependence of the physical properties. The value of \(Pr\) reduced from 183 to 181 for the same reason. (For ease of discussion, these small changes will be ignored henceforth, and the value at the position \('x = 0'\) will be quoted). The tube temperature continues to rise with increasing distance from the onset of heating, but the gradient decreases. The top of the tube becomes progressively hotter than the bottom as the distance increases, the maximum difference being equivalent to \(42\%\) of \((t_w - t_b)\), which is the difference used to calculate the local
coefficient of heat transfer. The peripheral distribution of the
tube temperature is given at axial distances of 1, 20, and 70 diameters
downstream, and these diagrams show an almost vertical symmetry.
The slight asymmetry could not be positively identified as being
related to perturbations in the tube's geometry or unevenness of
heating, but in other tests the 'right-handed' tendency was
reversed (see for example Fig: 9.3). No effort was made to
ensure a truly symmetrical temperature profile, since it was
clear that a relatively minor disturbance would change the tempera-
ture distribution and create major, practical difficulties, where-
as; there could be no significant benefits from having such an
idealised system. The only other pertinent feature of this
diagram is that the axial temperature gradient on the bottom of
the tube quickly reaches the same rate of increase as bulk
temperature of the fluid.

The temperature stratification in the fluid appears
to follow the proposals of part 3.5 in which a vertical, secondary
motion of the fluid near to the tube wall is supposed to carry the
less dense, heated fluid to the upper surfaces of the tube.
Some photographic evidence of these effects was obtained in the
flow visualization tests (described in 7.4(i)). Figure 9.4-1A.
shows water heated in a long tube with Re = 550. The fast moving
central core travels slowly downward, and the slow moving elements
near to the wall climb almost vertically to the top of the tube.
Figures 9.4B and C show how increasing Re suppresses the
secondary flow, and how increasing the rate of heat transfer
enhances this motion.

The temperature distributions in test 33s were obtained
with similar values of Re and Pr to test 32s already mentioned.
In this case however the heat flux was increased by 3\(\frac{1}{2}\) times.
The same general observations apply to the temperature profiles of both tests (see figure 9.2) but in 33s the stratification develops more rapidly with distance. The greatest temperature difference from top to bottom was equivalent to 59% of \((t_w - t_b)\). This is not such a dramatic increase over the previous 42%, considering that the heat flux was almost quadrupled.

From figure 9.3, the results of test 42s exhibit far less stratification than in the previous cases. The value of Re was 2,265 and Pr was 257. Re was high, although still in the laminar range, and it was evident that this caused the stratification to develop at a slow rate, as was indicated previously in the photographic study. The maximum degree of temperature stratification in this test was only 16% of \((t_w - t_b)\).

Increasing the Reynolds number even further caused the onset of turbulence and, as might be expected, the stratification was entirely suppressed by the random motions of the fluid. It may well be that free convection could have occurred at extremely high heat fluxes, but the subject was outside the scope of this work.

Figure 9.4 gives the temperature profiles for Re = 4,216 and Pr = 194 (test 45s). The temperature curves for the top and bottom of the tube are coincident, and these become parallel with the bulk temperature line after approximately 20 diameters from the start of heating.

9.5. THE SHORT STRAIGHT TUBE: HEAT TRANSFER COEFFICIENTS.

9.5.1 THE TUBE WITH A CALMING LENGTH.

Considering first the short, straight tube with a calming length, and restricting the discussion to transitional and turbulent flows, the initial objective was to determine
unique correlations between the local Nu and the various dimensionless parameters already discussed. Empirical and analytical techniques were utilised to this end. Suppose that

\[ Nu = f(Re, Pr, Gr, X, P, Q) \]

where P and Q are variables which take account of the 'viscosity - temperature' dependence, and 'dissipation' dependence of Nu, (The parameter X is \( (x/2r_w) \)).

In part 9.4. the free convectional contribution to heat transfer was shown to be small during turbulent flow from observation of the experimental temperature profiles. Gr can therefore be eliminated from the expression above.

An analytical investigation of dissipation in forced convection (part 11.1.(v)) indicated that the value of Nu was likely to be dependent on the parameter \( E = \left( \mu \bar{u}^2 / 2gr_w \right) \) - which corresponds to the variable Q. However, the neglect of this parameter was likely to cause errors of \(< \frac{1}{2}\%\) when estimating Nu, so the value of Q can be set at zero in the expression above.

A suitable variable P was postulated in the analysis of 'variable viscosity' convection (11.2.(iii) and (iv)) - the viscosity ratio \( M_{wall} = M_{bulk} / M_{wall} \). Some justification for the use of this parameter at small axial distances was given, but the present knowledge of turbulence precludes anything other than a tentative argument regarding its general applicability.

The conclusion at this juncture is that

\[ Nu = P(Re, Pr, X, M_{wall}) \]

Another result of the theoretical investigations, 11.2.(iv), was that for high Prandtl number fluids Nu is proportional to \( Pr^n \), where n is probably \( \frac{3}{4} \), but could be as low as \( \frac{1}{4} \). The preceding expression can be modified accordingly, and with only a small loss of generality the following is obtained.
for constant $X$, \[ \text{Nu} = F_1(M_{\text{wall}}) \cdot \text{Pr}^3F_2(\text{Re}) , \]
where $F_1 \to 1$ for a 'constant viscosity' fluid.

The functions $F_1$ and $F_2$ are to be determined empirically.

The exponent of Pr selected was $\frac{3}{4}$, which was to be justified experimentally. The advantage of using this value initially was that it was known to be valid for the laminar regime (as described in the theory 11.2(i)) and thus the correlation of the experimental parameters was simplified. Despite the substantial range of Pr utilised in these experiments (approximately 90 to 400 for turbulent flow) the difference between the exponents $\frac{3}{4}$ and $\frac{4}{4}$ was equivalent to only 12% of the Nusselt number for the purposes of correlation. Whereas such a discrepancy should have been detectable, it was unlikely that the exponent could be resolved to better than 10%.

In correlating the measured parameters it was noted that $M_{\text{wall}}$ varied considerably at large axial distances, so the function $F_1(M_{\text{wall}})$ was investigated initially at 70 diameters downstream of the onset of heating.

Figure 9.10 shows how the group $(\text{Nu}/\text{Pr}^{\frac{3}{4}})$ was plotted with Re as the abscissa. The correlation was poor because $F_1(M_{\text{wall}})$ was ignored; deviations of 20% in $(\text{Nu}/\text{Pr}^{\frac{3}{4}})$ were observed.

During tests 1s to 57s the heat flux was varied keeping Re almost constant (described in 7.1(i)) so that after making corrections for the small Re variations, it was possible to eliminate the heat flux, and therefore $M_{\text{wall}}$, dependence from the correlation by graphical procedure. The latter is illustrated in Figure 9.11.

where graphs of $(\text{Nu}/\text{Pr}^{\frac{3}{4}})$ versus the temperature difference $(t_w - t_b)$ are shown, for particular Re values. The magnitude of $(\text{Nu}/\text{Pr}^{\frac{3}{4}})$ for the 'zero heat-flux' condition could be obtained
by extrapolation. Returning now to figure 9.10, the line representing the function \( F_2(\text{Re}) \) could be drawn for \( F_1(M_{\text{wall}}) \) equal to unity. This was necessarily a first approximation to be refined in due course. Having obtained this function it was possible to determine the \( \text{Nu} \) dependence on the viscosity variation, or \( F_1(M_{\text{wall}}) \).

This was achieved by plotting \( (\text{Nu}/\text{Nu}_o) \) versus \( M_{\text{wall}} \) — where \( \text{Nu}_o \) is the value of \( \text{Nu} \) corresponding to \( M_{\text{wall}} = 1 \). The latter is given in figure 9.12 for the axial position of 70 diameters. The function \( F_1 \) was found to be well approximated by putting \( F_1 = M_{\text{wall}}^{a} \). From a 'least squares' regression, the most suitable value of 0.16 was calculated for 'a'. In the theoretical analyses 11.2(iii) and 11.2(vi) values in the range 0.14 to 0.17 were postulated for laminar flow, near to the start of heating, and it is proposed (in 11.2(vii)) that a similar value be utilised in the turbulent regime.

Sieder and Tate (Ref: D.1.) showed that for large axial distances a suitable value for 'a' would be 0.14. From the graph \( (\text{Nu}/\text{Nu}_o) \) versus \( M_{\text{wall}} \) it can be seen that \( M_{\text{wall}}^{0.14} \) differs only slightly from \( M_{\text{wall}}^{0.16} \) in these experiments; the maximum extent of the scatter in the results being +4%, -2.5% when \( F_1 = M_{\text{wall}}^{0.16} \), and +4.1%, -2.3% when \( F_1 = M_{\text{wall}}^{0.14} \).

To verify that the function \( F_1(M_{\text{wall}}) \) was independent of axial position, the entire procedure, for obtaining \( F_1 \) and \( F_2 \), was repeated at one diameter axial distance. The lower temperature differences obtained near to the inlet yielded smaller values for \( M_{\text{wall}} \). Figure 9.12 gives the resulting dependence of \( \text{Nu} \) on \( M_{\text{wall}} \), and it is apparent once more that, whereas \( 'F_1 = M_{\text{wall}}^{0.16}' \) gives the best fit to the experimental data, the
relationship \( F_1 = \mu \nu_{wall}^{0.14} \) adequately describes the results.

The group \( \left( \frac{\frac{Nu}{Pr^{0.14}}}{Re^{0.14}} \right) \) was chosen to be a function of \( Re \) and \( X \) for the purpose of correlating the experimental results in the turbulent regime. The exponent 0.14 being selected in keeping with the widely used Scalden-Tate correction. Figure 9.13 shows such a relationship, in which graphs of \( \frac{Nu}{Pr^{0.14}} \) versus \( Re \) are drawn at constant, axial distances of 1, 3, 6, 11, 20 and 70 diameters. The lines indicate the interpolation necessary to give a minimum deviation from the mean on the vertical axis; this is not necessarily the 'smoothest' curve through the experimental points. Although most data falls within 2% of the interpolated values, approximately 8% of the points plotted lie between 2% and 5%. These errors are not a simple function of the Reynolds number.

To ensure that the 'scatter' in the above mentioned correlation was not due to the wrongful selection of the \( Pr \) exponent, a graph of \( \frac{Nu}{Pr^{0.14}} \) versus \( Pr \) was plotted (see figure 9.14-), for a distance of 70 diameters, where \( F_2(Re) \) was obtained graphically from figure 9.10. The data plotted applies to tests 1s to 57s, which encompassed the approximate range of Prandtl number 180 to 400. The errors are seen to be independent of \( Pr \). A comparison between the hypothesis "\( Nu \sim Pr^{3} \)" and "\( Nu \sim Pr^{\frac{3}{2}} \)" shows that the exponent \( \frac{3}{2} \) is to be preferred; the maximum deviation from the mean being 6% in the latter case and 5% when \( \frac{3}{2} \) is chosen.

Specific data from tests 1s, 12s and 22s are given in
figure 9.15, which states graphically the measured values of the Nusselt number obtained at particular axial locations. Due to the effect of temperature on the viscosity, the Reynolds number increased with axial distance and the Prandtl number decreased. For this reason, and to a lesser extent because the ratio $M_{wall}$ increased with axial distance, the value of $Nu$ was expected to increase gradually after approximately 40 diameters. Any such increase was found to be immeasurably small, but this would not have been the case had $(Nu/Pr^3)$ been plotted instead of $Nu$. In these experiments the axial temperature gradient in the fluid was very small because high Prandtl numbers were encountered, and consequently the effects just mentioned were minimised.

The general shape of the $Nu$ functions described is such that a rapid diminution occurs with increasing distance, from a high initial value to an almost constant, limiting value. The limiting value is reached at approximately 40 diameters when $Re \approx 3,750$, and this reduces with increasing $Re$, so that the limit occurs at approximately 15 diameters when $Re \approx 9,200$. The way in which the Nusselt number varies, in the entrance region of a heated tube, can be expressed in a more practical form by plotting curves of $\frac{Nu^{5}}{Pr^{3} M_{wall}^{0.14}}$ versus the dimensionless, axial distance, $X = (x/2r_{w})$, for constant values of the Reynolds number. This information is recorded in figure 9.16. The effects of axial variations in the fluid properties have been eliminated from these results.

In the laminar regime the selection of the significant groups of parameters, required for correlating the experimental
results, was backed by a considerable amount of theory. Starting once again with the premise

$$\text{Nu} = \gamma \left( \text{Re}, \text{Pr}, \text{Gr}, X, P, \nu \right),$$

and considering the results obtained with a calming length upstream. The theoretical analyses 11.2.(ii) and (vii) indicated that dissipation could be neglected, its effect on the coefficient of heat transfer being $\ll 1\%$. The theoretical investigations to determine the dependence of $\text{Nu}$ on the variation in viscosity with temperature (11.2.(iii)) have already been mentioned. A complete solution to the energy and momentum equations was obtained which led to

$$\frac{\text{Nu}}{\text{Pr}^{0.3}} = \gamma_1(M_{\text{wall}}), 1.647 \cdot \text{Re}^{0.5} \cdot \left(\frac{x}{x_w}\right)^{-0.3},$$

where $\gamma_1(M_{\text{wall}}) = M_{\text{wall}}^{0.14}$ was found to be a good approximation.

The most significant omission from this solution is the dependence of $\text{Nu}$ on free convective heat transfer. Guided by the result stated, a correlation was attempted of the form

$$\left(\frac{\text{Nu}}{\text{Pr}^{0.3}M_{\text{wall}}^{0.14}}\right) = \gamma_2(\text{Re}, X),$$

where $X = (x/2x_w)$.

This relationship is shown graphically for $X = 1, 3, 6, 11, 20$ and 70 diameters, in the test series 1s to 57s, (see figure 9.17).

A maximum scatter of 50% occurred in the correlation and it was evident that the greatest errors occurred, in general, at 70 diameters, when $\text{Re}$ was low and $\text{Gr}$ was high. Clearly free convection could not be ignored.

Before proceeding with this analysis, a discussion concerning combined forced and free convection is in order, to help establish the important dimensionless groups which can be used to define the problem symbolically.
To give some insight into the process of combined free and forced convection in the entrance region of a horizontal tube, a simplified model of the system is to be proposed. By inspection of the governing equations, significant groups of variables can be obtained for application in the empirical analysis.

The axial velocity in the tube is assumed to be fully developed near to the entrance, and fluid with a high Prandtl number is considered. Free convection arises as a small perturbation in the flow pattern, and the uniform heat flux condition is imposed. Additional simplifications are expressed 'in loco'.

For the coordinate system shown below, the following parameters are defined.

\[ t_w(x, \frac{y}{r_w}) = \text{wall temperature, } t_{\infty} = \text{temperature at inlet}, \]
\[ u, \bar{u} = \text{axial, mean axial velocity, } w = \text{circumferential velocity}, \]
\[ v = \text{velocity towards axis, } \phi = \frac{(t - t_{\infty})}{(gr_w/K)}, \]
\[ U = \frac{(u/4\bar{u})}, V = \frac{(vRePr)/(2\bar{u})}, W = \frac{(wRePr)/(2\bar{u})}, Y = \frac{(y/r_w)}{,} \]
\[ \dot{X} = \frac{(x/2r_w)(RePr)}, \dot{Z} = \frac{(z/r_w)}, \text{Nu}_{FC} = \frac{2h(x, \frac{y}{r_w})}{K} \frac{r_w}{2} \phi(x, 0, \frac{y}{r_w}), \]
\[ \text{Gr}_{FC} = \frac{8\varepsilon^2 \beta(t_w - t_{\infty})}{\nu^2} v \frac{r_w^3}{,} \]
\[ \text{Nu} = \frac{1}{V} \int_{\pi/2}^\pi \text{Nu}_{FC} \, d\dot{Z}, \]
\[ \text{Gr} = \frac{1}{V} \int_{\pi/2}^\pi \text{Gr}_{FC} \, d\dot{Z}, \]

Suffix - 0 - without free convection.

The energy equation can be simplified if circumferential and axial conduction are neglected -

\[ U \frac{\partial \phi}{\partial X} + W \frac{\partial \phi}{\partial Z} + V \frac{\partial \phi}{\partial Y} = \frac{\partial^2 \phi}{\partial Y^2} \quad (9.8) \]

The axial velocity distribution close to the wall can be linearised,
and for a thin temperature layer will remain virtually unaffected
by free convection. Hence \( U \approx Y \).

If radial pressure gradients are considered small and viscous
forces predominate, the circumferential velocity is given approxi-
mately by

\[
\frac{\partial^2 W}{\partial Y^2} = - \frac{(\text{NuGrPr})}{16} \cdot \theta \cdot \cos Z. \quad (9.9)
\]

The radial velocity is assumed to be given by the continuity
equation \( \frac{\partial W}{\partial Z} + \frac{\partial V}{\partial Y} = 0 \), so that

\[
\frac{\partial^3 V}{\partial Y^3} = \frac{(\text{NuGrPr})}{16} \left( \frac{\partial \theta}{\partial Z} \cos Z - \theta \sin Z \right). \quad (9.11)
\]

Proceeding with the analysis as though the forced convection
solution were required, a similarity variable is derived from
inspection of the energy equation and consideration of the
temperature boundary conditions.

\[
\theta = \bar{X} \frac{3}{8} f(x, \eta, z) \text{ where } \eta = \bar{X} \frac{3}{8} Y. \quad (9.12)
\]

Substitution into the energy equation yields

\[
\frac{\eta^2}{6} \frac{\partial f}{\partial \eta} + \frac{\eta^2}{2 \bar{X}} \frac{\partial f}{\partial \eta} - \frac{(\text{NuGrPr})}{16} \bar{X}^{5/3} \left( \frac{\partial f}{\partial \eta} - \frac{\partial^2 f}{\partial \eta^2} \right) = \frac{\partial^2 f}{\partial \eta^2}, \quad (9.13)
\]

where \( \frac{\partial^2 f}{\partial \eta^2} = f \cos Z \), and \( \frac{\partial^3 f}{\partial \eta^3} = \left( \frac{\partial f}{\partial Z} \cos Z - \frac{\partial f}{\partial Z} \sin Z \right) \).

Without attempting to solve the equations for \( f \), it
can be appreciated that the group of parameters \((\text{NuGrPr}) \bar{X}^{5/3}\)
plays a significant role in the convection process. With \( \text{Gr} = 0 \),
the resulting value of \( f \) at the wall can be shown to be

\[
f_0 (\bar{x}, 0, z) = A, \text{ a constant.} \quad \text{The corresponding result for}
\]

the case \( \text{Gr} \neq 0 \) can be stated:

\[
f (\bar{x}, 0, z) = A \eta^{1/2} \left( \text{NuGrPr} \bar{X}^{5/3}, z \right). \quad (9.14-)
\]

The circumferential mean temperature \( \overline{\theta} \) becomes

\[
\overline{\theta} = A \bar{X}^{3/2} \eta^{1/2} \left( \text{NuGrPr} \bar{X}^{5/3} \right) = \overline{\theta}_0 \eta^{1/2}. \quad (9.15)
\]
The Nusselt number is written
\[
\frac{Nu}{Nu_o} = \Pi_3 \left( \frac{Nu}{Nu_o} \right) \frac{Gr.Pr.\bar{X}^{4/3}}{Pr^{2/3} (RePr)^{1/3} (2r_w)^{1/3}} \quad (9.16)
\]

Making the assumption that
\[
\Pi_3 = \bar{\Pi} \left( \frac{Nu}{Nu_o} \right) Gr.Pr.\bar{X}^{4/3} \quad (9.17)
\]

for a limited range of the independent variable, the final expression is determined
\[
\frac{Nu}{Nu_o} = \bar{\Pi} \left( \frac{Gr.Pr.}{RePr} \right)^{2/3} \left( \frac{x}{2r_w} \right)^{1/3} \frac{4m}{1-4} \quad (9.18)
\]

where \( \Pi \) and \( m \) are weak functions of the independent variable, such that \( \Pi \rightarrow 1 \) and \( m \rightarrow 0 \) as \( \left( \frac{Gr.Pr.}{RePr} \right)^{2/3} \left( \frac{x}{2r_w} \right)^{1/3} \rightarrow 0 \)

It will be shown that suitable values are \( \bar{\Pi} = 0.48 \) to 1.0, and \( m = 1/6 \) to 0.

From the argument just stated, a plausible method for correlating the laminar heat transfer coefficients ensued. Plotting the graph of \( \frac{Nu}{Pr^{5/3} M^{0.14}_{wall}} \) versus \( Re \) for particular axial locations (as described previously, Fig: 9.17) a line was drawn through those points corresponding to low Grashof numbers. The Nusselt number given by this function \( (Nu_o) \) was compared with that \( (Nu) \) obtained for all the tests in the range 1s to 57s. A graph of \( (Nu/Nu_o) \) versus \( (Gr.Pr.)^{2/3} / (RePr)^{1/3} \) was drawn and the whole procedure was repeated to improve the initial \( (Nu_o/Pr^{5/3} M^{0.14}_{wall}) \) function.
Figure 9.18 shows that the free convectional contribution to $\text{Nu}$ was small at distances of less than 20 diameters. At 70 diameters, up to 50% increase in $\text{Nu}$ was apparent when $\text{(GrPr)}^{\frac{1}{4}}/(\text{RePr})^{\frac{3}{4}}$ approached unity. When this value was less than 0.6, free convection did not cause $\text{Nu}$ to increase above $\text{Nu}_0$.

At 70 diameters downstream $\text{Nu}$ could be expressed

$$\left( \frac{\text{Nu}}{\text{Nu}_0} \right) = 0.28 + 1.2 \left( \frac{\text{GrPr}}{\text{RePr}} \right)^{\frac{1}{2}} \text{ for } \text{Nu} > \text{Nu}_0.$$  

In keeping with the derivation of the dimensionless groups (above) an alternative equation of the following type is obtained:

$$\left( \frac{\text{Nu}}{\text{Nu}_0} \right) = \frac{1}{\gamma} = 0.48 \left( \frac{\text{GrPr}}{\text{RePr}} \right)^{\frac{1}{2}} \left( \frac{x}{2r_w} \right)^{\frac{3}{2}}$$  

$$\text{ for } \text{Nu} > \text{Nu}_0,$$

and

$$\frac{1}{\gamma} = 1 \text{ for } (\text{GrPr})^{\frac{1}{2}}(x/2r_w)^{\frac{3}{2}}/(\text{RePr})^{\frac{3}{4}} < 2.5.$$  

Where the latter proposal applies to any axial position, and gives the critical condition below which free convection can be ignored altogether. It is worth noting the theoretical result, equation (11.17), gives $\text{Nu}_0 = \text{1.642 (RePr)}x/(x)^{\frac{3}{2}}$ (neglecting dissipation).

Now suppose at large values of $\frac{1}{\gamma}$ (outside the experimental range) the above exponent 0.8 tended to unity, this would lead to the expression $\text{Nu} = 0.623 (\text{GrPr})^{\frac{3}{2}}$ after substituting for $\text{Nu}_0$. There is a remarkable similarity between this and other well known expressions proposed for pure free convection processes. (See for example Ref. K2.)

The improved correlation was attempted -

$$\left( \frac{\text{Nu}}{\gamma \text{Pr}^{\frac{3}{2}} \text{Nu}_0 \text{wall}^{0.14}} \right) \text{ versus } (\text{Re}), \text{ at constant axial distance}.$$  

Figure: 9.19 shows the result for $x = 3, 20$ and 70 diameters. The experimental points fall within 10% of the mean lines, most data being within 5%. The exceptions to this statement were tests 23s and 34s which were 22% and 15% lower than the mean at 3 diameters downstream. No explanation could be found for these low measurements.
but it is probable that some uncertainty attaches to the results obtained with very low heat fluxes, and the lowest heat fluxes used throughout the experiments occurred in these tests, that is 600 J/m².

Figure 9.20 shows the interpolated Nusselt numbers plotted in a more useful form, viz:

\[
\left( \frac{\text{Nu}}{\Pr^{0.14} \text{Nu}_{\text{wall}}} \right) \text{ versus (Axial distance), at constant Re.}
\]

The value of Nu is seen to reduce with increasing axial distance from a high initial value. The magnitude of Nu does not tend towards a limiting value at large axial distances (<70 dia.), as was the case with the turbulent results (Fig. 9.16).

The use of the viscosity correction factor \( \text{Nu}_{\text{wall}}^{0.14} \) is justified empirically in figure 9.21 where the group \( \frac{\text{Nu}}{\frac{1}{Pr} \text{Nu}_{o}} \) is plotted against \( \text{Nu}_{\text{wall}} \). (Here, \( \text{Nu}_{o} \) is the Nusselt number without viscosity variation or free convection.) The data shown relates to an axial distance of 70 diameters.

With the bellmouth entrance fitted, higher Nusselt numbers were obtained in the laminar region, and lower values were obtained in the turbulent region. The analysis of these results proceeded on those lines already described, and a direct comparison was made with the 'calming length' results. Figure 9.22 shows the Nu variation with axial distance for tests 74sB to 85s. With \( \Pr = 91 \), a comparison is made between the two configurations at particular Re values.

In the transitional and turbulent regimes unique graphs of \( \left( \frac{\text{Nu}}{\Pr^{0.14} \text{Nu}_{\text{wall}}} \right) \) versus Re were obtained for particular axial locations. (Figure 9.23). The results included a range of Pr from 170 to 570 (as in tests 58sB to 72sB) and the dependent variable stated was found
to be adequate in describing the Pr dependence of Nu. The shape of these functions differed markedly from those obtained with a 'calming length' upstream, and the difference is clearly indicated when graphs of \( \frac{\text{Nu}}{\text{Pr}^{0.14}} \) versus (axial distance) are compared for both configurations (Fig: 9.24) at particular Re values. Significant reductions in Nu arise when flow development is present in the system. The maximum reduction is approximately 30% at Re = 3,000, but becomes 10% at Re = 10,000.

A comparison between 'calming length' and 'bellmouth' results was carried out for the laminar regime. The analysis of 'bellmouth' data followed the method used for the 'calming length' data. A graph of \( \frac{\text{Nu}}{\text{Pr}^{0.14}} \) versus (Re) for the particular axial positions, 3, 11, 20, and 70 diameters (Fig: 9.15) indicated that unique functions could only be plotted for specific Prandtl numbers. Figure 9.15 shows that in general Nu was greater for the case of developing velocity. This increase was greatest at small axial distances, and the magnitude was minimised at small Re. The values of Nu were highest when low Prandtl numbers were utilised, so that at 3 diameters downstream with Pr = 91, the 'bellmouth' results were 19% to 35% higher. At 20 diameters, the corresponding increases were 7% to 16%. For very high Prandtl numbers, say 500, velocity development caused an increase in Nu for Re = 0 (1,000) but its influence was not so marked as in the case Pr = 91.

Figure 9.26 shows the axial distribution of \( \frac{\text{Nu}}{\text{Pr}^{0.14}} \) for given Reynolds numbers, and with Pr \( \approx \) 91. The 'calming length'
results are given for comparison. In general, the effect of velocity development was to increase \( Nu \) significantly at small axial distances even when the fluid was highly viscous. This phenomenon is discussed theoretically in some detail in reference J.14 where the rate of velocity development is shown to depend on both the initial velocity profile and the variation in viscosity with temperature.

9.6. HEAT TRANSFER IN A CONVERGENCE.

Insofar as the convergence in diameter is qualitatively similar to the bellmouth entrance, it was assumed at the outset that it would be possible to correlate the experimental results in a similar way, i.e. graphs of \( Nu/Pr^3 \) versus (axial distance) were plotted for each test (therefore particular values of \( Re \) and \( Pr \)). The variation of \( Nu/Pr^3 \) with \( Pr \) was expected to be of second order magnitude. However, such a dependence was not precluded from the correlations because it was possible to classify the experimental results into groups having particular values of \( Pr \).

Moderate temperature differences were utilised in these experiments in order to minimise the effects of viscosity variation with temperature and free convection. Any increase in \( Nu \) due to these secondary considerations was neglected as insignificant from experience gained in the short tube experiments.

Figure 9.17 shows the axial distribution of \( Nu/Pr^3 \) downstream of a 3.34:1 contraction with \( Pr \approx 57 \). Comparing these results with figures 9.14 and 9.16, indicates a general agreement with the functions derived for the bellmouth tests. But at the higher Reynolds numbers downstream (\( Re_2 \)), the sudden contraction exhibited appreciably higher values for \( Nu/Pr^3 \) close to the change in section, whereas with the lower values of \( Re_2 \) \((2336)\) marginally smaller values were obtained. (NB. The Reynolds numbers at \( x = 0 \) quoted on the graphs are actually the values at thermocouple locations just upstream and downstream of
of the discontinuity).

Some substantiation of the assumption that the secondary effects due to viscosity variations and free convection can be neglected in these experiments is shown in figure 9.18. A large increase in heat flux was found to cause a comparatively small increase in \( \frac{Nu}{Pr^\frac{3}{2}} \) at distances of the order 1 diameter when turbulent flow prevailed \( (Re_2 = 30,550) \). In the laminar regime \( (Re_2 = 747) \) a small reduction ensued which diminished with increasing distance up to approximately 30 diameters. In the transition region \( (Re_2 = 5,156) \) the value of \( \frac{Nu}{Pr^\frac{3}{2}} \) was initially suppressed at 1 diameter, but increased in the range 2 to 6 diameters, exhibiting some of both the laminar and turbulent tendencies. The results obtained at less than 1 diameter were shown to be sensitive to 'tube-wall conduction' (see part 9.2) so some uncertainty attaches to the reliability of measurements taken in this region, with this in mind it was considered reasonable to suppose that heat-flux dependence was of second order magnitude.

The effects of reducing the heat flux to zero in the section of tube upstream are demonstrated for the 3.34:1 convergence in figure 9.19. The value of \( \frac{Nu}{Pr^\frac{3}{2}} \) was increased at small distances downstream. At 1 diameter the increase was \( 15\% \) with \( Re_2 = 29,325 \) rising to \( 100\% \) with \( Re_2 = 694 \). These differences reduced rapidly in the downstream direction, the corresponding values at 3 diameters being \( 2\% \) and \( 25\% \) respectively. Once again it should be noted that 'tube-wall conduction' could tend to distort the measurements at small axial distances (approximately < 2 diameters), (see parts 9.2. and 5:3. (iii)).

Figures 9.30, 9.31 and 9.32 give comparable results to the \( Pr \approx 57 \) case, but with \( Pr \approx 140, 190 \) and 340. In general, the effect
of increasing Pr was to reduce \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) in the laminar regime \( (\text{Re}_2 < 2,500) \) but no significant effect was observed in the turbulent regime \( (\text{Re}_2 > 10,000) \). With \( \text{Re}_2 \approx 700 \) and at 1 diameter downstream, \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) varied by a factor of three. The effect of Pr on \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) reduced with increasing axial distance. For transitional Reynolds numbers \( (2,500 \text{ to } 10,000) \) a peak occurred in \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) at approximately 1 diameter downstream. This was particularly well defined at high Prandtl numbers, and was probably a function of the heat transfer mechanism in the short, separated-flow region which must occur just downstream of the sharp discontinuity. In separated regions the relationship between Nu and Pr may differ from that in a region where the boundary layer is attached (for example see part 11.3.(vii)). It is also possible that the breakdown of the laminar boundary layer into turbulent flow which occurs close to a discontinuity could be dependent on the viscosity of the fluid and hence (indirectly) on the Prandtl number. It was not considered to be of sufficient significance to warrant a detailed investigation of the underlying mechanisms, but there was no reason to doubt the existence of maximum in the \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) function.

In figures 9.33 to 9.38 data is presented for 2:1 and 1.25:1 convergences, for a similar range of Prandtl numbers. Although there is close qualitative similarity with the results already described, the following differences were noted with reducing diameter ratio.

At all values of Pr, \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) close to the discontinuity was reduced by a factor of \( 1\frac{1}{2} \) to 2, with laminar Reynolds numbers, as the diameter ratio reduced from 3.34:1 to 1.25:1. The most part of this reduction occurred as the ratio was changed from 1:2 to 1:1.25, and the effect was less pronounced at high Prandtl numbers \( (\tilde{\text{Pr}} > 350) \). The reduction in \( (\text{Nu}/\text{Pr}^\frac{3}{2}) \) with increasing diameter ratio was small when \( \text{Re}_2 > 2,500 \) except at distances downstream of the order \( \frac{3}{4} \) diameter when a variation
similar in magnitude to the low $Re_2$ results was observed.

The differences between ($Nu/Pr$) at the various diameter ratios, as discussed above, where not so marked at distances downstream greater than approximately 10 diameters.

9.7. **HEAT TRANSFER IN A DIVERGENCE.**

9.7.(i) The mechanisms of heat transfer and fluid flow through a sudden divergence were found to be extremely complex, particularly at low Reynolds numbers. Extensive testing was required initially with the 1:3.34 diameter ratio, and some of the results obtained have been omitted from the graphical data presented for the sake of clarity. Such omissions have been made only where inexperience led to unnecessary elaboration of phenomena observed throughout testing.

9.7.(ii) **FLOW VISUALIZATION TESTS.**

It became apparent from preliminary experimentation that an investigation of the flow patterns through a divergence would be essential to the understanding of the heat transfer phenomena observed in the main experiments. The results of this investigation will now be presented in the form of a commentary on photographic data with a summary of some important results in graphical form. Brief notes are given on each of the photographs in figures 9.39 and 9.40.

$Re_2 = \text{Reynolds number downstream of step.}$

$X = \text{Distance (diameters) downstream of step.}$

$\Delta T = \text{Temperature water bath - temperature within tube (approximate).}$

All initial tests with water $Pr = 8$ to 9.5

**Figure 9.39. The 1:3 Divergence.**

A) $Re_2 = 199 \, \Delta T = 0$. Thin dye trace travels through centre of tube undisturbed.

B) $Re_2 = 457 \, \Delta T = 0$. Thin dye trace gives poor photograph of flow. Flood upstream leg with dye. Instability or localised turbulence
occurs in flow at $X \approx 6$. Heat transfer in turbulent region will probably be much higher than if flow were undisturbed.

C) $Re_2 = 700 \ \Delta T = 0$ \quad Increasing $Re_2$ causes instability to

D) $Re_2 = 925 \ \Delta T = 0$ \quad move upstream, until at onset of turb-

E) $Re_2 = 1450 \ \Delta T = 0$ \quad ulence in upstream leg flow develops as a

conical, turbulent jet issuing from upstream leg.

F), G), H) $Re_2 = 490 \ \Delta T = 0$. Shows development of flow entering
downstream leg. G) and H) are 10 and 30 seconds after F).

H) indicates diffusion of dye does not affect stability of flow.

I) $Re_2 = 148 \ \Delta T = 0.4 \degree C$ \quad I) through K) show effect of increasing

J) $Re_2 = 148 \ \Delta T = 0.6 \degree C$ \quad heat transfer with $Re_2$ constant, when

K) $Re_2 = 148 \ \Delta T = 0.6 \degree C$ \quad flow is laminar-stable. (N.B. It is

assumed that phenomena with constant

L) $Re_2 = 148 \ \Delta T = 1.0 \degree C$ \quad temperature bath will resemble uniform

heat flux imposed in main experiments.) J) and K) differ only

in type of dye trace used. Cold fluid from upstream drops to

bottom of downstream leg due to bouyancy, possibly causing a small

local peak in heat transfer coefficient versus distance. Rises

up wall of tube. J) shows tendency to recirculate upstream at
top of tube. Distance to 'bottoming' point reduces with increasing
temperature difference.

M) $Re_2 = 333 \ \Delta T = 2.4 \degree C$ \quad Shows effect of free convection (increasing

O) $Re_2 = 333 \ \Delta T = 5 \degree C$ \quad $\Delta T$ at higher value of $Re_2$. Instability

P) $Re_2 = 333 \ \Delta T = 14 \degree C$ \quad is present in flow.

Instability tends to occur at bottom of tube where cold fluid from

upstream leg 'bottoms' downstream, and hence probable location of

peak heat transfer coefficient moves upstream as heat transfer rate

increases. This will be a fairly prominent peak due to local
turbulence.
4) \( \text{Re}_2 = 190 \ \Delta T = 14 \). At a reduced \( \text{Re}_2 \) the free convection causes cold fluid from upstream leg to drop probably causing a small peak in local heat transfer coefficient at \( X = 1 \frac{1}{2} \) to 2. Then downstream, at \( X = 3 \) to 5, the flow becomes unstable which probably leads to a second, higher peak in the heat transfer coefficient versus distance.

**Figure 9.4-c. The 1:2 Divergence.**

A) \( \text{Re}_2 = 379 \ \Delta T = 0 \) Flood upstream leg with dye. Travels undisturbed through downstream leg. Flow stable. Slight fall in core due to buoyancy effect.

B) \( \text{Re}_2 = 703 \ \Delta T = 0 \) Instability starts at \( X = 11 \). This is much further downstream than for same \( \text{Re}_2 \) with 1:3 divergence.

C) \( \text{Re}_2 = 1092 \ \Delta T = 0 \). Shows how instability moves upstream with increasing \( \text{Re}_2 \).

D) \( \text{Re}_2 = 452 \ \Delta T = 4.3^\circ \text{C} \) Heat transfer causes core to fall with distance increasing. E) is 1 minute after D). Effects of recirculation are evident. Instability caused when core from upstream leg touches bottom downstream.

E) \( \text{Re}_2 = 452 \ \Delta T = 4.3^\circ \text{C} \) Distance increasing.

F) \( \text{Re}_2 = 422 \ \Delta T = 18.7^\circ \text{C} \). No severe instability when \( \text{Re}_2 \) is reduced slightly. Cold core entering downstream leg. Photograph taken 40 seconds after dye enters downstream leg. Recirculation evident.

G) \( \text{Re}_2 = 650 \ \Delta T = 18.6^\circ \text{C} \) Including F), shows effect of increasing flow-rate with \( \Delta T \) approximately constant.

H) \( \text{Re}_2 = 904 \ \Delta T = 17.8^\circ \text{C} \) Buoyancy effects suppressed with increasing \( \text{Re}_2 \). Position of local turbulence first at \( X \approx 4.5 \) because core drops quickly to bottom (G). In H) the instability moves downstream with increasing flow as buoyancy effects are suppressed. Further increase in flow rate causes instability to move back.
upstream, probably due to increased rate of shear (I).

Three characteristic flow patterns emerged from the visualization tests, first an undisturbed laminar flow, normally associated with characteristically low Nusselt numbers, second a disturbed laminar flow with localized turbulence which is associated with high Nusselt numbers, and third a fully turbulent flow where high Nusselt numbers are likely.

The position of the turbulent instability was a function of the heat-transfer rate and the Reynolds number. Figures 9.42 and 9.43 show how the position of the instability varied with $Re_2$ for the divergence ratios 1:3 and 1:2, with zero heat transfer. In the case of the 1:3 divergence a series of tests was carried out using a weak mixture of glycol and water, the purpose being to demonstrate that increasing the viscosity of the fluid did not significantly modify the onset of instability in the flow. The glycol mixture had a Prandtl number three times that of water.

As $Re_2$ was increased the instability moved rapidly upstream until a limiting position was reached, corresponding approximately to the onset of turbulence upstream.

It is proposed that a Critical Reynolds Number can be derived which will specify the location of the instability uniquely, for all Reynolds numbers below 2,500 upstream, and valid for all divergence ratios. This Reynolds number is defined by:

$$Re_{crit} = \left( \frac{\bar{u}_1}{D_2/D_1} \right) \cdot x_1$$

(Where the parameters are defined in figure 9.44.)

The actual value of $Re_{crit}$ was found to be approximately 64,000, and experimental corroboration of this is demonstrated in figure 9.44.)
Figures 9.45 and 9.46 give a comparison between the location of the turbulent flow instability during flow visualization, and the location of the maximum value of Nusselt number downstream of the two divergences (from the main heat transfer experiments, discussed in part 9.7(iii)). It is evident that the main effect of free convection, when the heat transfer rate is above zero, is to cause the instability, or peak Nusselt number, to move upstream. The effect was pronounced at the lower Reynolds numbers.

In an attempt to correlate the positions of the peak Nusselt numbers in a useful way, the following argument was derived.

It is assumed that the maximum Nusselt number occurs where the cold core of fluid from the tube upstream drops until it touches the bottom of the tube downstream. The approximate distance downstream can be determined as follows:

Assume core diameter $\approx D_1$, and viscous effects are negligible.

From simple buoyancy considerations

$$x_1 = \frac{\bar{u}_1^2}{\frac{\nu_1^2}{\rho A}} \left( \frac{D_2}{D_1} \right)^{\frac{1}{2}}$$

where

$$\frac{f_2}{f_1} = \frac{f_2}{f_1} = \frac{f_1}{f_1} = \Delta f$$

or

$$\left( \frac{x_1}{D_2} \right)^2 = \left( \frac{D_2}{D_1} \right)^{\frac{1}{2}} \left( \frac{Re_2}{Gr} \right)^{\frac{1}{2}}$$

From the preceding, crude analysis an attempt was made at correlating the distance to the peak Nusselt number for low Reynolds number tests. Figure 9.47 shows that the location of the peak Nusselt number is dependent on the local value of the group $(\alpha x/Re_2)$. Although
the results were inconclusive, the trend was as anticipated. A more
detailed analysis would be required to establish a reliable method of
correlating the different parameters involved. (The detailed discussion
of the experimental results is given in part 9.7.(iii)).

9.7.(iii) HEAT TRANSFER COEFFICIENT DOWNSTREAM OF A SUDDEN DIVERGENCE.

Figures 9.48 and 9.49 show the distribution of \((Nu/Pr^3)\)
in the downstream section of tube for the 1:3.34 divergence with \(Pr \leq 55\).
The results were plotted as described in order to eliminate most of the
effect of Prandtl number, for comparison purposes, and the only justi-
fication for selecting the exponent \(\frac{3}{5}\) was that \(Nu\) was known to be
proportional to \(Pr^{\frac{3}{5}}\) at large distances (from the tests on other configura-
tions). Deviations for particular tests from the values of \(Pr\) quoted
throughout part 9.7.(iii) were of the order 5\% and can be ignored for
all practical purposes. The tests were grouped into several values
of Prandtl number so that any secondary effects of Pr on Nu could be
readily determined. Some uncertainty attaches to the experimental
measurements made at distances of approximately 0.5 diameters or less
downstream because of the 'end effects' already discussed (5.3.(iii)).

The results obtained with turbulent flow upstream and turbu-
ulent or transitional flow downstream (i.e. Reynolds numbers > 2,500)
are presented in figure 9.48. The Nusselt number quickly reaches
a peak at 2.5 diameters, then falls to a constant value at 15 - 25
diameters downstream. The shape of this function is almost the same
at all Reynolds numbers, the peak value being 5 to 8 times the limiting
value.

With lower Reynolds numbers the distribution of \((Nu/Pr^3)\)
becomes more complex, as shown in figure 9.49. Initially, as the
Reynolds number downstream \((Re_2)\) is reduced - so that laminar flow
must occur at very large axial distances - the trend is for the large peak in the function, which is still present, to move downstream. However, it is evident that for a particular Reynolds number the maximum value of \( \frac{\text{Nu}}{\text{Pr}^3} \) can be one of two values, which differ by a factor of \( \sqrt{5} \), and that subsequent values of the function, at distances of the order 40 diameters, differ by a factor of 2 to 3. This phenomenon is explained in part 9.7(ii) as being caused by a localized region of turbulence which may or may not arise at low Reynolds numbers when the laminar flow pattern tends to be unstable.

The axial disposition of the peak Nusselt number appears to be dependent on the heat flux at the very low Reynolds numbers, which is in keeping with the arguments proposed in part 9.7(ii), where the location of the peak was shown to be dependent on \( \left( \frac{\text{Re}}{\text{Gr}^{\frac{1}{3}}} \right) \). To highlight the complexity of the heat transfer mechanisms at low values of \( \text{Re}_2 \), a graph was plotted showing \( \frac{\text{Nu}}{\text{Pr}^3} \) versus (axial distance) with \( \text{Re}_2 \approx 250 \) for five independent tests (figure 9.51).

It is clear that for low Reynolds numbers it is impossible to provide a unique relationship between heat transfer coefficient and distance, but it is probable that \( \text{Nu} \), for a given \( \text{Re}_2 \) and \( \text{Pr} \), can be either characteristically laminar or turbulent, will be a function of the critical length derived from \( \text{Re}_{\text{crit}} \) (see part 9.7(ii)), and will depend on \( \left( \frac{\text{Re}}{\text{Gr}^{\frac{1}{3}}} \right) \). Figures 9.42, 9.45 and 9.47 should be used in conjunction with the basic data for \( \frac{\text{Nu}}{\text{Pr}^3} \), discussed above, when it is necessary to interpolate from these experimental results to heat transfer coefficients at other operating conditions.

Figure 9.50 demonstrates how the Nusselt number downstream is affected by varying the rate of heat transfer in the section of tube upstream. The condition normally imposed, of uniform heat transfer per unit length in each section, is compared with the case of zero heat flux upstream. It was impracticable to conduct two tests with
different boundary conditions at exactly the same Reynolds number, but similar values of $\text{Re}_2$ were possible, and in comparing ($\text{Nu}/\text{Pr}^{\frac{3}{5}}$) an allowance for the small differences in $\text{Re}_2$ could be made by assuming $\text{Nu} \propto \text{Re}^n$. Whence, ($\%$ change in $\text{Nu}$) = $n$ (% change in $\text{Re}$) where $n$ can be estimated (as discussed later in this section). Bearing this in mind, it was shown that for fully turbulent flow ($\text{Re}_2 > 10,000$) ($\text{Nu}/\text{Pr}^{\frac{3}{5}}$) was within $2\%$ with the upstream lea either heated or unheated.

It is to be expected that $\text{Nu}$ will be unaffected by conditions upstream, since the very thin temperature layers close to the wall, which are normally associated with high values of Pr, are unlikely to be physically carried to the region of the wall downstream. It is more likely that the temperature boundary layers will become diffuse shortly after the discontinuity. With low values of Pr this might not be the case, Eds (Ref: H.1.), for example, showed that an increase in $\text{Nu}$ of the order $20\%$ could occur when the heat source was removed from upstream. The fluid utilised was water $\text{Pr} \approx 8$ and the configuration was similar to the present apparatus.

At low values of $\text{Re}_2$ a comparison was more difficult because of instability in the flow and the secondary effects of free convection. The maximum value of ($\text{Nu}/\text{Pr}^{\frac{3}{5}}$) was once again within $2\%$ for the heated and unheated case. When the heat source upstream was removed, there was a tendency for the location of the peak to move downstream slightly, this was particularly pronounced with low Reynolds numbers, but in view of the foregoing arguments it is apparent that slight variations in a number of parameters could easily lead to such phenomena.

Figures 9.52, 9.53 and 9.54 give the axial distribution of ($\text{Nu}/\text{Pr}^{\frac{3}{5}}$) for Prandtl numbers 140, 275 and 500. In general the shape and magnitude of the functions obtained with various values of Pr, when $\text{Re}_2$ was above approximately 2,000, was independent of the Prandtl number. At lower Reynolds numbers the trend was for the peak value of
($\text{Nu}/\text{Pr}^3$) to increase significantly with increasing Pr, although substantial changes were not apparent 40 diameters downstream.

A theoretical analysis of stable laminar motion through a 1:2 divergence (see part 11.3.) indicated that $\text{Nu} \propto \text{Pr}^n$ varied between $1/5$ and $1/3$ depending on axial position. This means the peak value of $(\text{Nu}/\text{Pr}^3)$ should reduce marginally with increasing Pr, but the experimental results were inconclusive in this respect, because the secondary effects of flow instability and free convection tended to distort the $(\text{Nu}/\text{Pr}^3)$ functions. The theory also indicated that close to the discontinuity the value of $(\text{Nu}/\text{Pr}^3)$ might reduce with increasing Reynolds number. Figure 9.49 (with Pr $\approx 55$) shows that $(\text{Nu}/\text{Pr}^3)$ could indeed reduce with increasing $\text{Re}_2$ ($\gtrsim 1,000$).

The figures 9.55 to 9.58 show comparable data on $(\text{Nu}/\text{Pr}^3)$ versus (axial distance), but with the divergence ratio 1:2. The same general observations made for the 1:3.34 divergence were applicable but the following differences were noted. The shape of the functions in the region of the peak value was such that the 1:2 curves exhibited a smaller maximum, whilst the length of this region was approximately the same as the 1:3.34 case - the basis of the comparison being particular values of $\text{Re}_2$ downstream. The ratio $(\text{Nu at peak})/(\text{Nu at 40 diameters})$ was 3 to 6 (compared with 5 to 8) for $\text{Re}_2 \lesssim 2,500$. $(\text{Nu}/\text{Pr}^3)$ at the lower Reynolds numbers was lower than the 1:3.34 case for particular values of $\text{Re}_2$. At high values of $\text{Re}_2$ the location of the peak was $\tilde{\text{2.0}}$ diameters downstream (compared with $\tilde{\text{2.5}}$).

For the 1:1.25 divergence the peak in the $(\text{Nu}/\text{Pr}^3)$ functions at $\text{Re}_2 \tilde{\gtrsim} 2,500$, was sharply defined and occurred at 0.5 diameters downstream; the value of the ratio $(\text{Nu at peak})/(\text{Nu at 40 diameters})$ was 2 to 4. The results are given in figures 9.59 to 9.62. There was a clear distinction between the magnitude of the apparently
'turbulent' and 'laminar' Nusselt numbers, and with lower values of \( \text{Re}_2 \), large variations in \( \text{Re}_2 \) were necessary to cause substantial changes in the magnitude of \( \left( \frac{\text{Nu}}{\text{Pr}^3} \right) \). In this region the functions tended to be almost a constant downstream of approximately 5 diameters.

To demonstrate the effects of diameter ratio, on \( \text{Nu} \) in the region of the discontinuity, over a wide range of ratios, a graph was plotted (Fig: §63 ) of \( \left( \frac{\text{Nu}}{\text{Pr}^3} \right) \) versus (axial distance) for particular Reynolds numbers and \( \text{Pr} \approx 60 \). From the theoretical findings of part 11.3., and from the conclusions of the flow visualization tests, it was decided that the peak value of \( \left( \frac{\text{Nu}}{\text{Pr}^3} \right) \), with either 'localised' or 'full' turbulence present in the downstream leg, was likely to be a function of \( \text{Re}_1 \) (The Reynolds number in the upstream leg). Consideration was given to the results of tests carried out with an extended range of diameter ratios (see part 7.4.(ii)).

\( \left( \frac{\text{Nu}}{\text{Pr}^3} \right) \) is given (Fig: §63 ) for \( \text{Re}_1 = 12,500 \) (± 4%). It is apparent from this graph that \( \text{Nu} \) is substantially a function of \( \text{Re}_1 \) for the first 4 diameters downstream, then largely a function of \( \text{Re}_2 \) downstream of 25 diameters. The range of diameter ratios considered was 1:1.25 to 1:14.4. Figure §64 gives a more detailed expression of how the peak value of \( \left( \frac{\text{Nu}}{\text{Pr}^3} \right) \), and the location of the peak, depends on the divergence ratio, at high Reynolds numbers. With low 'ratios', such that the flow close to the divergence is turbulent upstream and downstream \( (\text{Re}_1 \lesssim 2,500) \), \( \left( \frac{\text{Nu}}{\text{Pr}^3} \right) \) reduces by 10% with increasing divergence ratio. This could be because the distance between the exit of the upstream leg, and tube wall downstream, increases with the divergence ratio. The turbulence energy (or eddying) in the flow will probably decay with the increasing distance, which would cause a reduction in rate of transport of heat from the tube wall downstream.
At higher divergence ratios \((Re_1 < 2,500)\) the Nusselt number suddenly starts to increase with 'ratio', until \((Nu/Pr_{\text{max}}^3})\) is approximately 50\% higher (with the 1:14.4 divergence). The rate of increase in \(Nu\) with 'ratio' reduces as the divergence ratio increases. Although it was not demonstrated experimentally, the peak value of \((Nu/Pr_{\text{max}}^3})\) will eventually reduce with increasing diameter ratio because, as 'ratio' tends to infinity the turbulence energy transported from upstream will probably be damped by viscous action, before reaching the tube wall downstream. Such phenomena could only be observed with 'ratios' much greater than 1:14.4. In the range of 'ratios' considered, however, the effect of reducing the velocity downstream to a sub-turbulent level by increasing the ratio was to increase \((Nu/Pr_{\text{max}}^3})\), the mechanisms being too complex for an elementary explanation to be propounded. The relative rates at which turbulence energy is generated, diffused and damped in the laminar-unstable region downstream could lead to a complex relationship between divergence ratio and Nusselt number.

The axial position at which the peak value of \((Nu/Pr_{\text{max}}^3})\) occurs depends on the divergence ratio as follows. With the low 'ratios' (so that \(Re_2 \gg 2,500)\) the efflux from the upstream leg expanded downstream in a similar fashion to a submerged, round, turbulent jet \(\text{(e.g. Ref. K.1 \text{).}}\) As indicated by the flow visualization tests the initial mixing downstream occurred at a full angle of approximately 30° (see figure 9.39E) which is characteristic of turbulent jets and wakes in general. If it is assumed that the maximum value of \(Nu\) arises where the conical mixing region fills the downstream leg, then the distance to this point must be given approximately by

\[
(x/D_2)_{\text{max}} = (1 - D_1/D_2) \cot \infty
\]

where \(\infty\) = half angle of conical mixing region.
or  \( (x/D_2)_{\text{max}} = 3.73 \frac{(S - 1)}{S} \)

where  \( S = \text{diameter ratio} \).

This formula is shown plotted in figure 9.64 and is denoted "THEORY". Good agreement is demonstrated with experiment.

At the higher ratios the distance to the peak value of \((\text{Nu}/\text{Pr}^3)\) varies only marginally with diameter ratio, the position is nearer to the discontinuity than the above theory would indicate. Once more, the mechanisms of unstable laminar flow are too complex to permit a simple explanation of the phenomena.

Since the maximum value of \((\text{Nu}/\text{Pr}^3)\) was seen to be largely dependent upon the Reynolds number upstream \(\text{Re}_1\), graphs of the peak value \((\text{Nu}/\text{Pr}^3)_{\text{max}}\) versus \(\text{Re}_1\) were plotted for each individual diameter ratio. These are given in figures 9.65 to 9.67, and different values of \(\text{Pr}\) are specified. When \(\text{Re}_1\) exceeds approximately 2,500 unique functions can be defined which represent the experimental results with a maximum deviation of 40%. For two of the configurations these functions could be conveniently expressed; thus:

For 1:3.34 ratio, \( \text{Nu}_{\text{max}} = 0.292 \cdot \text{Re}_1^2 \cdot \text{Pr}^3 \).

For 1:2.00 ratio, \( \text{Nu}_{\text{max}} = 0.255 \cdot \text{Re}_1^2 \cdot \text{Pr}^3 \).

In the case of the 1:1.25 divergence a similar expression was found to be equally reliable when \(\text{Re}_1 > 3,000\), viz:

For 1:1.25 ratio, \( \text{Nu}_{\text{max}} = 0.255 \cdot \text{Re}_1^2 \cdot \text{Pr}^3 \).

However, it is apparent from figure 9.67 that the above equation will consistently overestimate \(\text{Nu}_{\text{max}}\) until \(\text{Re}_1\) exceeds 8,000.

Since the peak value of \((\text{Nu}/\text{Pr}^3)\) appeared to be a function of the Reynolds number upstream to a first approximation, a graph of \((\text{Nu}/\text{Pr}^3)_{\text{max}}\) versus \(\text{Re}_1\) was plotted with the experimental results derived from all three divergences. Figure 9.68 shows that in the range \(\text{Re}_1 \approx 700\) to 35,000 the following equation can be used to
represent the peak Nusselt number for flows which are characteristically turbulent or unstable-laminar:

$$\text{Nu}_{\max} = 0.28 \, \text{Re}^{2/3} \, \text{Pr}^{4/3}$$

The experimental deviations from the above function were as follows. With fully turbulent flow upstream, i.e. $\text{Re}_1 \gtrsim 10^4$, variations of 20% were observed. With lower values of $\text{Re}_1$, 90% of the data was within 50% of the equation given. In a few tests the flow was 'transitional', that is neither characteristically laminar nor turbulent. In these tests deviations from the above expression led to values of $\text{Nu}_{\max}$ which were of the order 40 to 50% of the proposed turbulent function.

When the Reynolds number upstream was in the laminar range ($\text{Re}_1 \lesssim 2,500$) the Nusselt numbers derived for stable-laminar motion were typically a factor of 1/5 to 1/7 of the turbulent, or unstable-laminar, values. The values of $(\text{Nu/Pr}^{4/3})_{\max}$ were scattered within 0.45 and 1.9 times the mean function when the results for all divergence ratios were compared (see figure 9.68). When particular divergence ratios were considered the results were more meaningful, (Figs: 9.65 to 9.67). For the 1:3.34 ratio a transition range of $\text{Re}_1$ was evident between 700 and 2,800, in which the flow was either characteristically laminar or turbulent (as indicated by the magnitude of $\text{Nu}_{\max}$). Below $\text{Re}_1 \approx 700$ the flow was never turbulent in character, and above $\text{Re}_1 \approx 2,800$ the flow was never laminar. For the 1:2.00 and 1:1.25 ratios, the transition region was not so clearly defined, At the lower end of the region it was found that characteristically turbulent flow could not be sustained below values of $\text{Re}_1$ of 1,700 and 1,500 respectively. Hence, the length of the transition region increased with divergence ratio (the upper limit of $\text{Re}_1$ can be considered invariant at approximately 2,500, the onset of normal
turbulence upstream.) The phenomenon described is consistent with the idea that the onset of the flow-instability depends on the shearing action arising from the difference between the velocities of the liquid upstream and downstream of the step. This difference in velocity, and therefore shearing rate, must be a function of the divergence ratio.

The 'laminar' values of \((\text{Nu}/\text{Pr}^3)_{\text{max}}\) for the 1:3.34 divergence are reasonably well defined by a unique function, as shown in figure 4.65 the experimental deviations did not exceed \(\pm 40\%\). With the 1:2.00 and 1:1.25 ratios, the results were scattered and did not correlate well. In view of the previous argument on transitional regions, and considering that free convectional contributions to \(\text{Nu}_{\text{max}}\) have been excluded from the correlation, it is understandable that a unique function representing these results could not be obtained using the methods described.
FIGURE 9.3
AXIAL TEMPERATURE DISTRIBUTION
TEST 42.5
Re = 27.65 to 277.6
Pr = 297 to 298

FIGURE 9.4
AXIAL TEMPERATURE DISTRIBUTION
TEST 45.5
Re = 42.16 to 4312
Pr = 194 to 190
Figure 9.5.

Tube temperature fluctuations with time at 1, 3, 4, 11, 20, 20 diameters downstream, for bellmouth entrance — derived from crude measurements.

Tests 605R to 725R.

\[ P_r = 175 \text{ to } 374. \]

Reynolds number.
(1:25:1, TESTS 5c2 to 11c2 and 17-18c2. — 3:34:1, TESTS 2c3 to 6c3 and 11c3 to 14c3).

\[ P_r = 57 \text{ to } 340. \]

- 1:25:1 CONVERGENCE.
- 3:34:1 CONVERGENCE.
- 3:34:1 CONVERGENCE AT 1 DIAMETER.

**Figure 9.6.** TUBE TEMPERATURE FLUCTUATIONS WITH TIME AT 15, 15.40 DIAMETERS DOWNSTREAM FOR Sudden CONVERGENCE EXPERIMENTS — DERIVED FROM MEASUREMENTS.
FIGURE 9.7

VARIATIONS IN TIME FOR TUBE TEMPERATURE IN TEST 4601.

DIVERGENCE RATIO = 1:334. Re = 497/1663. Pr = 2.71
AXIAL TEMPERATURE PROFILE MEASURED IN TEST 4821.

DIVERGENCE RATIO = 1:3:34, Re = 242/809, Pr = 275.
MEASURED VARIATIONS IN TUBE WALL TEMPERATURE WITH TIME. \( 1:3.34 \) DIVERGENCE. 5 AND 40 DIAMETERS DOWNSTREAM.

(TESTS 1 DI. TO 51 DI.)

FLOW REGIME: HIGHLY UNSTABLE.

---

FIGURE 9.9

- O - MEASURED 5 DIAMETERS DOWNSTREAM.
- X - MEASURED 40 DIAMETERS DOWNSTREAM.

PERCENTAGE VARIATION IN TUBE - FLUID TEMP. DIFF. %.

REYNOLDS NUMBER:

- 20% - 66.8
- 100% - 334
- 1000% - 3340

LARGE AXIAL DISTANCE.
The dimensionless coefficient of heat transfer 70 diameters downstream, with calming length, without viscosity correction.

\[
\frac{N_{ul}}{Pr^{1/3}} \quad \text{against} \quad \text{Reynolds Number}
\]

The function \(f_2(R_e)\) at 70 diameters.

Tests 13 to 573 (for \(Re \geq 2,500\))
Prandtl number 180 to 400.
FIGURE 9.11.

EXTRAPOLATION OF NUSSELT NUMBER TO THE
ZERO HEAT FLUX CONDITION ELIMINATING THE
PROPERTY VARIATION WITH TEMPERATURE.
(CALMING LENGTH FITTED. MEASUREMENTS AT 70 DIA.)

\[
\frac{N_u}{\rho r^{1/2}}
\]

\( \text{(LOCAL WALL TEMP. - BULK TEMP.)} \quad (t_w - t_b) \quad \text{(K)} \)

- \( R_e = 9,274 \)
- \( R_e = 7,567 \)
- \( R_e = 5,822 \)
- \( R_e = 4,471 \)
- \( R_e = 3,791 \)
Figure 9.12

The dependence of Nusselt number on the ratio (viscosity at bulk temp)/(viscosity at wall temp) — tests 1.5 to 5.7.

Axial distance = 70 diam.
Excluding laminar data.

Viscosity ratio $M_{\text{wall}} \equiv \mu_b/\mu_w$.

($N_u = \text{Nusselt number when viscosity is independent of temperature}$)

Axial distance = 1 diam.
Excluding laminar data.
Figure 9.13

The dependence of Nusselt number on Reynolds number for particular axial positions. Calming length fitted. Re = 2,500 to 10,000.

Tests 1 to 5731 (for Re = 2,500).

Points plotted for 1, 3, 5, 7, 10 diameters.

Axial positions 1, 3, 5, 7, 10, 20.

Reynolds number Re = 5,000 to 10,000.
Figure 9.14.

The dependence of Nusselt Number on Prandtl Number.

\[
\frac{N_u}{\left(\frac{M_{\text{ave}}}{f_1(R_e)}\right)} \text{ versus } (Pr) \text{ for tests } 15 \text{ to } 575.
\]

Excluding laminar flow data. Calming length fitted.
Figure 9.15

The variation in Nusselt number with axial position. Calming length fitted tests 16, 123, 226. $Re > 2500$.

- $Re = 9076$ to 9160, $Pr = 134$ to 132.
- $Re = 5784$ to 5814, $Pr = 274$ to 273.
- $Re = 3744$ to 3751, $Pr = 401$ to 402.
Figure 9.16

Nusselt Number in the Entrance Region, Calming Length Fitted, Re > 2,500

Test Series 1% to 5%,
Prandtl Number 180 to 400.

\( \frac{Nu}{Me^{0.12}} \)

- Re = 10,000
- Re = 6,000
- Re = 5,000
- Re = 4,000
- Re = 3,000
- Re = 2,500

Axial Distance (Diameters) 0 5 10 15 20

70 Diameter Value

Thermocouple Locations.
Laminar flow data, test range 15 to 57.

The variation in Nusselt number with Reynolds number, Prandtl numbers from 160 to 650.

Calming length upstream.

Figure 9.17.

\[ \frac{Nu}{(M^{*}P^{2})} \]

**Maximum deviation approx. 50%**

Axial distance \( X = 70 \) diameters.

Broken lines indicate intermediate values. Only mean line given for clarity.
The increase in Nusselt number by free convection as indicated by the group \((\text{Gr} \cdot \text{Pr})^{1/4}/(\text{Re} \cdot \text{Pr})^{1/5}\). Laminar flow data for tests in the range 1.5 to 825, on the short tube. Reynolds number = 100 – 2,000, Prandtl number = 90 – 650.
LAMINAR FLOW DATA. TEST RANGE 18 to 57.

THE VARIATION IN NUSSELT NUMBER WITH
REYNOLDS NUMBER AFTER CORRECTING WITH
THE FREE CONVECTION FUNCTION 2.

PRANDTL NUMBERS FROM 160 to 650.
CALMING LENGTH UPSTREAM.

FIGURE 9.9

$N_u (h_{ref} P^{1/3} x)^{-1}$

$X = 3$

$X = 2.0$

AXIAL DISTANCE $X = 70$ DIAMETERS.
LAMINAR FLOW DATA. TEST RANGE 15 to 57s. VARIATION OF NUSSELT NUMBER WITH AXIAL DISTANCE AFTER CORRECTING WITH FREE CONVECTION FUNCTION φ. SMOOTHED AND INTERPOLATED FUNCTIONS GIVEN. PRANDTL NUMBERS 160 to 650 CALMING LENGTH UPSTREAM.
THE RATIO OF NUSSELT NUMBER WITH VARIABLE VISCOSITY TO NUSSELT NUMBER WITH CONSTANT VISCOSITY (AFTER ELIMINATING THE CONTRIBUTION OF FREE CONVECTION USING THE FUNCTION $\Phi$) — THE DEPENDENCE ON VISCOSITY RATIO $M_{wall}$ FOR LAMINAR FLOW. TEST RANGE 1s to 57s.

Prandtl Numbers 160 to 650.
Reynolds Numbers 100 to 2,300.
Calming Length Entrance.
Axial Distance = 70 diameters.

$\frac{N_u}{(N_u,0,$(\Phi)\$)}$ $= N_u/[(N_u,0,\Phi)]$ $\frac{M_{wall}^{0.14}}{M_{wall}^{0.14}}$

$M_{wall} = \frac{[\text{viscosity at bulk temp.}]}{[\text{viscosity at wall}]}.$
FIGURE 9.22.

NUSSLETT NUMBER DISTRIBUTION IN THE ENTRANCE REGION OF A SHORT TUBE: A COMPARISON BETWEEN DEVELOPED AND UNDEVELOPED FLOW CONDITIONS. TESTS 74.5B to 85.5.

PRANDTL NUMBER APPROX. 0.91.

- ○○ CALMING LENGTH FITTED.
- ○-.. BELLMOUTH ENTRANCE.

At $x = 0$, $Re = 14286$

$N_u$ (values in the curves) vary with $x$ (in terms of diameters downstream from onset of heating).
THE DEPENDENCE OF NUSSELT NUMBER ON REYNOLDS NUMBER FOR PARTICULAR AXIAL POSITIONS, BELLMOUTH PERTURBATION.

Tests 583.8 to 72.8 (with Re > 2500)
Prandtl Number 170 to 400
Axial positions 1, 3, 6, 11, 20, 70 diameters
Points plotted for 1 and 70 diam. only for clarity.
Figure 9.24

Nu:SEL NUMBER IN THE ENTRANCE REGION. BELL-MOUTH ENTRY FITTED. Re > 2,500

Test series 58.5 to 72.5.
Prandtl number 170 to 400.

\[ \left( \frac{Nu}{D_{in}} \right) \]

- \( Re = 10,000 \)
- \( Re = 8,000 \)
- \( Re = 6,000 \)
- \( Re = 5,000 \)
- \( Re = 4,000 \)
- \( Re = 3,000 \)
- \( Re = 2,500 \)

Broken lines with calming length.
LAMINAR FLOW DATA. TEST RANGE 5858-7958.

BELLMOUTH ENTRANCE RESULTS COMPARED WITH FUNCTIONS OBTAINED WITH CALMING LENGTH.
NUSSELT NUMBER VARIATION WITH REYNOLDS NUMBER IS SHOWN, AFTER CORRECTING WITH THE FREE CONVECTION FUNCTION \( \Xi \).

**FIGURE 9.25.**

- \( Pr = 91 \)
- \( Pr = 570 \)
- \( Pr = 130 \) to 370

Lines show equivalent results with calming length fitted at a particular axial distance \( X \) diameters.
Laminar flow data tests 5858 to 7958 with bellmouth entrance. The variation in Nu with distance from inlet.

\[ Pr = 91 \]
\[ \phi = \text{the free convection function.} \]

Full lines show smoothed and interpolated bellmouth results.

Broken lines show calming length results.

Value 70 dia. downstream.

\begin{align*}
\text{Nu} & = \frac{hD}{\kappa} \\
\text{Re} & = \frac{\rho UV D}{\mu} \\
\end{align*}

\text{0} \quad \text{AXIAL} \quad 10 \quad \text{DISTANCE} \quad 15 \quad \text{(DIAMETERS)} \quad 20.
Figure 9.27.

Variation in Nusselt number with distance downstream of a sudden convergence ratio 3.34:1.
Prandtl number = 57. Tests 1c3 to 6c3.

Reynolds number at $(x/D_2) = 0$: 10,380/34,700
6967/23,293
3530/11,800
1624/5431
699/2336
237/791

$(x/D_2)$ 0 10 Axial 20 Distance, 30 (Diameters), 40

Flow $\rightarrow$ $D_2$ $\rightarrow$
**Figure 9.28.**

The effect of doubling the heat flux at the tube on the Nusselt number downstream of a 3:34:1 contraction, Pr = 57.

Tests 19c3 to 21c3.

Reynolds No \((\xi/d_2) = \cdot\)

- 10380/34700
- 9137/30550
- 1624/5431
- 1542/5156
- 237/791
- 223/747

Broken lines show results obtained with half the value of heat flux in tests 1c3 to 6c3.
**Figure 9.29.**

The effect of having an unheated section of tube upstream of a 3:34:1 contraction. The variation in Nusselt number downstream of the discontinuity. Pr ≈ 57.

Tests 22c3 to 24c3.

Reynolds number at \( \frac{x}{D_2} = 0 \):
- 10,380/34700
- 6,967/23293
- 1,624/5431
- 1,421/4745

Broken lines show results obtained with a heated section of tube upstream.

\( \frac{N_u}{Pr^{\frac{1}{3}}} \)

\( \frac{x}{D_2} \) 1/10 axial 2/10 distance 3/10 (diameter) 4/10
VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 3.34:1. PRANDTL NUMBER = 140. TESTS 7C3 to 10C3

REYNOLDS NUMBER AT ($x/D_2$) = 0.
3,530/11,600
2,205/7,372
3,870/12,939

BROKEN LINES SHOW RESULTS OBTAINED WITH Pr = 57.
FIGURE 9.31.

VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 3:34:1. PRANDTL NUMBER \( \approx 190 \). TESTS 15c3 to 18c3.

![Diagram showing variation in Nusselt number with distance downstream of a sudden convergence ratio.](image)

Broken lines show results obtained with \( Pr \approx 57 \).
Figure 9.32. Variation in Nusselt number with distance downstream of a sudden convergence ratio 3.34:1. Prandtl number $\approx 340$. Tests 11c3 to 14c3.

Broken lines show results obtained with $Pr = 57$. 
FIGURE 9.33
VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 2:1
PRANDTL NUMBER = 60. TESTS 1C1 TO 6C1

REYNOLDS NO AT (x/D) = 0
13301 / 26488
7472 / 14879
3745 / 7457
1725 / 3434
740 / 1474
247 / 491

FLOW [D_2]
0 (x/D) 10 AXIAL 20 DISTANCE 30 (DIAMETERS) 40
FIGURE 9.34.
VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 2:1
PRANDTL NUMBERS 150 AND 200. TESTS 7c1 to 14c1.

Pr ≈ 150.
Pr ≈ 200.

REYNOLDS NO AT
(x/D2) = 0.
5002/3.861
4119/6.202
2223/4.428
1593/3.172
742/1.478
625/1.244
287/5.72
218/4.33

FLOW

0 (x/D2) 10 AXIAL 20 DISTANCE 30 (DIAMETERS) 40
FIGURE 9.35
VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 2:1.
PRANDTL NUMBER $\approx 350$. TEST 15C1 to 18C1.

$\frac{N_u}{Pr^{1/3}}$

374.5/7457
2274/4529
1725/3434
1331/2650
247/491
1675/1343
240/477

REYNOLDS NO AT $(x/D_2) = 0$

BROKEN LINES SHOW
RESULTS OBTAINED WITH
Pr $\approx 60$.  

FLOW

$D_2$

$D_1$

$0$ $(x/D_2)$ 10 AXIAL 20 DISTANCE. 30 (DIAMETERS) 40
Figure 9.36

Variation in Nusselt number with distance downstream of a sudden convergence ratio 1:25:1. Prandtl number = 60. Tests 9c2 to 11c2 and 16c2 to 18c2.

Reynolds No. at 
$$(x/D_2) = 0$$

20319 / 26091

12180 / 15192

6133 / 7650

2787 / 3577

1208 / 1507

431 / 537

FLOW

$$(x/D_2)$$ 0 10 AXIAL 20 DISTANCE 30 (DIAMETERS) 40
FIGURE 9.37.
VARIATION IN NUSSELT NUMBER WITH DISTANCE
DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 1.25:1
PRANDTL NUMBERS 150 AND 200. TESTS 1c2 to 4c2 and 12c2 to 15c2

\[ Nu \] vs \( \frac{x}{D_2} \)

- \( Pr \approx 150 \)
- \( Pr \approx 200 \)

REYNOLDS No
AT \( (x/D_2) = 0 \):
8,097/10,099
6786/8464
3669/4576
2,564/3,197
1211/1,510
472/589
988/1,232
354/441

FLOW
\( D_2 \)
VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN CONVERGENCE RATIO 1.25:1.
PRANDTL NUMBER $\approx 34.0$. TESTS 5c2 TO 8c2.

BROKEN LINES SHOW RESULTS OBTAINED WITH Pr $\approx 60$.

REYNOLDS No. AT $(x/D_2) = 0$.

FLOW
Figure 9.40: cont...

G.

Figure 9.41.

Re = 550,
\(\Delta T = 14^\circ C\).
\(\text{Static temp} - \text{bulk temp inside tube} = \Delta T\).

A.

Re = 1370,
\(\Delta T = 14^\circ C\).

B.

Re = 246,
\(\Delta T = 9^\circ C\).

C.
THE POSITION OF THE ONSET OF UNSTABLE FLOW DOWNSTREAM OF A SUDDEN EXPANSION RATIO 1:3.

UTILISING WATER $Pr = 9.2$ AND GLYCOL-WATER MIXTURE $Pr = 24.1$.

$x_1$ = DISTANCE TO THE ONSET OF UNSTABLE FLOW AT THE TUBE WALL. $x_2$ = DISTANCE TO ONSET OF INSTABILITY AT TUBE AXIS.

$X_1 = x_1 / D_2$  
$X_2 = x_2 / D_2$

THE ONSET OF TURBULENCE IN TUBE SECTION UPSTREAM.

FIGURE 9.42:

- WATER $Pr = 9.2$
- PROPYLENE GLYCOL/WATER 25:1% $Pr = 24.1$

$Re_2$ REYNOLDS NUMBER IN DOWNSTREAM TUBE SECTION.
Figure 9.43.

The position of the onset of unstable flow downstream of a sudden expansion ratio 1:2, utilising water Pr = 9.2.

\[ X_1 = \text{Distance to the onset of unstable flow at the tube wall} \]

\[ X_2 = \text{Distance to the onset of instability at tube axis} \]
The position of the onset of unstable flow downstream of a sudden expansion — ratios 1:2 and 1:3 — as a function of Reynolds number upstream. Pr=9.2 (water).

Critical Reynolds number \( Re_{\text{crit}} = \left( \frac{\rho U_1 (D_2 / D_1) x_1}{\mu} \right) \approx 64,000 \).

\[ S = \frac{D_2}{D_1} \]
\[ X_1 = \frac{x_1}{D_2} \]

**Figure 9.44:**

- **○** = divergence ratio 1:3
- **×** = ratio 1:2

Onset of turbulence upstream of step.

\[ X_1 = \frac{64,000 S^2}{Re_1} \]
The axial distance (downstream of a 1:3.34 sudden increase in diameter) to the location of the maximum Nusselt number. Measurements from tests 101 to 5001, Prandtl numbers 55, 140, 275, 500 (approx.) the dependence on Reynolds number is shown and a comparison is made with flow visualisation data.
The axial distance (downstream of a 1:2 sudden increase in diameter) to the location of the maximum Nusselt number. Measurements from tests 1D2 to 22D2. Prandtl numbers 55, 150, 275, 400 (approx). The dependence on Reynolds number is shown and a comparison is made with flow visualisation data.
Distance to peak value of Nusselt number downstream of a sudden divergence. The variation in length due to free convective effects at low Reynolds numbers.

- ○ Data for 1:3.34 divergence when $Re_2 < 250$.
- × 1:2 divergence when $Re_2 < 600$.

**Figure 9.47**

Distance downstream to peak Nusselt N° (diameters)

The value of $(Gr/Re_2^2)$ at location of peak Nusselt N°.
FIGURE 9.48

VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN DIVERGENCE RATIO 1:3.34.
HIGH REYNOLDS NUMBERS.
PRANDTL NUMBER ≈ 55. TESTS 1D1 and 7D1 to 10D1.

FLOW

REYNOLDS NUMBER AT (x/D₁) = 0
35903/10.72 2
25642/7.668
19252/5.759
12827/3.836
6310/1.867

AXIAL DISTANCE (x/D₁)
**Figure 9.49.**

Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:3.34:

Low Reynolds numbers,

Prandtl number \( \sim 5.5 \)

Tests 4D1, 5D1, 12, 14, 17, 26, 28D1

Reynolds number at \( (x/D_2) = 0 \)

\[ \frac{Nu}{Pr^{1/3}} \]

AXIAL DISTANCE \( (x/D_2) \)

FLOW
**Figure 9.50.**

Variation in Nusselt number with distance downstream of a sudden divergence. Ratio 1:3.34. Upstream tube unheated. Prandtl number \( \approx 55 \).

Tests 52D1 to 54D1 and 7D1, 9D1, 20D1.

Reynolds number at \( (x/D_2) = 0 \):
- 19258/5759
- 18640/5578
- 6310/1887
- 6110/1829
- 1613/482
- 1553/465

Broken lines show similar tests with upstream section heated.

Axial distance \( (x/D_2) \) vs. \( N_u / Pr^{1/3} \).
Figure 9.51

Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:3.34.
A comparison between tests with similar Reynolds numbers. Tests 2D1, 17, 28, 41 and 48D1.

Reynolds number and Prandtl number at \((x/D) = 0\):

- 888/266 (4.96)
- 807/241 (5.5)
- 809/242 (27.4)
- 836/250 (5.4)
  - Nominal flux = 185 W/m²
- 832/249 (5.4)
  - Nominal flux = 237 W/m²

Axial distance \((x/D)\)
Figure 9.52.

Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:3.34
Prandtl number ≈ 14.0.
Tests 30D1 to 32D1 and 34D1.

Reynolds number at (x/D) = 0:

11,981 / 3583
7,444 / 2226
3,658 / 1,094
1,837 / 54.9

Axial distance (x/D)

Flow
Figure 9.53.

Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:3.34. Prandtl number ≈ 275.
Tests 44D1, 46D1 and 48D1 to 50D1.

Reynolds number at $(x/D_2) = 0$.

7.078/2.116
3.850/1.151
1.663/4.97
8.09/24.2
4.20/12.6
FIGURE 9.54.

VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN DIVERGENCE RATIO 1:3.34.

PRANDTL NUMBER ≈ 500.

TESTS 35D1, 37D1, 38D1, and 40D1 to 43D1.

REYNOLDS NUMBER AT (x/D) = 0.

AXIAL DISTANCE (x/D)
Figure 9.55.
Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:2. Prandtl number $\approx 57$.
Tests 1d2, 3d2 and 5d2 to 9d2.

$\left[ \frac{N_u}{Pr^{\frac{1}{3}}} \right]$ vs. axial distance $(x/D_2)$

- 28122 / 14122
- 18319 / 9199
- 7845 / 3940
- 4135 / 2077
- 2061 / 1035
- 1048 / 52.6
Figure 9.56

Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:2.
Prandtl number \( \approx 1.44 \).
Tests 10D_2 to 13D_2.

Reynolds number at \((x/D_2) = 0\):
- 10,281 / 5,163
- 4,822 / 2,424
- 1,589 / 798
- 604 / 304

Axial distance \((x/D_2)\)
Figure 9.57.
Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:2.
Prandtl number \( \approx 2.30 \).
Tests 14D2 to 18D2.

Reynolds number at \((x/D_2) = 0\):

- 7,111/3,571
- 3,436/1,725
- 651/327
- 376/129

\[ \frac{Nu}{Pr^{1/3}} \]

Axial distance \((x/D_2)\):
FIGURE 9.58.
VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM
OF A SUDDEN DIVERGENCE RATIO 1:2.
PRANDTL NUMBER ≈ 4:10.
TESTS 19D2 TO 22D2.

\[ \frac{N_u}{Pr^{1/3}} \]

FLOW.

REYNOLDS NUMBER AT \( \frac{x}{D} = 0 \):

- 3117/1565
- 2252/1131
- 1050/527
- 403/202

AXIAL DISTANCE \( \frac{x}{D} \)
Figure 9.59.

Variation in Nusselt Number with distance downstream of a sudden divergence ratio 1:1.25.
Prandtl number ≈ 60.
Tests 10D3, 12D3 to 18D3.

\[
\left( \frac{N_a}{Pr \nu} \right)
\]

Reynolds number at \((x/D) = 0\):
7,565/6,065
3,920/3,143
1,969/1,578
994/797
521/418

Axial distance \((x/D)\)
Figure 9.60

Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:1.25.
Prandtl number ≈ 150.
Tests 1913 to 2213.

Reynolds number at \((x/D_z) = 0\):

- 3,955 / 7,982
- 4,677 / 3,750
- 1,524 / 1,222
- 567 / 454

Axial distance \((x/D_z)\) vs. \(\frac{Nu}{Pr^{1/3}}\)
Figure 9.61
Variation in Nusselt number with distance downstream of a sudden divergence ratio 1:1.25.
Prandtl number \( \approx 2.20 \).
Tests 1D3 to 5D3.

\[
\left( \frac{Nu}{Pr^{1/3}} \right)
\]

- 7,520/602.9
- 3,571/2,863
- 1499/1202
- 655/525
- 388/311

Reynolds number at \( (x/D_z) = 0 \).
**FIGURE 9.62.**

VARIATION IN NUSSELT NUMBER WITH DISTANCE DOWNSTREAM OF A SUDDEN DIVERGENCE RATIO 1:1.25.

PRANDTL NUMBER ≤ 385.

TESTS 6D₃ to 9D₃.

\[
\frac{Nu}{Pr^{\frac{1}{3}}} = \frac{1}{Tr}
\]

FLOW.

REYNOLDS NUMBER AT \( (x/D) = 0 \):
- 3,958 / 3,204
- 2,411 / 1,933
- 1,230 / 986
- 423 / 339

AXIAL DISTANCE \( (x/D) \).
FIGURE 9.63.
Nusselt number downstream of various divergences with Reynolds number upstream approximately $Re_1 = 12.500$ ($\pm 4\%$).
Prandtl number $\approx 60$.
Tests 1DX to 3DX, 8D1, 4D2, and 13D3.

$\frac{Nu}{Pr^{1/2}}$ vs $x/D_2$ for different divergence ratios.
Maximum local value of \( \frac{Nu}{Pr^{1/2}} \) versus diameter ratio of divergence. Reynolds number upstream approximately 12,500. Prandtl number \( \approx 60 \).

Reynolds number downstream.

Divergence ratio.

Onset of turbulence upstream.

Distance \( x/D \) downstream of divergence to position of maximum value of \( Nu/A^{1/3} \).

Reynolds number downstream.

Theory.

Onset of turbulence upstream.
Maximum value of \( \frac{N_u}{Pr^{\frac{1}{3}}} \) versus \( Re_{1 \text{ (upstream)}} \).

Divergence ratio 1:3.34. Tests 1D1 to 50D1.

- \( Pr = 55 \)
- \( Pr = 140 \)
- \( Pr = 275 \)
- \( Pr = 500 \)

\[ y = 0.292 \times Re_{1 \text{ (upstream)}}^{\frac{2}{3}} \]

Figure 9.65.
Maximum value of \( \left( \frac{\text{Nu}}{\text{Pr}^{\frac{1}{3}}} \right) \) versus \( \text{Re}_1 \) (upstream).
Divergence ratio 1:2.0. Tests 1D2 to 22D2.

- \( \text{Pr} \approx 57 \)
- \( \sim 144 \)
- \( \sim 230 \)
- \( \sim 410 \)

\( \text{Re}_1 \) Reynolds number upstream of divergence.

Figure 9.66.
Maximum value of \( \frac{N_v}{Pr^{\frac{5}{3}}} \) versus \( Re_1 \) (upstream).

Divergence ratio 1:1.25, tests 1D3 to 22D3.

- \( Pr \approx 60 \)
- \( Pr \approx 150 \)
- \( Pr \approx 220 \)
- \( Pr \approx 385 \)

Figure 9.67.
MAXIMUM VALUE OF \((\frac{Nu}{R^{3/2}})\) VERSUS \(Re_1\) (UPSTREAM).
DIVERGENCE RATIOS 1:3.34, 1:2.0, 1:1.25.
TESTS 1D1 to 50D1, 1D2 to 22D2, and 1D3 to 22D3.
PRANDTL NUMBERS 55 to 500.

\[
\left(\frac{Nu}{Pr^{1/3}}\right)_{\text{MAX}}
\]

\[
Re_1, \text{ REYNOLDS NUMBER UPSTREAM OF A DIVERGENCE.}
\]
10. COMPARISON OF RESULTS WITH PREVIOUS WORK.

10.1. THE SHORT TUBE.

For the laminar regime, none of the publications discussed in part 3 contained experimental data which could be compared with the results of these investigations. With the fully developed flow arrangement, the theoretical results of Sellars (Ref: C.13) were used for this purpose, and the experimental values of Nu (or Nu') were first corrected for variations in viscosity, and the contribution of free convection as discussed in part 9. The 'smoothed' experimental data was interpolated to suitable values of Re. Figure 10.1 shows this comparison between experiment and theory. The experimental values of Nu' were mostly between 2% and 10% less than the theoretical estimate, but in the first few diameters downstream the experimental data was a maximum of 20% less. The influence of conduction in the tube wall has been demonstrated to have a significant effect on measurements made in the first few diameters, and this could account for the more severe differences between the two cases. It is considered that the agreement is fairly good for the remainder of the data.

With the bellmouth fitted, the experimental results obtained for Pr = 91 can be compared with the theoretical results of Roy (Ref: C.12), who considered the effects of developing flow with Pr = 100. Figure 10.2 shows this comparison, and it is evident that the experimental and theoretical values of Nu' were in good agreement. The disparity between the two cases could be expressed as a maximum of ± 7% deviation in Nu'.

In the transitional and turbulent flow regimes few data were available for comparison. Figure 10.3 shows the results of Hartnett (Ref: B.2), for an electrically heated tube with a
calming length; the fluid was oil. The experimental work discussed in this thesis showed markedly lower heat-transfer coefficients near to the start of heating. No satisfactory explanation of this could be found. The theoretical work described in part 11 (see figure 11.16) and the analysis of Diessler (Ref: C.2) — summarised in figure 10.4 — tends to confirm the findings of this work in preference to those of Hartnett, and indicates that the substantially higher values of $h$ reported by Hartnett for the entrance region are probably misleading.

With undeveloped transitional or turbulent flow at the inlet, no alternative source of $Nu$ for the entrance-region could be found in which the worker had used a viscous liquid. Mills provided some data for Reynolds numbers in excess of 10,000 where the fluid considered was air. Figure 3.6 shows the distribution of $h$ near the inlet of the tube. Comparing the case of $Re = 10,000$ it was evident, from figure 9.24, that the location of the minimum in the 'dip' region was different. The minimum value of $h$ for air occurred much further downstream than with viscous liquid. No reasonable basis for comparison was possible.

* The results of a recent paper by Martin (Ref: J.14), incorporating both experimental and analytical work (on similar short-tube configurations to those discussed herein) compare more favourably with the results of this author than do the previous comparisons made with data from Sellars and Roy (See Figs: 10.1 & 10.2.)
10.2. **THE CONVERGENCE.**

A direct comparison was possible between the experimental coefficients of heat-transfer and those of Ede (Ref. H.1) for the convergence system. The fluid used by Ede was water, and it was to be expected that some differences might occur between the results as a consequence of the higher Prandtl numbers covered in these tests.

A comparison for the 2:1 convergence ratio is shown in figures 10.5 to 10.7. At the higher Reynolds numbers good agreement was evident when \( h/h_\infty \) was plotted versus axial distance (fig. 10.5), this was generally the case for fully turbulent flow downstream \( (Re_2 > 10,000) \). As \( Re_2 \) was reduced to the laminar level \( (Re_2 \approx 2,500) \) considerable disagreement between the two sets of data occurred. Severe differences were apparent when \( Re_2 \) was laminar rather than transitional \( (Re_2 = 2,500 \text{ to } 10,000) \). The discussion in part 9 indicated that the shape of the \( (Nu/Pr^3) \) - distance functions was strongly dependent on \( Pr \) at low Reynolds numbers, for the tests carried out. This was not unexpected since the theoretical analysis in part 11 (which discusses separation and divergence systems) showed that in flows embodying a region of boundary-layer separation, the distribution of temperature in the fluid can vary considerably with changes in \( Pr \).
10.3. THE DIVERGENCE.

The measured distribution of $h$ in the region of a sudden divergence was compared directly with the data presented by Ede (Ref: H.1), for heat-transfer to water. Figures 10.8 to 10.10 show the axial variation in $(h/h_o)$. With fully turbulent flow ($Re_2 > 10,000$) the results for water compared favourably with those for glycol. Figures 10.8 and 10.9 give data for the 1:3:3 and 1:1.25 diameter ratios. It was concluded generally from the analysis of the test results (part 9) that the shape of the $(Nu/Pr^3)$ - distance functions was invariant with $Pr$. This is consistent with the preceding comparison.

Figure 10.10 shows that, for the 1:2 divergence, substantial differences between the results of Ede and the present test data were apparent at low Reynolds numbers (of the order 1,000 downstream). It was extremely difficult to make any reasonable comparison between the values of $h$ obtained from different sources where $Re$ was small. The complex nature of the flow pattern and its dependence on the heat-flux (as discussed in part 9.) make it virtually impossible at this moment in time, to arrive at a unique function which can be used to define $h$. All that can be said is that the shape of both graphs in figure 10.10 are in qualitative agreement.
LAMINAR FULLY DEVELOPED FLOW.
Nusselt number (corrected to constant properties) variation with axial distance. Comparison with Sellars' theory.

Graph showing the variation of Nusselt number with axial distance for different Reynolds numbers (Re = 100, 500, 2000). The lines represent the predictions of Sellars, McIntyre, and Martin.
LAMINAR UNDEVELOPED FLOW AT ENTRY. NUSSELT NUMBER (CORRECTED TO CONSTANT PROPERTIES) VARIATION WITH AXIAL DISTANCE: COMPARISON WITH ROY'S THEORY.

\[ \frac{Nu}{Pr^{1/3}} \]

- McIntyre (Exptl.), \( Pr \approx 91 \)
- Roy (Theory), \( Pr = 100 \)
- Martin, \( Pr = 100 \)

**Figure 10.2.**
Comparison with the results of Hartnett. $(\frac{h}{h_\infty})$ versus axial distance.


- Hartnett
- McIntyre

$Re = 3.340$

Axial distance (diameters).
THE ANALYTICAL RESULTS OF DEISSLER FOR ENTRANCE-
-REGION HEAT TRANSFER. TURBULENT FLOW.

\[ \text{Nud} = \text{FULLY DEVELOPED VALUE OF Nu} \]
2:1 CONVERGENCE COMPARISON WITH RESULTS OF CDE (WATER).

Pr > 60. \( \Re_z = 10,000 \)

\[ \frac{h}{h_\infty} \]

---

CDE
MC Intyre

FIGURE 10.5
2:1 CONVERGENCE COMPARISON WITH
RESULTS OF EDE (WATER).

$\text{Pr} = 200.$

($h_{\infty} = h$ at 4.0 dia)

$R_{c} = 3000.$

$R_{c} = 1000.$
1:3:3 divergence comparison with results of EDE (water).

$Pr = 55$,

$Re_z = 10,000$. ($h_\infty \approx h$ at 40 diam.)
1:1.25 DIVERGENCE. COMPARISON WITH RESULTS OF EDE (WATER).

$Pr = 60.$

$Re_z = 10,000.$ ($h_\infty = h$ at 40 dia.).

---

EDE

McINTYRE.
1:2 DIVERGENCE. COMPARISON WITH RESULTS OF EDE (WATER).

Pr = 57.

Re-z = 1,000 \( (h_\infty = h \text{ at } 40 \text{ diam.}) \)

--- EDE.
--- MCINTYRE.

AXIAL DISTANCE (DIAMETERS)
11. Theory Analyses and Comparison with Experiments.

11.1 Introduction.

The reasons for carrying out the theoretical investigations discussed in part 11 can be summarised as follows: to permit a fuller appreciation of the mechanisms of heat transfer in the experimental systems, to justify the selection of certain significant variables used in correlating experimental data, to support the empirically derived coefficients of heat transfer, to help realise inherent limitations in certain results which are presented for practical use, and to derive simple formulae where possible to explain experimental behaviour.

The analyses have been combined into a single section (part 11) because the different problems considered bear close relationship to one another. In practice however the theoretical and experimental work progressed simultaneously, and reference to the theoretical findings is made throughout the thesis whenever appropriate, (as mentioned in the Preface).

Initially, heat transfer in a short tube is considered, after which some calculations appertaining to a sudden divergence in diameter are described.

11.2 The Short, Straight Tube.

The local coefficient of heat transfer in a short tube may well be dependent on the dissipation of mechanical energy into heat if the heat-transfer fluid is highly viscous. Such effects may be significant even at low Reynolds numbers.

It is characteristic of highly viscous fluids that the viscosity varies considerably with temperature, and the dependence of 'h' on the viscosity variation may be significant.

A detailed investigation of these phenomena is now given.
NON-DISSIPATIVE LAMINAR FLOW WITH CONSTANT VISCOSITY.

It is well known that for a fluid having a high Prandtl number, the rate of increase in thickness (in the flow direction) of a thermal boundary layer is very much less than the rate at which the associated velocity layer thickens. This means that the axial velocity profile near the entrance of a tube develops much more rapidly than the temperature profile (see for example ref: K.5). It can be assumed therefore that fully developed flow conditions prevail at the entrance of a tube for the purposes of this analysis, but that the coefficients of heat transfer calculated provide a reasonably good approximation for any other entry conditions.

Leveque further approximated the form of the axial velocity profile for the case of a short tube at constant temperature (as discussed in part 2.4(ii)), and calculated local Nusselt numbers for high values of Pr. The simplifying assumption used was the the distribution of axial velocity near to the tube-wall was a linear function of radial distance. Morgan (Ref: I.1) shows this to be a good approximation with high Prandtl numbers because the temperature layer is much thinner than the velocity layer.

Sellars (Ref: C.13) modified the result of Leveque for the 'uniform heat-flux' boundary condition. The value of the local Nusselt number was given by -

\[
Nu = 2^{1/3} \cdot 9^{2/3} \cdot \frac{\Gamma(\frac{5}{3})}{3} \cdot \left( \frac{x}{\nu R_e Pr} \right)^{-1/3}
\]

or

\[
Nu = 1.639 \left( \frac{x}{\nu R_e Pr} \right)^{-1/3}
\]

(11.1)

In appendix (B) a similar problem is solved by means of the Sellars technique, which arises elsewhere (11.2(vii)).
11.2(ii) DISSIPATIVE LAMINAR FLOW WITH CONSTANT VISCOSITY.

A solution to the problem of dissipation in laminar high speed gas flows, with heat transfer from a constant temperature plate, can be found in many of the standard text books (e.g. Refs: K.5, K.6). A solution to the apparently similar problem, of forced convection with dissipation in tubes, cannot be obtained using the same sort of mathematical procedure. Kudryaev (Ref: F.1, F.2), investigated a tube at constant temperature, by modifying the solution of Graetz (described in part 2.4(ii)). The result contained the sum of two eigenvalue series, which were slowly convergent at high Prandtl numbers.

The following analysis relates to the 'uniform heat-flux' tube, with a 'high Prandtl number' fluid, the flow is 'fully developed' and 'viscous dissipation' is considered. The approach to the problem embodies the simplifications made by Leveque.

11.2(ii)a.

Consider first the limiting behaviour for large axial distances $x$.

The momentum and energy equations (as in Part 2.2(ii)) can be written as:

\[
\frac{\lambda}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = \frac{dp}{dx}
\]  
\[\text{and} \quad c \left( \frac{\partial u}{\partial x} = \frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial t}{\partial r} \right) + \nu \left( \frac{\partial u}{\partial r} \right)^2 \right)
\]  
\[\text{where} \quad \frac{\partial t}{\partial x} = \frac{dt}{dx} = \frac{dt_b}{dx} \quad \text{as} \quad x \to \infty,
\]

with the boundary conditions:

\[
\frac{\partial u}{\partial r} = 0 = \frac{\partial t}{\partial r} \quad \text{at} \quad r = 0,
\]

and $u = 0$, $t = t_w$ at $r = r_w$. 

The integrations are straightforward.

\[ u = 2u \left(1 - \left(\frac{r}{r_w}\right)^2\right) \]

and

\[ t - t_w = \frac{3}{4} \left(\frac{r}{r_w}\right)^2 - \left(\frac{r}{r_w}\right)^4 \left(\frac{16}{30} - \frac{11}{14} \frac{r}{r_w}\right) \] (11.5)

where

\[ \bar{T} = r_w \text{Re Pr} \frac{dT}{dx} \]

From the definition of bulk temperature

\[ (t_b - t_w) = - \left(\frac{11}{96} \frac{dt}{dx} r_w \text{Re Pr} - \frac{5}{6} \frac{\mu \bar{T}}{K}\right)^2 \] (11.6)

Since

\[ \left(\frac{2 h r_w}{k}\right) = \left(2 r_w \frac{dt}{dx} \left|_{r_w}^{\infty}\right.\right) / (t_w - t_b) \]

the limiting Nusselt number \( \text{Nu}_\infty \) becomes

\[ \text{Nu}_\infty = \frac{48}{11} \left(\frac{dt}{dx} - \frac{\mu \bar{T}}{r_w} \left(\frac{\partial}{\partial r}\right)\right) / \left(\frac{dt}{dx} - \frac{40}{11} \frac{\mu \bar{T}}{r_w} \left(\frac{\partial}{\partial r}\right)\right) \]

which reduces to

\[ \text{Nu}_\infty = \frac{48}{11} \left(1 + \frac{24}{11} \frac{\mu \bar{T}}{r_w} \right)^2 \] (11.7)

Kudryashev pointed out that \( \text{Nu}_\infty = 9.6 \) with the 'constant temperature' constraint. However, it is apparent here that for 'uniform heat-flux, the \( \text{Nu}_\infty \) tends to 48/11 with high heat fluxes, and zero at low heat fluxes. This indicates that tube temperature can easily be underestimated when small heating rates are encountered in practice provided the fluid is highly viscous and the Reynolds number large.

At small negative fluxes (when cooling the fluid) \( \text{Nu}_\infty \) becomes large and negative (or \( \text{Nu}_\infty \rightarrow -\infty \) as \( q \rightarrow -(24/11)(\mu \bar{T}^2/r_w) \)). This implies that heat flows out of the system whilst the wall temperature is greater than the bulk temperature of the fluid. The phenomenon is easily explained by the fact that a point of inflection occurs in the radial temperature distribution, in the region of high shear stress close to the tube wall. At higher cooling rates the effect is insignificant and \( \text{Nu}_\infty = 48/11 \).
11.2(ii)b Consider now the effects of dissipation at small axial distances \( x \). The energy equation (see part 2.2(ii)) is now written

\[
\frac{u}{\partial t} + \left( \frac{V}{\partial x} \right) = \frac{K}{c} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{H}{c} \left( \frac{\partial T}{\partial x} \right)^2
\]

If \( Pr \) is high, and the thickness of the temperature layer is consequently much less than the radius of the tube, a simplification can be made:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \approx \frac{\partial^2 T}{\partial r^2}
\]

With the following substitutions,

\[
X = \frac{x}{\bar{r}_w \ln Pr}, \quad Y = r_w - x, \quad \theta = \frac{(t - t_0)}{Kr_w}, \quad V = \frac{u}{\bar{u}}
\]

where \( t_0 \) = the temperature of the fluid at the start of heating \((x = 0)\), the energy equation for fully developed flow becomes

\[
\frac{V}{2} \frac{\partial^2 \theta}{\partial x^2} + 16 \left( \frac{J^* n}{q_w} \right)^2 \left( \frac{1}{4} \frac{\partial V}{\partial Y} \right)^2 = \frac{\partial \theta}{\partial X}
\]

(11.8)

The velocity is \( V = 4(Y - Y^2) \) for fully developed flow, and from the Leveque approximation, \( Y^2 \ll Y \), hence,

\[
V = 4Y
\]

(11.9)

The final form of the energy equation is therefore

\[
2Y \frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} + \phi
\]

(11.10)

where \( \phi = 16 \left( J^* n \right)^2 \left( q_w \right) \).

Equation 11.10 is to be solved with the boundary conditions

\[
\frac{\partial \theta}{\partial Y} = -1, \quad Y = 0, \text{ all } X, \quad \theta = 0, \quad Y = Y_w (\text{the limit of the temp. layer}), \quad \text{all } X, \quad \theta = 0, \text{ all } Y, \quad X = 0.
\]

The first condition specifies uniform heat-flux, the second implies the existence of a temperature boundary layer, such that
the temperature outside this layer is equal to the inlet temperature, and the third condition states that the temperature at the start of heating \((x = 0)\) is uniform. The latter may not be achieved in practice because dissipative heating could occur upstream of \(x = 0\). The condition imposed however is the only reasonable one.

A similarity solution is sought, putting

\[ f = x^m \cdot \Phi \quad \text{and} \quad g = x^n \cdot \Psi. \]

By transforming equation (11.10), and considering the boundary conditions, the values of \(m\) and \(n\) are chosen to be \((-\frac{1}{3})\).

A solution which satisfies the required boundary conditions is

\[ f(\eta, x) = \xi(\eta) + \delta(x) \cdot \eta(\eta). \quad (11.11) \]

Putting equation (11.11) into (11.10) yields

\[ \frac{2}{3} \eta ( - \eta \eta' - \frac{2}{3} \eta \delta \eta' + 3 \eta \delta' \delta + \eta' + \delta \eta' \delta') = \rho x^{\frac{1}{3}} + \xi'' + \delta \xi'' \quad (11.12) \]

whereupon by separating the variables

\[ \frac{\eta''}{\frac{2}{3} \rho^2 \xi'} - \frac{2}{3} \rho \xi = 0 \quad (11.13) \]

\[ \delta'' + \frac{4}{3} \rho^2 \delta' - \frac{4}{3} \rho \delta' + 1 = 0 \quad (11.14) \]

\[ \delta = \rho x^{\frac{1}{3}} \quad (11.15) \]

The transformed boundary conditions are

\[ \xi(\infty) = 0, \quad \xi'(0) = -1, \]

\[ \eta(\infty) = 0, \quad \eta'(0) = 0. \]

The solution to equations (11.13) and (11.14) were obtained by a finite difference method, and the computer programme used for this purpose is shown in figure 11.2.

The procedure used in the programme was to add the results of two arbitrary, single point boundary-condition solutions in such a way that the final function satisfied the two point boundary-conditions as follows -

Putting \(L\) a linear operator, the differential equations to be solved can be written
\[ L(f) = \gamma(\mathcal{C}) \]

Putting \[ f = \alpha y_1 + \beta y_2 \]

it can then be shown that \( \alpha L(y_1) + \beta L(y_2) = \gamma \).

This means that \( f \) can be found from the solutions of

\[ L(y_1) = 0, \quad L(y_2) = \beta^{-1} \gamma \]

The starting conditions required for \( y_1 \) and \( y_2 \) are arbitrary, whence \( \alpha \) and \( \beta \) are chosen to satisfy the original boundary conditions.

The functions \( g \) and \( \mathcal{C} \) are illustrated in figure 11.3. The wall values were quite insensitive to the selection of \( \gamma(\mathcal{C}) \) (the boundary layer limit) and a suitable value was \( \gamma(\mathcal{C}) = 3.6 \). The dissipation function \( \gamma \) is greatest in the fluid layers close to the heated surface. This provides some justification for the selection of \( \gamma(Y_0) = 0 \) in the first place.

The main purpose of the analysis is to determine the local Nusselt number

\[ \text{Nu} = 2 \left( g(0) \right) \frac{x^\frac{3}{2}}{\mathcal{Q}^\frac{3}{2}} \]  

(11.16)

The bulk temperature is assumed to be similar to \( t_\infty \), which is reasonable at high Prandtl numbers, because the axial temperature gradient along the tube wall is always much greater than the axial bulk-temperature gradient.

The final form of the local Nusselt number is

\[ \text{Nu} = 1.642 \left( \frac{r_w}{\text{RePr}} \right)^\frac{5}{3} \left[ 1 + 11.26 \left( \frac{\text{Nu}}{q r_w} \right) \left( \frac{x}{\text{RePr}} \right)^\frac{3}{2} \right]^{-1} \]  

(11.17)

Equation (11.17) compares favourably with the result of Sellars (Ref: C.13), \( \text{Nu} = 1.639 \left( \frac{r_w}{\text{RePr}} \right)^\frac{5}{3} \), when the heat flux is large.

Putting \[ E = \left( \frac{\mu^2}{q r_w} \right) \]

and \[ \text{Nu}_o = \text{the theoretical value of Nu if dissipation is neglected, the following relationships are obtained.} \]
\[
\frac{N_u}{N_u_0} = \left[ 1 + 18.51 \frac{E^{\frac{1}{3}}}{N_u_0} \right]^{-1} \text{ at small distances } \quad (11.18) \\
\frac{N_u}{N_u_0} = \left[ 1 + 2.182 E^{\frac{1}{3}} \right]^{-1} \text{ at large distances } \quad (11.19)
\]

\(X < 0.0072\)

The effects of dissipation on the local Nusselt number are therefore vanishingly small for short tube lengths. An indication of the magnitudes can be gained from the simple equations (11.18) and (11.19). Part 11.2(vii) contains a discussion of how these findings were applied in the experimental work.

11.2(iii) NON-DISSIPATIVE, LAMINAR FLOW WITH VARIABLE VISCOSITY.

Several attempts have been made at solving the problem of laminar heat transfer with temperature dependent viscosity in circular tubes. Particular consideration has been given to the evaluation of Nusselt numbers at large axial distances \(N_u_0\) (see Ref: D6, D7, D9, D10). Few investigators have been concerned with short tubes. Yang (Ref: D3) used an iterative, integral procedure to solve the equations of momentum and energy, with uniform heat-flux, which yielded a complicated expression for the local Nusselt Number. \(N_u\) could be related to the corresponding value for a fluid with constant viscosity \(N_u_0\) by an expression of the kind

\[
N_u = \left( \frac{H_b}{H_w} \right)^{0.14} N_u_0 \quad (11.20)
\]

where \(N_u_0\) = the local Nusselt number with constant viscosity.

This form of expression was first postulated by Sieder and Tate (Ref: D1) in their empirical analysis (details are given in Part 2.4(ii)) but was based on mean values of Nusselt number \((N_u_m)\) rather than local values \((N_u)\). The equation

(11.20) represents the data of Yang approximately, but \(\frac{N_u}{N_u_0}\) was not a unique function of \(\frac{H_b}{H_w}\).
Rosenberg (Ref: D8) investigated the tube with uniform wall temperature by a finite difference method. The axial velocity profile was undeveloped at the inlet of the tube, and results were obtained at different Prandtl numbers. The few data given showed reasonable agreement with equation (11.20), but the value of \( \left( \frac{\text{Nu}}{\text{Nu}_0} \right) \) was weakly dependent on \( \left( \frac{x/r_w}{(Re.Pr)} \right) \) the dimensionless axial distance. Inertial effects were shown to be insignificant at high Prandtl number (100) and Nu was similar to the "fully developed flow" value.

Test (Ref: D5) found a solution similar to Rosenberg's uniform wall temperature case. Instead of starting with the simplified boundary-layer equations (as in part 2.2.(ii)), the full Navier-Stokes equations were considered. The scope of the results was limited by the increased complexity of the solution, and large spatial differences were necessary in the interests of economy of computer time. Test found that equation (11.20) overestimated Nu considerably during heating.

Shannon (Ref: D10) investigated the uniform heat-flux boundary condition with fully developed flow. Most of the data reported refers to large axial distances, and the equation (11.20) was in good agreement with these. The results pertaining to small axial distances were generally in disagreement with the analysis of Yang and with equation (11.20). Insufficient information was recorded to form any detailed conclusions, but the value of \( \left( \frac{\text{Nu}}{\text{Nu}_0} \right) \) was found once more to be a function of the dimensionless axial distance \( \left( \frac{x/r_w}{(Re.Pr)} \right) \).

The analysis now to be presented was an attempt to fulfil the following -

(a) To prove or otherwise the validity of the expression
\[
\left( \frac{\text{Nu}}{\text{Nu}_0} \right) = \left( \frac{A}{A_w} \right)^{0.14}
\]

(b) To show that \( \text{Nu} = \text{Nu}_0 \cdot f \left( \frac{A}{A_w} \right) \).
(c) To find the dependence of Nu on $(\mu_y/\mu)$ at very small tube lengths.

(d) To determine the relationship in (b) for the inlet region of a tube with fully developed flow.

The generality of the numerical solution is improved by selecting developed flow at the inlet, then assuming that inertia terms arising from the viscosity-temperature dependence are negligible. These conditions are most likely to be well approximated with highly viscous fluids, but the solution is an asymptotic one and strictly only applies to a hypothetical fluid in which Pr tends to infinity.

The boundary-layer equations of momentum and energy (described in part 2.2.(ii) can be simplified for non-dissipative, fully developed laminar flow in a short tube. The following assumptions are made:

1. Radial velocity components are small.
2. The flow is fully developed at $x = 0$, the start of heating.
3. Inertia effects due to viscosity variations are negligible.
4. Radial pressure gradients are negligible.

The equations become (after referring to the notation used in part 11.2.(ii) b)

$$\frac{\partial}{\partial Y} \left[ \frac{(1-Y)}{M} \cdot \frac{\partial V}{\partial Y} \right] = \left[ \frac{dP}{dx} \cdot \frac{r_w^2}{\mu_0 \nu} \right] (1-Y) \tag{11.21}$$

where $\mu_0$ = the viscosity at the start of heating $x = 0$,
and $M = (\mu_y/\mu)$, the viscosity ratio.

$$\frac{V \cdot \partial \Theta}{2} = \frac{1}{(1-Y)} \frac{\partial}{\partial Y} \left[ \frac{(1-Y)}{M} \cdot \frac{\partial \Theta}{\partial Y} \right] \tag{11.22}$$

Integrating equation (11.21) with $(V)/(Y) = 0$ at $Y = 1$,

$$\frac{\partial V}{\partial Y} = \left[ -\frac{1}{2} \frac{dP}{dx} \cdot \frac{r_w^2}{\mu_0 \nu} \right] (1-Y) M \tag{11.23}$$

and differentiating equation (11.22),

$$V \frac{\partial \Theta}{\partial Y} = 2 \left[ \frac{\partial \Theta}{\partial Y} \right] - \frac{1}{(1-Y)} \frac{\partial \Theta}{\partial Y} \right] \tag{11.24}$$

The equation of continuity is required for a solution to be obtained, and this can be stated

$$\int_{0}^{1} V (1-Y) dY = 1 \tag{11.25}$$
To define the problem fully it is necessary to specify how the viscosity ratio \( M \) varies with temperature \( \Theta \). An exponential relationship is proposed: 
\[
\log(M) = (A + B\Theta)
\]
which is reasonably general, and well represents viscosity in practice for a moderate temperature range. Figure 8.4 shows that \( \log(M) \) is almost linear in temperature for propylene glycol-water mixtures over a small interval. For 100% glycol the viscosity can be determined within \( \pm 2\% \) by such a relationship within the temperature range 20 to 40°C.

The resulting viscosity ratio \( M \) is given by 
\[
M = \exp\left(\frac{q^2 \Omega}{\delta^2}\right)
\]
so that the effect of varying the heat flux is the same as varying the viscosity-temperature dependence. A single parameter \( \Omega \) is used in 
\[
M = \exp\left(\frac{\Omega}{\delta}\right)
\]  
(11.26)

The momentum equation (11.23) is non-linear, and a finite difference solution was sought, being the most straightforward approach.

Central differences were used in the \( Y \)-direction, and forward differences in the \( X \)-direction. 'n' is the \( Y \)-wise suffix, and 'm' the \( X \)-wise suffix. Computations were carried out solving \( G \) as a function of \( X \), for particular values of the parameter \( \Omega \).

Defining the new variable \( G \) as
\[
G = \sqrt{\frac{1}{2} \frac{dp}{dx} \frac{x^2}{\delta\varrho}}
\]  
(11.27)

first, the momentum equation is solved for \( G \) at a particular axial position.

\[
G[n, m] = G[n-1, m] + \frac{\Delta Y}{4} (2 - Y[n] + Y[n-1])(M[n, m] + M[n-1, m])
\]  
(11.28)

with the condition \( G[0, m] = 0 \).

The values of \( M \) are derived by reference to equation (11.26) requiring \( \Omega \), which is known because forward differences are used axially in calculating \( G \).

Next, the variable \( G \) is converted into axial velocity with
the continuity equation from
\[ V[n,m] = G[n,m] \left( \sum_{V=0}^{1} G[n,m] \frac{1-Y[n]}{\Delta Y} \right)^{-1} \]  
(11.29)

The energy equation can now be solved giving \( \Theta \) at the next axial increment
\[ \Theta[n,m+1] = \Theta[n,m] + 2 \Delta X \frac{\Theta[n+1,m] + \Theta[n-1,m] - 2 \Theta[n,m]}{V[n,m]} \Delta Y \]
(11.30)
The condition for uniform heat-flux is \( \Theta[0,m] = \Theta[1,m] + \Delta Y \),
and for symmetry at the axis \( \Theta[n_{\text{max}},m] = \Theta[n_{\text{max}}-1,m] \),
with the inlet condition \( \Theta[n,0] = 0 \).

Finally, the local Nusselt number is calculated from
\[ Nu[m] = \frac{2}{\Theta[0,m]} \]  
(11.31)
where the local bulk temperature \( TT \) is given by
\[ TT = 2 \sum_{Y=0}^{1} \Theta[n,m] V[n,m] \frac{1-Y[n]}{\Delta Y} \]
The choice of radial increment affected the accuracy of the calculated wall temperature at small axial distances \( X \). By definition, \( Nu = \infty \) at \( X = 0 \), but a maximum calculable value of \( Nu \) limited the results, the value of which depended on the computation time available and the smallest value of \( X \) required. For an increment \( \Delta Y \) at the tube wall, the maximum possible value of \( Nu \) was \( (2/\Delta Y_{w}) \), and in most of the calculations carried out the maximum value of \( Nu \) was 210.

An acceptable computation time was achieved with 20 radial increments \( \Delta Y \). The size of increment increased linearly with distance from the tube wall, because the temperature changed rapidly in the wall region, but slowly near the axis. Typically, the increments were increased from 0.0095 at the wall to 0.146 at the axis. If smaller axial distances had been investigated, the value 0.0095 would have been too large and the Nusselt numbers underestimated.
A suitable grid spacing was determined by trial.
The axial increments $\Delta X$ expanded linearly with the distance downstream. The first and smallest step was determined by the shortest axial distance for which $Nu$ was required, and a suitable value was found to be $\Delta X = 6 \times 10^{-9}$. The maximum step size was limited by the stability of the solution, and for distances up to $X = 10^{-3}$ stability was ensured if $\Delta X$ was always less than $10^{-6}$. A maximum value of $\Delta X = 7.5 \times 10^{-7}$ was specified at the maximum dimensionless axial distance of $X = 10^{-3}$.

$Nu$ as a function of $X$ was calculated on an ICL 1905 computer for the values of the viscosity-variation parameter $G$ of 0, 10, 20, 50, 100 and 500. The Nusselt numbers calculated were unreliable at dimensionless axial distances $X$ of less than $10^{-6}$, because the radial grid spacing was too large. The calculated Nusselt numbers are compared with Sellars' theory (part 11.2.(i)) putting $G = 0$, (see figure 11.4). A good agreement is indicated. The results obtained with $G > 50$ were found to be sensitive to the radial step size, and these Nusselt numbers are presented merely to show the trend with very considerable viscosity variations. With $G < 50$ a reduction of 20% in the step size was found to have negligible effect on the local Nusselt number when $X \geq 2 \times 10^{-6}$.

The results of the preceding analysis are given in figures 11.4 and 11.9 to 11.11 and the theoretical result of Sellars is superimposed on the graphs for comparison. The calculated Nusselt numbers for slug-flow (uniform velocity) based on Sellars method (see appendix (B)) are plotted for reasons described in a later section. The significance of these results is discussed in 11.2(vii).

11.2.(vii). The preceding finite difference analysis 11.2.(vi) will be unreliable when the temperature layer is thin, as occurs when $X$ tends to zero. The following analytical approach was used to enable
Nu to be determined in the limit as $x \to 0$. The method described
has the advantage of being able to show that $Nu = Nu_0 \frac{\sqrt{\nu}}{\sqrt{\mu_w}}$
in the region considered.

Starting with the energy equation in the form used in the
dissipation analysis, equation (11.8), but neglecting the dissipation
term $\beta$ for the purposes of this analysis -

$$\frac{V}{2} \frac{\partial \theta}{\partial x} = \frac{1}{2} \frac{\partial^2 \theta}{\partial y^2}$$

(11.32)

and restating the momentum equation (11.23) already discussed -

$$\frac{\partial V}{\partial y} = \left( -\frac{1}{2} \frac{d\rho}{dx} \left( \frac{1}{\mu_0} \frac{d\rho}{dy} \right) \right)(1-x^2) \exp \left[ \frac{\alpha_0}{\mu_0} \right] \left[ \xi, \eta \right]$$

(11.23)

These equations are transformed in a similar way to part 11.1(ii)b,

$$f = x^{-1} \frac{1}{\rho}, \quad s = x^{-1} v \quad \text{and} \quad \eta = x^{-1} y.$$

A series solution is attempted which is convergent at small $x$, as
follows -

$$f(\eta, x) = f_0(\eta) + \xi x^\frac{2}{3} f_1(\eta) + \xi x^\frac{3}{2} f_2 ++$$

(11.33)

Substituting equation (11.33) into the energy equation (11.32)

$$\frac{s_s}{\eta^2} \left[ \eta^2 \frac{d}{d\eta} \left( \xi x^{\frac{2}{3}} \right) \frac{d}{d\eta} \left( \xi x^{\frac{2}{3}} \right) \right] \left[ \xi x^{\frac{2}{3}} \eta^2 \right] \left[ \eta^2 \right] = \int_{\eta}^{\eta} \left( \xi x^{\frac{2}{3}} \eta^2 \right) \left[ \xi x^{\frac{2}{3}} \eta^2 \right]$$

(11.34)

The velocity is expressed as the series -

$$S(\eta, x) = S_0(\eta) + \xi x^{\frac{2}{3}} S_1(\eta) ++$$

(11.35)

which can be substituted into the momentum equation (11.23)

$$S'_{0} + \xi x^{\frac{2}{3}} S_{1}' = \left( -\frac{1}{2} \frac{d\rho}{dx} \left( \frac{1}{\mu_0} \frac{d\rho}{dy} \right) \right)(1-x^2) \left[ \xi x^{\frac{2}{3}} \left( f_{0} + \right) \right] \left( \xi x^{\frac{2}{3}} \left( f_{0} + \right) \right)^2 \left( \xi x^{\frac{2}{3}} \left( f_{0} + \right) \right)^2$$

(11.36)

When $x^3$ is small, terms in $x^3$ must be negligible, so the
series (11.32) and (11.35) can be truncated after two terms.

In (11.36) $d\rho/\rho$ is a weak function of $x$. By inspection
of the continuity equation viz -

$$\rho = 2 \int_{\eta}^{\eta} (1-\gamma) \int_{\eta}^{\eta} \left( 1-\gamma \right) M \left( \frac{1}{2} \frac{d\rho}{dx} \left( \frac{1}{\mu_0} \frac{d\rho}{dy} \right) \right) \gamma \eta \eta$$

(11.37)
it can be shown that \[ \frac{-1}{2} \frac{d}{dx} \left( \frac{x^2}{\eta} \right) = 4 \] to the order \( \left( \frac{x^3}{\eta} \right)^2 \),

\( (d\eta/dx) \) can therefore be considered constant in generating the first two terms of the series.

Separating the variables in equation (11.36) yields

\[ S'_o = 4 \quad \text{and} \quad S'_1 = 4 f'_o, \]

after making the approximation \( (1 - x^3 \eta) \approx 1 \), which is the Leveque approximation already discussed.

Integrating with the boundary conditions \( S_o(0) = 0, \quad S_1(0) = 0, \)

\[ S_o = 4 \eta \quad \text{and} \quad S_1 = 4 \int_0^\eta f_o \, d\eta \quad . \quad (11.38) \]

The velocity functions (11.38) can be substituted into the energy equation (11.34), and after separating the variables -

\[ \int_0^\eta \frac{x^2}{\eta} f'_o \, d\eta - 2 \int \frac{x^3}{\eta^2} \, d\eta = 0 \quad \text{with} \quad \int f'_o (\infty) = 0 \]

(11.39)

and

\[ \int_0^\eta \frac{x^2}{\eta} f'_1 \, d\eta = 2 \int \frac{x^3}{\eta^2} \, d\eta \quad \text{with} \quad \int f'_1 (\infty) = 0 \]

(11.40)

Equations (11.39) and (11.40) were solved in a similar way to the dissipation equations of part 11.1.(ii)b. The boundary layer limit was set at \( \gamma(\infty) = 4 \). The function \( f'_o(0) \) should correspond with the dissipation function \( g(0) \) (part 11.2.(ii)b) but a small error is apparent being attributable to the computation error involved in setting \( \gamma(\infty) \) at a slightly different value.

The functions \( f'_o \) and \( f'_1 \) are shown in figure 11.5. The local Nusselt number with viscosity variation at very small axial distances becomes

\[ \text{Nu} = 2 x^{-3} \left[ f'_o(0) + C x^{\frac{3}{2}} \int_0^1 f'_1(\eta) \, d\eta \right]^{-1} \]
The parameter is eliminated straightforwardly, giving

\[ \text{Nu} = 2X^{-3} \left[ f_0(o) + f_1(o) \left( H_{\text{wall}} - 1 \right) \right] \]

where \( X = (x)/(r_w \Re \Pr) \) as usual and \( H_{\text{wall}} = \left( \mu'/\mu \right) \approx \left( \mu'/\mu \right) \).

The final result of the calculations is

\[ \frac{\text{Nu}}{\text{Nu}_o} = \left[ 1 - 0.170 (H_{\text{wall}} - 1) + \right]^{-1} \]

(11.41)

where \( \text{Nu}_o = 1.647 \times 10^{-3} \), the theoretical value of local Nusselt number without viscosity variation.

Equation (11.41) can be considered as the first two terms of a binomial expansion of the function

\[ \left( \frac{\text{Nu}}{\text{Nu}_o} \right) = H_{\text{wall}}^{0.17} \]

(11.42)

Numerical results of the preceding analysis are given in figure 11.2, where a comparison is made with the finite difference solution of part 11.2(iii).

11.2. (iv) NON-DISSIPATIVE TURBULENT FLOW WITH CONSTANT VISCOSITY.

From the literature survey (part 3) it is clear that a considerable amount of research has been done into solving problems of heat transfer with turbulent flow in tubes, theories of turbulence and analytical methods are evolving continually. A detailed study of 'turbulence' is outside the scope of this work, but reference can be made to papers by Wolfshliten (Ref: H.23) and Spalding (Ref: H.22) where some of the most recent advances in this field are discussed.

The following analysis is an attempt to solve the particular problem at hand - that is the investigation of heat transfer with turbulent flow, at high Prandtl numbers, in the entrance region of a tube, at uniform heat-flux - and is based on well established models of turbulent flow.
The hypotheses of Reichardt (Ref: I.10), Deissler (Ref: C.2) and Van Driest (Ref: I.6) have been widely used in analysing internal fluid flows. The second and third of these are simple to apply and have a great deal in common. These will be described in more detail and form the basis of the subsequent analysis.

The difficulty in solving turbulent flow problems is in specifying the 'apparent' transport properties. Prandtl's mixing-length theory states that the turbulent viscosity component is

\[ \nu_t = A^2 y^2 \frac{du}{dy} \]

in boundary-layer flow, where

- \( u \) = velocity parallel to the heated surface,
- \( y \) = distance from the heated surface,
- \( A \) = a constant,

suffix \( t \) = turbulent component.

Both Deissler and Van Driest propose similar formulae, which can be described as follows -

\[ \nu_t = \left( A^2 y^2 \frac{du}{dy} \right) f \]

where \( f \) is a damping factor; this is required because, although turbulence has been shown to be present as \( y \to 0 \) (see Ref: I.4, I.5) the turbulence eddies are damped by viscous action near to the wall.

According to Deissler, \( f = \left( 1 - \exp \left[ - \left( \frac{B^2 y^2}{\nu} \frac{du}{dy} \right) \right] \right) \) (B constant)

where \( \frac{du}{dy} \) is replaced by \( \frac{u}{y} \), and from Van Driest

\[ f = \left( 1 - \exp \left[ - \left( \frac{B^2 y^2}{\nu} \frac{du}{dy} \right)^{1/2} \right] \right)^2 \]

In the limit as \( y \to 0 \), both hypotheses give

\[ \nu_t = C \frac{\nu^2}{\nu'} \left( \frac{du}{dy_o} \right)^2 \cdot y^4 \]

(C constant)

where the index 4 is a consequence of the arbitrary damping factor.
Other workers (for example Harriott, Ref: I.9) and Hubbard, Ref: I.8) have proposed that the index 4 causes $H_t$ to vary too rapidly with $y$, and a value of 3 would be more suitable. Little experimental evidence is available to support either case because velocity cannot be measured easily close to a wall (discussed in Ref: I.4, I.5). The hypotheses discussed above have been used in many of the tube-flow analyses, (Ref: C.2, C.3, C.4, C.10) but there has been comparatively little experimental work done with high Prandtl number fluids where the value of the index described is critical (because thin temperature layers occur when Pr is large.)

Deissler and Sparrow (Ref: C.2, C.4) considered high Prandtl number fluids analytically. The former used an integral procedure depending on an approximate radial temperature profile in the inlet region of the tube. Sparrow found a more satisfactory result to the problem posed by Deissler, by finding an eigenvalue series solution. The 'exact' result of Sparrow made use of Deissler's model of turbulence, but the range of results was somewhat limited. Reynolds numbers above 50,000 were considered with Prandtl numbers less than 100. The very short thermal entrance lengths predicted were of little practical value in the case of Pr = 100, and results at lower Reynolds numbers would have been valuable.

The energy and momentum equations for fully developed turbulent flow with constant viscosity and no dissipation (from Part 2.2.(iv)) can be stated -

$$\frac{C_K}{K} \frac{\partial U}{\partial x} = \frac{\partial}{\partial y} \left( \left[ 1 + \left( \frac{K_e}{K} \right) \right] \frac{\partial T}{\partial y} \right)$$  \hspace{1cm} (11.43)

$$\frac{1}{\mu_t} \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left( \left[ 1 + \left( \frac{\mu_t}{\mu} \right) \right] \frac{\partial U}{\partial y} \right)$$  \hspace{1cm} (11.44)

(subscript $t = $ turbulent component, as usual)
Putting \( M_t \left( \frac{M_t}{M_t} \right) \), \( \sigma' = Pr \),
then \( (K_t/K) = (\sigma' / \sigma_t') \cdot M_t \approx \sigma' \cdot M_t \) (Reynolds analogy)

Introducing the dimensionless parameters -
\[ V = (u/u_1), \quad X = (x/r_w \ Re \sigma'), \quad Y = (y/r_w), \quad \Theta = (t - t_\infty)/(g r_w/k) \]
where \( t_\infty \) is the inlet temperature of the fluid,
equations (11.43) and (11.44) can be written
\[
\frac{V}{2} \frac{\partial \Theta}{\partial X} = \frac{1}{V} \left( \frac{1}{1 + \sigma' M_t} \right) \frac{\partial \Theta}{\partial Y} \quad (11.45)
\]
and
\[
V = \left( \frac{\partial V}{\partial Y} \right)_0 \int_0^Y \frac{dY}{1 + \sigma' M_t} \quad (11.46)
\]
where the approximation \( \frac{\partial \Theta}{\partial X} \approx \frac{\partial \Theta}{\partial X} \cdot r_w \) is made in (11.46) and
suffix \( o \) = wall value.

Now suppose
\[
U = V \left( \frac{\partial V}{\partial Y} \right)_0^{-1}, \quad R = \left[ \frac{Re}{2} \left( \frac{\partial V}{\partial Y} \right)_0^{-1} \right]^{1/2}, \quad Z = X \left( \frac{\partial V}{\partial Y} \right)_0^{-1}
\]

The turbulent viscosity component is chosen to be
\[ M_t = a R^3 Y^3, \quad \text{as} \ Y \to 0, \quad \text{where} \quad a = \text{constant}. \]

This expression permits temperature to be calculated as \( Pr \to \infty \),
but will be inaccurate for the calculation of velocity profiles
except within the temperature layer. The limitation is a consequence
of the boundary conditions \( \Theta = 0 \) and \( U = U_c \) at large values of \( Y \).
Equations (11.45) and (11.46) now become
\[
\frac{U}{2} \frac{\partial \Theta}{\partial Z} = \frac{1}{V} \left( \frac{1}{1 + \sigma' M_t R^3 Y^3} \right) \frac{\partial \Theta}{\partial Y} \quad (11.47)
\]
\[
U = \int \frac{dY}{1 + a R^3 Y^3} = \sum_{n=0}^{\infty} \frac{(-a R^3)^n Y^{3n+1}}{(3n+1)} \quad (11.48)
\]
The series in equation (11.48) converges for \( (a R^3 Y^3) < 1 \) only, but
the product \( U \frac{\partial \Theta}{\partial Z} \) must always converge so this presents no problem.
The equations are transformed by introducing
\[ \tilde{\gamma} = z^{-3/2} \gamma \quad \text{and} \quad f = z^{-3/2} \phi. \]
Hence
\[ z^{-3/2} \frac{U}{2} \left( \frac{\partial \tilde{\gamma}}{\partial z} + \frac{f}{z} - \frac{\tilde{\gamma}}{z} \frac{\partial \phi}{\partial z} \right) = \frac{d}{d\tilde{\gamma}} \left( \left[ 1 + \sigma (z\alpha^2 \gamma)^3 \right] \frac{\partial \tilde{\phi}}{\partial \tilde{\gamma}} \right) \]
(11.49)
and
\[ z^{-3} U = \partial \gamma + \frac{\gamma^4}{4} (z\alpha^2) + (z\alpha^2)^2 \frac{\gamma^7}{7} \]
(11.50)
The temperature variable is expressed as a series which converges at small \( z \),
\[ f(\gamma, z) = f_0(\gamma) + (\sigma^2 \alpha^2 z) f_1(\gamma) + (\sigma^2 \alpha^2 z)^2 f_2(\gamma) + \]
and the first two terms are generated by combining equations (11.49) and (11.50):
\[ \left( \gamma - \frac{\epsilon^3}{4 \sigma} \frac{f_0}{f_1} \right) 
\left( 4 \epsilon f_1 + f_0 - \gamma f_1^0 - 8 \epsilon f_1^1 \right) = 6 \frac{d}{d\tilde{\gamma}} \left( \left[ 1 + \frac{\epsilon f_0^3}{4 \sigma} \right] \left[ f_0^1 + \epsilon f_1^1 \right] \right) \]
where \( \epsilon = (\sigma^2 \alpha^2 z) \) (independent of Prandtl number).

This leads to
\[ \gamma^2 f_0 - \gamma f_0^1 = 6 f_0^2 \quad \text{where} \quad \begin{cases} f_0^1(\infty) = 0 \\ f_0^1(0) = -1 \end{cases} \]
and
\[ \frac{1}{240} \left( \gamma^4 f_0 - \gamma^5 f_0^1 \right) + \frac{1}{6} \left( 4 \gamma f_1 - \gamma^2 f_1^1 \right) = f_1'' + \gamma^3 f_0' + 3 \gamma^2 f_0 \\
\text{where} \begin{cases} f_1(\infty) = 0 \\ f_1(0) = 0 \end{cases} \]
(11.52)

The first term of equation (11.52) is identical to \( \frac{\epsilon^3}{4 \sigma} f_0^2 \), and is therefore small when \( \sigma^2 > 0.25 \).

The local Nusselt number is given by
\[ \text{Nu} = 2 \frac{f_0(0)}{f_0(\infty)} \left( \frac{\epsilon x}{Y_0} \right)^{3/2} \left( 1 + f_1(0) \right) \left[ \frac{a}{2^{3/2}} \left( \frac{x}{r_w} \right)^{3/2} \right]^{-1} \]
(11.53)
The functions \( f_0 \) and \( f_1 \) are shown in figures 11.6, and the computations were carried out as in part 11.2.(ii).

The effect of function \( f_1 \) is to cause a flattening of the radial temperature profile, and this is characteristic of turbulent
flows in general. The change in sign of $f_1$ has no particular significance since at a given axial location the temperature is everywhere less than that based on the $f_0$ term only. The $(f_0)$ term has close similarity to the $(g)$ term in the laminar solution of part 11.1(ii).

In deriving $\text{Nu}$ it is necessary to know $\left(\frac{\partial Y}{\partial Y_0}\right)$. For fully turbulent flows a well known expression can be used (as described in Knudson & Katz Ref. 1.3) which has been derived from measurements of pressure drop through tubes.

$$\left(\frac{\partial Y}{\partial Y_0}\right) = 0.0115 \text{ Re}^{0.8}$$

The local Nusselt number becomes

$$\text{Nu} = 0.1855 \text{ Pr}^{3/2} \text{ Re}^{0.6} (x/2x_w)^{-3} (1 - 0.0645 a (x/2x_w) \text{ Re}^{0.9})^{-1}$$

(11.54)

A limiting Nusselt number is reached at large axial distances when $(\partial t/\partial x)$ becomes constant. Equation 11.54 must be matched to this solution.

The energy equation 11.45 is simplified at large axial distances because $(\partial \theta/\partial x)$ becomes small, and can be set to zero.

$$0 = \frac{\partial}{\partial Y} \left[ (1 + a \sigma Y^3) \frac{\partial \theta}{\partial Y} \right]$$

(11.55)

Integrating with the boundary conditions

$$\theta (\infty) = 0 \quad \text{and} \quad \theta' (0) = -1$$

gives

$$\theta (0) = \pi (3 a^{3/2} \sigma^{3/2} R \sin(\pi/3))^{-1}$$

From the definition of $\left(\frac{\partial Y}{\partial Y_0}\right)_0$, the parameter $R$ can be stated

$$R = 0.07583 \text{ Re}^{0.9}$$

and since $\text{Nu}_b = 2/\theta (0)$ if $t_b \equiv t_\infty$,

the Nusselt number at large axial distances $\text{Nu}_\infty$ is

$$\text{Nu}_\infty = 0.1254 a^{3/2} \text{ Pr}^{3/2} \text{ Re}^{0.9}$$

(11.56)

This equation must be used in conjunction with 11.54 so that $\text{Nu}$ is not underestimated at large axial distances.
A value is required for the constant "a", and this is discussed in part 11.2.(vii).

11.2.(v) TURBULENT FLOW WITH CONSTANT VISCOSITY AND DISSIPATION.

The effect of dissipation on heat transfer in turbulent flow of viscous fluid in tubes has apparently not been investigated theoretically. As in the laminar flow case some work has been done for cases flowing over flat plates (refs: K.5, K.6) which is of little help in the problem under consideration. The following analysis is a simplified theory for assessing the magnitude of the thermal effect caused by dissipation. There is little point in refining such an analysis since the result can be no more accurate than the turbulence model permits. Most of the dissipative contribution to the temperature profile is concentrated in the laminar sub-layer so it is probable that the solution gives a reasonable indication of the orders of magnitude involved.

Once again the limiting case is discussed first, and the energy equation as formulated in part 11.1.(iv) (see equation 11.55).

\[ 0 = \frac{\partial}{\partial Y} \left( \left[ 1 + a \cdot R^3 Y^3 \right] \frac{\partial \phi}{\partial Y} \right) + \left( \frac{\nu}{\nu_w} \right)^2 \left[ 1 + a R^3 Y^3 \right] \left( \frac{\partial \phi}{\partial Y} \right)_o \]  

(11.57)

The symbols have the usual definitions.

Putting \[ E = \left( \frac{\nu}{\nu_w} \right)^2, \quad \frac{\partial \phi}{\partial Y} = \left( \frac{\partial \phi}{\partial Y} \right)_o \]

then integrating 11.57 with \( \phi'(0) = -1 \) and \( \phi(\infty) = 0 \), the result is \[ \phi(\infty) = I_1 + I_2 \]

(11.58)

where \[ I_1 = \int_0^\infty \left[ 1 + a \cdot R^3 Y^3 \right]^{-1} dY, \]

and \[ I_2 = E \left( \frac{\partial \phi}{\partial Y} \right)_o \int_0^\infty \left[ 1 + a \cdot R^3 Y^3 \right]^{-1} \int_0^Y \left[ 1 + a R^3 Y^3 \right]^{-1} dY dY \]

Now \( I_1 = \pi (a R^3 \sigma)^{-\frac{3}{2}} / 3 \sin(\pi/3) \), as in 11.2.(iv).
Integrating
\[ I_2 = E \left( \frac{\partial V}{\partial Y} \right)^2 \int_0^\infty \left[ 1 + \alpha \sigma R^3 Y^3 \right]^{-1} \left( \sum_{n=0}^\infty \frac{(-1)^n \left( a \sigma R Y \right)^{3n+1}}{a \sigma^3 R (3n+1)} \right) dY \]
which becomes
\[ I_2 = E \left( \frac{\partial V}{\partial Y} \right)^2 \frac{\pi (a \sigma R)^{3/2}}{3 \sin (2\pi / 3)} \sum_{n=0}^\infty \frac{\alpha \sigma^n}{(3n+1)} \]  \hspace{1cm} (11.59)
As \( \sigma \) tends to infinity the result becomes
\[ \Theta(\sigma) = 1.210 \left( a R^3 \sigma \right)^{-3/2} \left[ 1 + E \left( \frac{\partial V}{\partial Y} \right)^2 \left( a R^3 \sigma \right)^{-3/2} \right] \]  \hspace{1cm} (11.60)
and substituting for \( \frac{\partial V}{\partial Y} \) as in equation 11.54, with \( \frac{\text{Nu}_\infty}{\Theta(\sigma)} = 2 \)
\[ \text{Nu}_\infty = 0.1254 a^3 \frac{Pr}{Pr^3} Re^{0.9} \left[ 1 + 1.745 \times 10^{-3} a^3 \frac{Re^{0.7}}{Pr^{3/2}} \right] \]  \hspace{1cm} (11.61)
Another way of expressing 11.61 is as follows
\[ \frac{\text{Nu}_\infty}{\text{Nu}_o} = \left[ 1 + 2.188 \times 10^{-4} \frac{E}{\text{Nu}_o} Re^{1.6} \right]^{-1} \]  \hspace{1cm} (11.62)
where \( \text{Nu}_o \) is the Nusselt number at large axial distances without dissipation.

For the entrance region of the tube the energy equation
\[ \frac{U}{2} \frac{\partial \Theta}{\partial z} = \frac{\partial}{\partial Y} \left( \left[ 1 + aR^3 Y^3 \right] \frac{\partial \Theta}{\partial Y} \right) + E \left( \frac{\partial V}{\partial Y} \right)^2 \left[ 1 + aR^3 Y^3 \right]^{-1} \]  \hspace{1cm} (11.63)
where the parameters still have the definitions of part 11.2.(iv).

If a small perturbation solution were sought for small values of \( z \) (dimensionless axial distance), the perturbation parameter would be defined by \( \Delta = (a \sigma R^3)^{3/2} z^{3/2} \) (independent of \( \sigma \)) in
\[ \Theta = z^{3/2} \left[ b_0 \left( \gamma / z^3 \right) + \Delta b_1 + \Delta^2 b_2 \right] \]
It becomes evident after inserting this expression in 11.63 that the turbulent components of viscosity and conductivity are of order \( \Delta^3 \), and this implies that the first order solution will be identical to the laminar solution of part 11.2.(ii)b, but the velocity gradient at the wall will be different. The range of axial distance over which this kind of result is applicable will be smaller than for
the corresponding laminar flow result because the perturbation parameter selected is different.

Omitting all terms of higher orders than $\Delta^1$, the energy equation becomes

$$\frac{U}{2} \frac{\partial \varphi}{\partial z} - \frac{\partial^2 \varphi}{\partial y^2} = E \left( \frac{\partial V}{\partial y} \right)_o^2$$  \hspace{1cm} (11.64)

and the integrated momentum equation (order $\Delta^1$) is

$$U = Y$$  \hspace{1cm} (11.65)

Stating $Z = 4Z$

then

$$2Y \frac{\partial \varphi}{\partial Z} = \frac{\partial^2 \varphi}{\partial y^2} + E \left( \frac{\partial V}{\partial y} \right)_o^2$$

which is solved in part 11.2.(ii), i.e. equation (11.10), for the boundary conditions $\varphi(0,x) = 0$, $\frac{\partial \varphi(0,x)}{\partial y} = -1$, $\varphi(Y,0) = 0$.

Writing the earlier result in terms of $Z^2$

$$\varphi(0,x) = 1.217 + 0.857 \frac{E(\partial V)}{\partial y}_o^2 Z^2$$  \hspace{1cm} (11.66)

The local Nusselt number in the entrance region is therefore

$$Nu = 0.1855 \frac{Pr^3}{Re} \frac{0.6}{\left( x/2r_w \right)^{-3}} \left[ 1 + 8.26 \times 10^{-4} E Re(x/2r_w)^3/Pr^3 \right]$$

or

$$\frac{Nu}{Nu_o} = \left[ 1 + 1.532 \times 10^{-4} E \frac{Re}{Nu_o}^{1.6} \right]^{-1}$$

where the usual substitution is made for $\left( \frac{\partial \varphi}{\partial y} \right)_o$, and $Nu_o$ is the local value of $Nu$ when there is no dissipation.

The equation (11.67) may be compared with the large $X$ solution, equation (11.62). The coefficient $1.532 \times 10^{-4}$ in (11.67) is close to the value $2.188 \times 10^{-4}$ in (11.62). This indicates that in an equation applicable to all regions, the coefficient would have to be a weak function of axial distance. A constant value would be a reasonable approximation.

Further discussion of the above analysis is included in part 11.2.(vii).
11.2.(vi) TURBULENT FLOW WITH VARIABLE VISCOSITY, AND NO DISSIPATION

The main theoretical work - on variable viscosity with turbulence at high Prandtl number, in tube-flow - has been carried out by Deissler (Ref: 0.2). The relationship between Nusselt number at large axial distances and viscosity variation was accounted for by using a weighted reference temperature for determining the viscosity. If the results are expressed in the exponential form used for the laminar flow cases discussed previously i.e.

\[ Nu = Nu \text{(constant viscosity)} \cdot M^n_{wall} \]

then the value of index 'n' increases with Prandtl number. For Pr increasing from 1 to 100, n increases from 0.2 to 0.4. The theory of Deissler was not justified experimentally with high Prandtl number fluids.

Sieder and Tate proposed a value of n of 0.14 (as discussed in part 2.3.(iii)) but this was based on mean Nusselt numbers and a crude experimental technique. Kalina (as in part 3.2.(ii)) carried out local measurements with water and oil in long tubes. It was concluded that a satisfactory value of n would be 0.05 for Prandtl numbers up to 75.

Clearly, the effects of viscosity variation must depend strongly on the model of turbulence proposed in a theoretical analysis. Since there is disagreement on the magnitude of these effects (on the Nusselt number) it would be outside the scope of this work to attempt to provide an analytical solution. However, it may be said that for very small axial distances Nu depends on \( l_{wall} \) in the same way as in the laminar flow solution. This can be concluded after attempting a perturbation solution of the energy and momentum equations for small axial distances in a similar way to the dissipation solution of part 11.2.(v). It is implied in such an analysis that the only
significant variations in viscosity occur in the laminar sublayers, and that for small axial distances the value of exponent \( n \) would correspond to the laminar result. Such values were shown in part 11.2.(iii) to be in the range 0.14 to 0.17.

11.2.(vii) DISCUSSION ON THEORETICAL RESULTS AND COMPARISON WITH EXPERIMENT.

The value of the dissipation parameter \( E = \left( \frac{z}{\nu} \right) \frac{u^2}{\rho r_w} \) was always small in the laminar flow experiments with the short straight tube. The range of \( E \) was \( 3.5 \times 10^{-5} \) to \( 2.86 \times 10^{-2} \), and the maximum effect was observed at \( Re = 1,690, Pr = 625 \). The theoretical reduction in \( Nu \) due to dissipation must be maximum at the greatest axial distance (70 diameters) and this was calculated to be 1.7%. Had the tube been of infinite length the result would have been 6.2%. At all other test conditions the reduction was always less than 1%.

It was not possible to justify the theory experimentally because of practical limitations. To increase the value of \( E \) suitably it would be necessary to reduce the tube to fluid temperature difference or the cross sectional area of the tube by a factor of approximately 100.

Figure 11.7 shows the theoretical effects of dissipation for constant values of parameter \( E \) at large and small values of the \( (Re \times Pr) \) product. Dissipation is more pronounced at low values of \( (Re \times Pr) \), and the greatest reduction in \( Nu \) occurs at large axial distances.

In the turbulent regime the maximum value of \( E \) was \( 2.6 \times 10^{-3} \), and this occurred at \( Re = 7,750, Pr = 186 \). Dissipation was shown theoretically, to cause a reduction in \( Nu \) of 0.5% at large axial distances. It was impracticable to attempt an empirical study of dissipation during turbulence as explained above in the discussion of
laminar flows.

A map of the theoretical results on dissipation is given in figure 11.8. Nusselt numbers greater than 8.48 are considered and both the laminar and turbulent regimes are shown.

The theoretical work on the variable viscosity dependence of Nusselt number was confined to the laminar regime, but it was argued in 11.2.(vi) that the results could be applied to turbulent flow for very short tube lengths. Figure 11.9 records the numerical data of the finite difference solution: values of Nusselt number \( \approx \text{Nu} \), the ratio \( (\text{Nusselt number})/(\text{constant viscosity Nusselt number}) \) \( (\approx \text{Nu}/\text{Nu}_0) \), the ratio \( (\text{viscosity at local bulk temperature})/(\text{viscosity at local wall temperature}) \) \( (\approx M_{\text{wall}}) \). The group \( (\text{Re}.\text{Pr}) \) appears in the dimensionless axial distance term \( (\approx X) \), and data is shown for specific values of the viscosity variation parameter \( (\approx \zeta) \) of part 11.2.(iii).

The functions \( \text{Nu} = f(X) \) are plotted in figure 11.4. The effect of increasing \( \zeta \) is seen to be an increase in \( \text{Nu} \), and this is pronounced at large value of \( X \). When \( \zeta \) is very large, say 500, there is a limit to the extent to which \( \text{Nu} \) can be augmented. An inspection of the velocity and temperature profiles (Fig: 11.10) shows that velocity is parabolic in radius when \( \zeta = 0 \), but tends towards a uniform distribution as \( \zeta \) becomes large. The temperature profiles corresponding to the two extreme cases give rise to an upper and lower bound on \( \text{Nu} \), the former are illustrated in figure 11.4.

The way in which \( \text{Nu} \) varies with axial distance is demonstrated in figure 11.11 for \( (\text{Re}.\text{Pr}) = 10^5 \) and \( (\text{Re}.\text{Pr}) = 10^6 \). For a particular value of \( \zeta \), and at a given location, the ratio \( (\text{Nu}/\text{Nu}_{\zeta=0}) \) decreases as \( (\text{Re}.\text{Pr}) \) increases. It is only at considerable axial
distances that \( Nu \) depends significantly on the parameter \( \zeta \).

Therefore, in tests designed to investigate viscosity variations, large axial distances must be used (when uniform flux is imposed). It was found (see part 3.5.) that under such circumstances temperature stratification could occur in the fluid due to free convection, and some difficulty was encountered in carrying out the tests.

An attempt at correlating \( (Nu/Nu_0) \) with \( k_{wall} \) is shown in figure 11.12; the enormous viscosity variations when \( \zeta = 500 \) are omitted. \( (Nu/Nu_0) \) is evidently a very weak function of \( X \), and nearly all the data fall between \( (Nu/Nu_0) = k_{wall}^{0.14} \) and \( (Nu/Nu_0) = k_{wall}^{0.15} \). The exponent 0.14 is identical to that proposed by Sieder and Tate (def: D.1) for estimating mean values of Nusselt numbers, \( Nu \). A comparison made between \( k_{wall}^{0.14} \) and calculated values of \( (Nu/Nu_0) \) - figure 11.13 - indicated a maximum difference of approximately 2\% at \( X \approx 10^{-4} \). If the exponent 0.15 had been considered, the corresponding difference would have been 0.1\%, but this would be combined with a loss of accuracy at larger values of \( X \). For moderate variations in viscosity (say \( \zeta < 20 \)) the errors incurred in using the approximation \( k_{wall}^{0.14} \) were much less than 1\%.

When \( \zeta = 500 \), \( (Nu/Nu_0) \) was found to be a function of axial distance \( X \). In this region the numerical results became suspect (see part 11.1.(iii)) and merely indicated the trend for very large viscosity variations. Figure 11.14 shows how \( Nu \) became asymptotic to the case of uniform velocity heat transfer:

\[
Nu = 1.252 X^{\frac{1}{3}}.
\]

(see appendix B).

An alternative expression used is stated

\[
(Nu/Nu_0) = 0.895 \left( \frac{\zeta}{\log k_{wall}} \right)^{\frac{2}{3}}.
\]

It is improbable that, in practice, \( k_{wall} \) would be great enough to approach the asymptotic case described above.
The analytical solution obtained for small values of \( X \)
indicated that \((\text{Nu}/\text{Nu}_o)_X \to 0 = \text{Nu}_{\text{wall}}^{0.17}\). This was compatible with the
finite difference results which showed the exponent to increase from
0.14 to 0.15 with decreasing \( X \). In the analysis it was considered
unnecessary to generate more than the linear perturbation terms in
the solution, because the main objectives were adequately demonstrated
as follows. The dependence of Nu on \( M_{\text{wall}} \) was substantially the same
for large and small values of axial distance, and the assumption
\((\text{Nu}/\text{Nu}_o) = f(M_{\text{wall}})\) was valid in the entrance region.

Both the findings of Yang (Ref: D.3) and Shannon (Ref: D.10)
are at variance with the above theoretical results. The data of
Yang were interpreted and compared graphically in figure 11.12; the
data of Shannon could not be extracted in the same form, but it
suffices to say that at small values of \( X \) the exponent of \( M_{\text{wall}} \)
tended to 0.3 approximately.

The experimental dependence of Nu on \( M_{\text{wall}} \) could not be
derived readily. This was due to the effects of free convection.
In part 9.5. of the thesis the empirical findings are discussed.

In analysing heat transfer with turbulent flow in the
entrance region, two expressions were derived which enabled Nu to be
estimated at small and large values of axial distance. The wall-
gradient \( \frac{\partial \text{Nu}}{\partial y} \) and constant "a" appear in both results. The former
can be evaluated from the well known empirical relationship
\( \frac{\partial \text{Nu}}{\partial y_o} = 0.0115 \text{Re}^{0.8} \) proposed in part 11.1.(iv), but the reliability of this
expression comes into question at the lower values of \( \text{Re} \) in the
turbulent regime. Such Reynolds numbers (say \( 10^4 \)) were encountered
in the experiments, so a new expression was sought based on measure-
ments of pressure-drop in tubes. Figure 11.15 shows the function $\frac{\partial V}{\partial Y_0}$ versus $\text{Re}$ according to Keys (Ref: K.20). For the range $4,000 < \text{Re} < 10,000$, the following formula was determined

$$\frac{\partial V}{\partial Y_0} = 3.18 \times 10^{-4} \text{ Re}^{1.2}$$

Whereas, the existing formulae suffice for $\text{Re} > 10,000$ (i.e. Eq. 11.54 and 11.56) their counterparts, incorporating the new $\frac{\partial V}{\partial Y_0}$ permit a comparison between theory and experiment.

The revised formulae are:

at large $X$, $\text{Nu}_w = 0.02087 a^3 \text{ Re}^{1.1} \text{ Pr}^{3}$

at small $X$, $\text{Nu} = \frac{0.05625 \text{ Re}^{0.733} (x/2r_w)^{-3} \text{ Pr}^{3}}{1 - 0.01077 a (x/2r_w) \text{ Re}^{1.1}}$ (11.69)

A suitable value for "a" was derived by comparing equation 11.68 with experiment. Inserting $a = 5.22 \times 10^{-4}$ into equations 11.68 and 11.69, results in:

at large $X$, $\text{Nu}_w = 1.68 \times 10^{-3} \text{ Re}^{1.1} \text{ Pr}^{3}$

at small $X$, $\text{Nu} = \frac{0.0562 \text{ Re}^{0.733} (x/2r_w)^{-3} \text{ Pr}^{3}}{1 - 5.62 \times 10^{-6} (x/2r_w) \text{ Re}^{1.1}}$ (11.71)

In figure 11.16 a comparison between experiment and theory is shown for a small distance (1 diameter) and a large distance (70 diameters). Reasonable agreement is demonstrated. The experimental Nusselt numbers used were corrected for viscosity variation and the graph shown represents the best fit through the data.

The most important conclusion which can be drawn from the theory is that $\text{Nu}$ can be represented by a relationship of the kind

$$\text{Nu} = \text{Pr}^{3/4} f(\text{Re}, X)$$

at small and large values of axial distance. This simplifies the analysis of experimental data, as a comparison with initial proposal
\( \text{Nu} = f (\text{Pr}, \text{Re}, X) \) indicates. Prandtl number is eliminated as an independent variable.

In addition, the asymptotic behaviour of Nu with distance is shown. This helps to justify the experimental findings near the start of heating where heat conduction in the tube wall becomes significant, and may be used in the absence of experimental data for distances of less than one diameter.

The asymptotic behaviour is demonstrated in figure 11.17 where the experimental variation in Nu with distance, Re and Pr is shown. As Re increase from 4,000 to 10,000, the temperature profile develops more rapidly, the fully developed state being reached in 15 and 4 diameters respectively. Since the 'small X' result can only be valid for about 25% of this length, it was not possible to obtain experimental corroboration of the theory at high Reynolds numbers. Nevertheless, a reasonable agreement is shown, even when the 'small X' result is taken to its limit (the function is minimised when the denominator becomes 0.75). The maximum difference between theory and experiment increased from 10% to 12% as Re was increased. The error is consistent with the accuracy of the approximate expression for \( \frac{\text{Nu}}{3Y_0} \), which overestimated the data of Kays by 10% at Re = 10,000.

Finally, the theory may be used to produce a semi-theoretical expression for Nu valid at all distances along the tube. An interpolation is carried out between small and large distance asymptotes, in the region 4,000 < Re < 10,000 (similar expressions can easily be determined for any other range of Re).

In the preceding analysis for small values of \( \varepsilon (x) \)

\[
f (o) = f_0 (o) + \varepsilon f_1 (o).
\]

Now suppose for moderate values of \( \varepsilon \) a better value of \( f(o) \) is given by

\[
f(o) = f_0 (o) + \lambda \varepsilon f_1 (o)
\]
where $\lambda$ is a correction factor of the order unity. Selecting a suitable value for $\lambda$ the condition $\frac{\partial \theta}{\partial x} = 0$ at $Nu = Nu_w$ can be satisfied. Hence, $\lambda$ is chosen to be 0.512.

This leads to an expression for entrance length as follows:

$$\left( \frac{x}{2r_w} \right)_{\text{max}} = 8.69 \times 10^4 \cdot \text{Re}^{-1.1}$$  \hspace{1cm} (11.72)

It is possible that a more accurate result for $f(\theta)$ could be obtained by generating further terms in the series, though this would make the problem much more complicated, and was not considered to be a suitable approach.

The resulting Nusselt number becomes

$$Nu = \frac{0.0562 \cdot \text{Re}^{0.733} \cdot (x/2r_w)^{-3} \cdot \text{Pr}^{1/3}}{1 - 2.87 \times 10^{-6} \left( \frac{x}{2r_w} \right) \cdot \text{Re}^{1.1}}$$  \hspace{1cm} (11.73)

This equation is plotted in figure 11.18 together with the experimental results, and reasonable agreement is indicated.

11.3. THE DIVERGENCE.

11.3.(i) Introduction.

The measured values of the local heat-transfer coefficient in the region of a sudden divergence in diameter exhibited certain characteristics which could not be readily explained. In an attempt to understand the mechanisms of heat transfer the following theoretical analysis of the system was carried out. Initially, the work was motivated by the fact that very low coefficients of heat transfer were obtained at small Reynolds numbers ($10^2$). These differed markedly in magnitude and axial distribution to the measured values at moderate $Re (< 10^3)$. It was considered necessary to show that such low coefficients were not simply indicative of some unacknowledged limitation on the means of measurement.
FIGURE 11.1

HYPOTHETICAL DESCRIPTION OF FLOW THROUGH A DIVERSION
AND TYPICAL, LOCAL COEFFICIENTS OF HEAT TRANSFER.

FLOW

CORE

MIXING LAYER

OUTER

ANNULUS

RADIAL DISTRIBUTION OF VELOCITY.

\[ \text{DISTANCE} \]

(MODERATE \( R_e \) (<10^3))

(SMALL \( R_e \) (~10^2))

LOCAL COEFFICIENTS OF HEAT TRANSFER.
Figure 11.1 illustrates the probable flow development through a sudden divergence, and shows the axial distribution of the local coefficient of heat transfer in general terms. The flow pattern is similar to that proposed by Abramovitch for turbulent wakes behind enclosed, bluff bodies (Ref: K.7), but the diagram can be taken as applicable to both turbulent and laminar flows provided they are stable. Abramovitch envisaged a simple, boundary-layer model in which the initial part of the divergence was at constant pressure, and contained an inner, fast moving core having uniform velocity. A slow reverse flow occurred in the outer annulus, which also had a uniform, radial velocity distribution, and a mixing layer was positioned between the two uniform streams in which the shape of the velocity profile was assumed. Further downstream the velocity profile was supposed to flatten as the source of inertia in the core became depleted, and the mixing layer thickened. This region of the flow was solved by an inviscid analysis.

Omissions have been made in the above description for brevity, but a qualitative description is given for the way in which a standing eddy is formed downstream of a step. However, it is doubtful whether such boundary layer simplifications can be used to determine reliable temperature distributions in a divergence. A recent attempt by Filetti (Ref: H.8) to describe heat transfer to air in a divergence, indicated that such simplifications were unacceptable.

The analysis of a two-dimensional flow with recirculation invariably leads to a numerical solution of the momentum equations. This implies the finite-difference solution of two elliptic, partial differential equations. Until recently, the difficulty in obtaining a stable result, with recirculation present, limited such solutions to extremely small Reynolds numbers. The work of Gosman et al
(Ref: J.13) has made available a general procedure for solving such problems with the likelihood of a much improved stability.

The latter method was applied to the problem at hand, i.e. flow through a divergence with heat transfer. The ratio of upstream to downstream diameters was 1:2, but the computer programme discussed in this section can be used for other ratios with small modifications. An extensive theoretical investigation was not the objective of this analysis, but the mathematical procedure was utilised as a tool with which to explain the mechanisms of heat transfer.

In deriving the particular form of the momentum and energy equations required for a numerical solution (given in the next part 11.3(ii)) the operation described is simpler than the original explanation of Gosman (Ref: J.13). The reason for this is because the physical properties are assumed to be constant, and the system is specified at the outset as having 'cylindrical, polar co-ordinates'. The flow is further assumed to be axi-symmetrical.

11.3.(ii) The derivation of the momentum and energy equations.

11.3.(ii)a. The symbols used throughout are similar to those favoured by Gosman (Ref: J.13), so as to eliminate confusion when referring to the original source. These definitions are local to the present section 11.3 only.

Consider the co-ordinates below -

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<table>
<thead>
<tr>
<th>Direction 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>A</td>
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<tr>
<td></td>
</tr>
<tr>
<td>Direction 1</td>
</tr>
<tr>
<td>-------------</td>
</tr>
</tbody>
</table>

VECTOR \vec{V} IS WRITTEN \vec{V}.
```

The nomenclature is first specified -
<p>| Programming |</p>
<table>
<thead>
<tr>
<th>Symbols</th>
<th>Text Symbols</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1(I)</td>
<td>Z</td>
<td>axial distance</td>
</tr>
<tr>
<td>X2(J)</td>
<td>r</td>
<td>radial distance</td>
</tr>
<tr>
<td>NV1,NV2</td>
<td>V1,V2</td>
<td>axial, radial velocities</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>NW</td>
<td>( \omega/r )</td>
<td>(vorticity/radius)</td>
</tr>
<tr>
<td>NF</td>
<td>( \gamma )</td>
<td>stream function</td>
</tr>
<tr>
<td>NT</td>
<td>t</td>
<td>temperature</td>
</tr>
<tr>
<td>NH0</td>
<td>( \zeta )</td>
<td>density</td>
</tr>
<tr>
<td>NMU</td>
<td>( \mu )</td>
<td>viscosity</td>
</tr>
<tr>
<td></td>
<td>C'</td>
<td>specific heat capacity</td>
</tr>
<tr>
<td></td>
<td>K'</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td></td>
<td>( \phi )</td>
<td>generalised dependent variable in text</td>
</tr>
<tr>
<td></td>
<td>a( \phi ), b( \phi )</td>
<td>coefficients in generalised form of differential equation</td>
</tr>
</tbody>
</table>

- A(I,J,K) | array for the general dependent variable, the coefficients A\( j \) defined in text. |
- AE, AW, AN, AS | |
- APP | The coefficient a\( \phi \) in integrated form of general equation. |
- BB(I), BW(I), BN(J), BS(J) | the coefficients B\( j \) defined in text. |
- BPP | the coefficient b\( \phi \) in integrated form of general equation. |
- CC | convergence criterion (maximum residual). |
- I | index for constant Z lines. |
- IE | number of differential equations to be solved. |
- IMAX(J), IMIN(J) | the I - indices denoting nodes adjacent to the wall and axis. |
- IN | total number of constant Z grid lines |
- IMM | IN - 1 |
- IP | number of successive printouts required |
- IV | number of variables to be output. |
- J | index for constant r lines |
- JN | total number of constant r grid lines. |
- JNM | JN - 1 |
- NEBEGIN | index of first variable to be output |
- NCORD | a number given to particular co-ordinate system. |
- NITER | number of iterations carried out. |
- NMAX | maximum number of iterations. |
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPRINT</td>
<td>number of iterations between printout</td>
</tr>
<tr>
<td>NTOTAL</td>
<td>number of variables output.</td>
</tr>
<tr>
<td>PR(K)</td>
<td>Prandtl number for variable K.</td>
</tr>
<tr>
<td>QK</td>
<td>(Heat flux)/(thermal conductivity).</td>
</tr>
<tr>
<td>R(J)</td>
<td>distance from axis.</td>
</tr>
<tr>
<td>RES</td>
<td>maximum residual for all ( \phi ) equations.</td>
</tr>
<tr>
<td>DOREF</td>
<td>reference density.</td>
</tr>
</tbody>
</table>
| RP(K) | relaxation parameters \( \alpha \), used in updating \( \phi \) viz: \[
\phi = \alpha \phi^{(N)} + (1 - \alpha) \phi^{(N-1)},
\]
where \( \phi^{(N)} \) and \( \phi^{(N-1)} \) are values computed at consecutive iterations. |
| RS | residual, or \( (1 - \phi_p^{(N)})/\phi_p^{(N-1)} \). |
| RSU(K) | maximum value of RS for each \( \phi \). |
| SOURCE | -d\( \phi \) in generalised differential equation. |
| ZMUREF | reference viscosity. |

* Index corresponding to \( K \) in array A.*
The momentum equations in directions 1 and 2 can be written down

\[
\text{div} \left( \nabla_2 \vec{V} - \vec{T}_1 \right) + \frac{\partial n}{\partial z} = 0
\]

\[
\text{div} \left( \nabla_2 \vec{V} - \vec{T}_2 \right) + \frac{\partial n}{r^2} + \frac{\partial n}{\partial r} = 0
\]

where \( \text{div} (\vec{r}) = \frac{1}{r} \frac{\partial}{\partial r} (r^2 \vec{r}) + \frac{1}{r} \frac{\partial}{\partial z} (f_1 \vec{r}) \), and the stress components are

\[
\begin{align*}
T_{1,1} &= 2\gamma \frac{\partial V_1}{\partial z} , & T_{1,2} &= \gamma \left( \frac{\partial V_1}{\partial r} + \frac{\partial V_2}{\partial z} \right) \\
T_{2,1} &= T_{1,2} , & T_{2,2} &= 2\gamma \frac{\partial V_2}{\partial r} .
\end{align*}
\]

The continuity equation is written

\[
\text{div} \vec{V} = 0 .
\]

The equations above are quoted in many standard text books and require no proof.

It is convenient to eliminate the pressure gradients by introducing vorticity into the equations, as defined by

\[
\omega = \frac{\partial V_2}{\partial z} - \frac{\partial V_1}{\partial r} .
\]

Differentiating the direction 1 equation with respect to \( r \), and the direction 2 equation with respect to \( z \), gives rise to similar pressure terms, which are eliminated by combining the equations thus

\[
\left( r \frac{\partial}{\partial z} \left( \text{div} (\nabla_2 \vec{V}) \right) \right) - \left( \frac{\partial}{\partial r} \left( \text{div} (\nabla_2 \vec{V}) \right) \right) = \frac{1}{r^2} \left( \text{div} \vec{T}_2 \right) - \frac{1}{r} \left( \text{div} \vec{T}_1 \right) - \frac{1}{\partial z} \left( 2\gamma \frac{\partial V_2}{r^2} \right) ,
\]

or after introducing the vorticity

\[
\left( r \frac{\partial}{\partial z} \left( \frac{r}{\omega} \right) \right) - \frac{1}{r} \left( \frac{r}{\omega} \right) = -\frac{1}{r^2} \left( \text{div} \vec{T}_2 \right) - \frac{1}{r} \left( \frac{r}{\omega} \right) - \frac{1}{\partial z} \left( 2\gamma \frac{\partial V_2}{r^2} \right) = -\omega , \text{ say.}
\]

Expanding the right hand side of the equation, \( \omega \), the following is obtained
\[ \frac{\mu}{r} \frac{\partial}{\partial r} \left( \frac{2}{r} \frac{\partial V_2}{\partial r} \right) \]

The first and third terms (denoted by a square bracket) can be combined to introduce vorticity; the second and fourth terms are combined in a similar way, hence

\[ \mu^{-1} \Omega = \frac{\partial^2 \omega}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\omega}{r} \frac{\partial}{\partial r} \right) + 2 \left[ \frac{1}{r} \frac{\partial V_2}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{V_2}{r^2} \right) \right]. \]

The next step is to replace the remaining velocity terms on both sides of the equation by the stream function, which is an expedient to eliminate the continuity equation from the calculations. The velocities are

\[ V_1 = \frac{1}{r} \frac{\partial Y}{\partial r}, \quad V_2 = -\frac{1}{r} \frac{\partial Y}{\partial z} \]

Now, the final form of the momentum equation is

\[ r^2 \left( \frac{\partial}{\partial z} \left( \frac{\omega}{r} \frac{\partial Y}{\partial r} \right) - \frac{\partial}{\partial r} \left( \frac{\omega}{r} \frac{\partial Y}{\partial z} \right) \right) - \frac{\partial}{\partial z} \left( r^2 \frac{\partial}{\partial z} \left( \frac{\mu Y}{r} \right) \right) - \frac{\partial}{\partial r} \left( \frac{r^3}{\partial r} \left( \frac{\mu Y}{r} \right) \right) = 0. \]

This rather complicated form of the momentum equation has close similarity to the energy equation, soon to be derived, hence its unconventional appearance.

The vorticity equation can be written in terms of the stream function

\[ \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial Y}{\partial z} \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial Y}{\partial r} \right) + \omega = 0. \]
11.3.(ii)c. The energy equation is simplified by neglecting the
dissipation of mechanical energy into heat, hence
\[
\frac{\partial}{\partial Z} \left( \rho c^I V_1 \text{tr} \right) + \frac{\partial}{\partial r} \left( \rho c^I V_2 \text{tr} \right) = \frac{\partial}{\partial Z} \left( K^I \frac{V_1}{\text{tr}} \right) + \frac{\partial}{\partial r} \left( K^I \frac{V_2}{\text{tr}} \right).
\]
Replacing the velocity by the stream function, then defining
the exchange coefficient \( \Gamma' = K' / \rho' \), the equation becomes
\[
\frac{\partial}{\partial Z} \left( \frac{t \psi'}{\partial Z} \right) - \frac{\partial}{\partial r} \left( \frac{t \psi'}{\partial r} \right) - \frac{1}{\rho} \left( \Gamma' \frac{\partial \psi'}{\partial Z} \right) - \frac{1}{\rho} \left( \Gamma' \frac{\partial \psi'}{\partial r} \right) = 0.
\]

11.3.(iii) The General Equation.

Three differential equations are to be solved in \( \omega \), \( \gamma \) and \( t \). The velocity components can be readily determined from this
information. A single, elliptic, partial differential equation

\[
c' \left[ \frac{\partial}{\partial Z} \left( \rho \frac{\partial \psi'}{\partial Z} \right) - \frac{\partial}{\partial r} \left( \rho \frac{\partial \psi'}{\partial r} \right) \right] - \frac{\partial}{\partial Z} \left[ b \frac{\partial}{\partial Z} (c' \rho) \right] - \frac{\partial}{\partial r} \left[ b \frac{\partial}{\partial r} (c' \rho) \right] + \rho \frac{\partial \psi'}{\partial t} = 0.
\]

where the parameters are

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( a' )</th>
<th>( b' )</th>
<th>( c' )</th>
<th>( d' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>1</td>
<td>( \Gamma' )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( (\omega / \rho) )</td>
<td>( x^2 )</td>
<td>( \rho (x^2)^{-1} )</td>
<td>( \rho )</td>
<td>0</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0</td>
<td>( \rho )</td>
<td>0</td>
<td>( - (\omega / \rho) )</td>
</tr>
</tbody>
</table>

11.3.(iv) Outline of the Mathematical Solution.

The numerical solution of the general equation - after
Gosman (Ref. J.13) - will be described briefly.

The equation is integrated approximately over small, finite
elements in the field of interest. Parallel grid lines intersect
at nodal points, \( P \), which are not necessarily equi-spaced. The notation is given below.

\( N, E, W, \) and \( S \) are adjacent nodes.

The integration required is

\[
\int_{r_s}^{r_a} \int_{Z_w}^{Z_e} \left\{ \frac{\partial}{\partial Z} \left( \phi \frac{\partial \psi}{\partial r} \right) - \frac{\partial}{\partial r} \left( \phi \frac{\partial \psi}{\partial Z} \right) \right\} \, dZ \, dr - \\
- \int_{r_s}^{r_a} \int_{Z_w}^{Z_e} \left\{ \frac{\partial}{\partial Z} \left( b_r \frac{\partial}{\partial Z} \left( c_r \phi \right) \right) + \frac{\partial}{\partial r} \left( b_r \frac{\partial}{\partial r} \left( c_r \phi \right) \right) \right\} \, dZ \, dr + \\
+ \int_{r_s}^{r_a} \int_{Z_w}^{Z_e} (r d\phi) \, dZ \, dr = 0 .
\]

To illustrate the process, one term of each integral is considered.

The first part of the first integral, \( I_c \), becomes

\[
I_c = \int_{r_s}^{r_a} \int_{Z_w}^{Z_e} a_p \frac{\partial}{\partial Z} \left( \phi \frac{\partial \psi}{\partial r} \right) \, dZ \, dr \approx \alpha \frac{\phi}{r} \left( \psi_{ce} - \psi_{se} \right)
\]

where \( \phi \) is the value of \( \phi \) upstream of the c-face, i.e., \( \phi_p \) or \( \phi_E \) depending on the flow direction. Hence

\[
I_c \approx \frac{\alpha \phi}{2} \left( \psi_{ce} - \psi_{se} \right) \left( \psi_{ce} - \psi_{se} \right)
\]

The form of \( I_c \) ensures that upstream differences are always selected, and the stability of the solution improves considerably because of this.

The inter-nodal values are approximated by

\[
\frac{\psi_{se}}{\psi_{ce}} \approx \left( \frac{\psi_{se} + \psi_{p} + \psi_{s}}{2} \right) / 4
\]

The first term of the second integral, \( I_d \), is

\[
I_d = \int_{r_s}^{r_a} \int_{Z_w}^{Z_e} \left( b_r \frac{\partial}{\partial Z} \left( c_r \phi \right) \right) \, dZ \, dr \\
\approx \frac{(b_{\phi_e} + b_{\phi_p})}{2} \left( \frac{r_e + r_p}{2} \right) \left( c_{\phi_{se}} - c_{\phi_{p}} \phi \right) \left( \frac{r_e - r_s}{Z_e - Z_p} \right)
\]
The third integral, \( I_{\text{SOR}} \), becomes

\[
I_{\text{SOR}} = \int_{r_s}^{r_N} \int_{Z_N}^{Z_E} r \, d\phi \, dz \, dr = d_{\phi p} V_p
\]

where \( V_p = \frac{(Z_E - Z_W)}{2} \cdot \frac{(r_N - r_s)}{2} r_p \).

The general differential equation can now be replaced by an algebraic equation, as follows.

\[
A_E (\phi_p - \phi_E) + A_w (\phi_p - \phi_w) + A_N (\phi_p - \phi_N) + A_S (\phi_p - \phi_S) - B_E (c_{\phi_p} - c_{\phi_E}) - B_w (c_{\phi_p} - c_{\phi_w}) - B_N (c_{\phi_p} - c_{\phi_N}) - B_S (c_{\phi_p} - c_{\phi_S}) + d_{\phi p} V_p = 0.
\]

Where the coefficients of \( \phi \) are as stated at the end of this section 11.3.(iv).

The algebraic equation is solved for \( \phi_p \) by an iterative procedure, in which the field is scanned repeatedly updating the value of \( \phi_p \) at each node. The order of scanning is - increasing \( Z \) then increasing \( r \). Initially, a guess at the \( \phi \) distribution is necessary, then the data from any five adjacent nodes is used to obtain a better value of \( \phi_p \) at each position.

The form of the numerical equation must now be changed to permit the procedure to be implemented. Therefore

\[
\phi_p = C_E \phi_E + C_w \phi_w + C_N \phi_N + C_S \phi_S + D
\]

where

\[
C_E = (A_E + B_E c_{\phi_N})/\Sigma_{A_E}, \quad C_w = (A_w + B_w c_{\phi_w})/\Sigma_{A_w}, \quad C_N = (A_N + B_N c_{\phi_N})/\Sigma_{A_N}, \quad C_S = (A_S + B_S c_{\phi_S})/\Sigma_{A_S}, \quad D = -d_{\phi p} V_p / \Sigma_{A_E},
\]

and

\[
\Sigma_{A_E} = A_E + A_N + A_N + A_S + c_{\phi_p} (B_E + B_w + B_N + B_S), \quad V_p = \frac{r_p}{\xi} (Z_E - Z_W) (r_N - r_s).
\]

The coefficients \( A \) are in terms of the stream function, and the coefficients \( B \) are in terms of spatial parameters only.
\[ A_E = \frac{a_{fE}}{8} \left( \psi_E - \psi_N - \psi_S + \psi_S \right) + \left( \begin{array}{c} \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \\ \psi_E - \psi_N - \psi_S \end{array} \right) \]

\[ A_W = \frac{a_{fW}}{8} \left( \psi_W - \psi_N - \psi_S + \psi_S \right) + \left( \begin{array}{c} \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \\ \psi_W - \psi_N - \psi_S \end{array} \right) \]

\[ A_N = \frac{a_{fN}}{8} \left( \psi_N - \psi_E - \psi_W + \psi_E \right) + \left( \begin{array}{c} \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \\ \psi_N - \psi_E - \psi_W \end{array} \right) \]

\[ A_S = \frac{a_{fS}}{8} \left( \psi_S - \psi_W - \psi_E + \psi_E \right) + \left( \begin{array}{c} \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \\ \psi_S - \psi_W - \psi_E \end{array} \right) \]

\[ B_E = \frac{(b_{fE} + b_{fP})}{8} \left( \begin{array}{c} r_N - r_S \\ r_E + r_P \end{array} \right) + \left( \begin{array}{c} r_N - r_S \\ r_E + r_P \end{array} \right) \]

\[ B_W = \frac{(b_{fW} + b_{fP})}{8} \left( \begin{array}{c} r_N - r_S \\ r_E + r_P \end{array} \right) + \left( \begin{array}{c} r_N - r_S \\ r_E + r_P \end{array} \right) \]

\[ B_N = \frac{(b_{fN} + b_{fP})}{8} \left( \begin{array}{c} r_N - r_P \\ r_W + r_P \end{array} \right) + \left( \begin{array}{c} r_N - r_P \\ r_W + r_P \end{array} \right) \]

\[ B_S = \frac{(b_{fS} + b_{fP})}{8} \left( \begin{array}{c} r_S + r_P \\ r_E - r_W \end{array} \right) + \left( \begin{array}{c} r_S + r_P \\ r_E - r_W \end{array} \right) \]

The complicated problem posed initially reduces to three solutions of the finite difference equation above.

In the original text (Ref: J.13) considerable space was devoted to the discussion of convergence, and of truncation errors. In this application it will simply be stated that small truncation errors are inevitable, but these have been minimised by the selection of small finite elements at the expense of considerable computer time.

No attempt was made to under or over-relax the solution, (i.e. correcting the updated values of \( \phi_p \) by a multiple of the difference between successive iterations, the multiple being the relaxation parameter) and the relaxation parameter was set at unity.

Conditions at the boundary must be specified to define the specific problem. The next section deals with these conditions.

11.3 (v) Boundary conditions.

For uniform property flow through the divergence the boundary conditions, with uniform heat flux can be specified as
in the diagram below.

At the inlet (2) the velocity is specified as parabolic in \( r \) for laminar flow, or uniform in \( r \) for turbulent flow.

At the outlet (5), the boundary conditions are not critical because being far downstream they were unlikely a priori to affect the temperature and flow near to the divergence. The boundary values selected were found by experience to give a tolerable rate of convergence, whereas gradient-type conditions were found to be very slow. The temperature was put equal to the bulk temperature, and the flow was assumed to be fully developed (i.e. parabolic, axial velocity for laminar flow, and uniform velocity for turbulent flow).

The conditions posed are readily converted into vorticity and stream function, as follows:

At (1) \( V_1 = \frac{1}{C_r} \frac{\partial \psi}{\partial r} \) therefore \( \psi = \psi_T \), constant along the wall.

The vorticity at the wall is replaced by an equation relating vorticity to stream function -

Suppose that \( P \) is a wall-node, and \( NP \) is the nearest node to the wall. Linearising the vorticity near the wall gives

\[
\omega_{NP} \approx \frac{A'(Z_{NP} - Z_P) + B'}{M}.
\]
From the definition of stream function
\[
\left( \frac{\psi_{np} - \psi_p}{\rho} \right) = -\rho r_p \int \int \frac{A_l(z-z_d)+B}{\mu} d\zeta \, d\zeta,
\]
\[
= -\rho r_p \left[ \frac{A_l}{\mu} (Z_{np} - Z_p)^3 + \frac{B}{2} (Z_{np} - Z_p)^2 \right]
\]
Combining the two expressions
\[
\left( \frac{\omega}{r} \right)_p = -\left( \frac{3}{r_p^2 (Z_p - Z_{np})^2} \frac{\psi_{np} - \psi_p}{2} \right) + \frac{\omega_{np}}{2 r_p}
\]
At (2) \( V_1(0, r) \) is expressed in terms of stream function.
\[
V_1(0, r) = V_1(0, 0) \left[ \left( \frac{r}{r_1} \right)^2 - \frac{r}{r_1^2} \right]
\]
so that
\[
\psi = r \cdot V_1(0, 0) \left[ (r^{3/2}_1 - (r^{4/3}/4 r_1^2) \right)
\]
The vorticity is 
\[
\omega = -\partial V_1/\partial r
\]
therefore
\[
\left( \frac{\omega}{r} \right) = \frac{2 V_1(0, 0)}{r_1^2}
\]
At (3) the stream function is again \( \psi = \psi_p \), and the vorticity equation is derived in the same way as at the shoulder (1).
Hence
\[
\left( \frac{\omega}{r} \right)_p = \left( \frac{3}{r_p^2 (r_p - r_{np})^2} \frac{\psi_{np} - \psi_p}{2} \right) + \frac{\omega_{np}}{2 r_p}
\]
At (4) the stream function is zero (any constant suffices) for symmetry, and the vorticity at the axis is found by approximating the axial velocity with a parabola.
\[
V_1 = \overline{Ar} + \overline{B}r^2
\]
From the definition of stream function \( \psi = V_1(0, 0) \frac{r^2}{2} + \frac{B}{8} r^4 \),
and since
\[
\omega = -\partial V_1/\partial r,
\]
\[
\left( \frac{\omega}{r} \right)_p = \frac{\Delta}{r} \left[ \left( \frac{\psi_{np}/r_{np}^2}{\psi_{np}/r_{np}^2} \right) - \left( \frac{\psi_{np}/r_{np}^2}{\psi_{np}/r_{np}^2} \right) \right] / \left( \psi_{np}/r_{np}^2 - \psi_{np}/r_{np}^2 \right)
\]
where NP and NPI are nodes which are once and twice removed from the axis.

The bulk temperature is given by
\[
T_{\text{bulk}} = \left( \frac{4 q}{K'} \right) \left( \frac{R_e^2 Pr}{R_e^2 Pr} \right) Z
\]
The computer programme.

The programme for solving the equations described is given in this section. It is very similar to that proposed by Gosman, named ANSWER, therefore it is unnecessary to describe the general construction. A step by step description of the component parts has been included in the programme itself, and the language ALGOL 60 has been utilised instead of the original FORTRAN IV. Close inspection of the programme would be necessary before modifying it to suit a different problem. The particular example given applies to laminar flow with uniform flux heat-transfer and constant physical properties. The computations were carried out on an ICL 1905 computer.

Before rewriting the equations in programming symbols the general substitution formula must be slightly altered, for convenience, by putting

$$\frac{\phi_p}{d_{\phi\rho}} = \frac{\sum_{j=N,S,E,W} \left[ A_j' + c_{\phi\rho} (b_{\phi\rho} + b_{\phi\phi}) B_j' \right] \phi_j - d_{\phi\rho}}{\sum_{j=N,S,E,W} \left[ A_j' + c_{\phi\rho} (b_{\phi\rho} + b_{\phi\phi}) B_j' \right]}$$

where

$$A_j' = A_j / V_\rho$$

and

$$B_j' = B_j / V_\rho (b_{\phi\rho} + b_{\phi\phi})$$

To improve the stability of the solution when a non-uniform grid is specified, the vorticity at the wall is eliminated from those calculations appertaining to the 'near wall' nodes. This is achieved by introducing another form of the substitution equation in which the vorticity at the wall is replaced by that boundary condition, (11.3. (v)) which relates vorticity to stream function. This is referred to in the programme itself.
In making use of the programme described, a converged solution was obtained after approximately 130 iterations, each requiring 6 seconds of the total time. The number of iterations could have been drastically reduced by selecting fewer nodes, with an ensuing loss of accuracy. The total number required for convergence was found to increase with Reynolds number and Prandtl number. It is generally known that comparatively slow convergence ensues when uniform flux is specified rather than uniform wall temperature, so substantial computing times were anticipated.
PROCEDURE VISCOS(NHU)

INTEGER NHU

COMMENT COMPUTE EFFECTIVE VISCOSITY

BEGIN
  FOR J=1 STEP 1 UNTIL JN DO
    FOR I=1 STEP 1 UNTIL IN DO
      A[I,J,NHU]=ZHUREF
    END FOR
  END FOR
END VISCOS

PROCEDURE SOURCE(SOURCE)

REAL SOURCE

COMMENT CALC SOURCE TERMS

BEGIN
  IF K=2 THEN SOURCE=A[I,J,NW]
  IF K#2 THEN SOURCE=0
END SOURCE

PROCEDURE CONVEC(K)

COMMENT CALC AE,AN,AS

INTEGER K

BEGIN
  REAL DV,G1PW,G1PE,G2PS,G2PN
  COMMENT CALC MEAN MASS FLOW RATES THRO 4 TUBES OF ELEMENT
  DV=RE(J)*X[I+1]-X[I-1]-X2(J+1)-X2(J-1)
  G2PN=A[I+1,J,NF]=A[I+1,J,NF]
  COMPUTE AE,AN,AS
  APP=1
  IF K=NW THEN APP=RE(J)*R[J]
  AE=0.5*APP*(ABS(G1PE)-G1PE)
  AN=0.5*APP*(ABS(G1PW)+G1PW)
  AS=0.5*APP*(ABS(G2PS)+G2PS)
END CONVEC
PROCEDURE BOUND(DX, QX, OKP);  
REAL QX, OKP;  
BEGIN  
COMMENT COMPUTE ONE ITERATION FOR BOUNDARY NODES NEEDING UPDATING;  
DX1 := X1(JN) - X1(IN);  
DX2 := X2(JH) - X2(IN);  
BEGIN  
COMMENT COMPUTE VORTICITY/RADIUS NEAR WALL, NEAR AXIS, THEN TEMP NEAR WALL, NEAR AXIS;  
FOR J1 := 2 STEP 1 UNTIL JNH DO  
BEGIN  
A1(J1, JNH, NWW) := -3 * (A1(J1, JNH, NF) - A1(J1, JNH, NF))/A1(J1, JNH, NF)  
+ 0.5 * A1(J1, JNH, NWW)/A1(J1, JNH, NF);  
A1(J1, JNH, NWW) := 3 * (A1(J1, JNH, NF) - A1(J1, JNH, NF))/A1(J1, JNH, NF)  
+ 0.5 * A1(J1, JNH, NWW)/A1(J1, JNH, NF);  
END;  
COMMENT NEXT DO OUTLET END STREAM FUNC, VORT/RADIUS TEMP;  
A1(J1, JNH, NT) := A1(J1, JNH, NT);  
COMMENT NEXT DO ALONG SHOULDER VORT/RADIUS AND TEMP;  
DX2 := X2(JH);  
FOR J1 := P STEP 1 UNTIL JH DO  
BEGIN  
A1(J1, JNH, NWW) := -3 * (A1(J1, JNH, NF) - A1(J1, JNH, NF))/A1(J1, JNH, NF)  
+ 0.5 * A1(J1, JNH, NWW)/A1(J1, JNH, NF);  
END;  
A1(J1, JNH, NT) := A1(J1, JNH, NT);  
END_BOUND;  
PROCEDURE EQ(NH, NF, NT);  
BEGIN  
END;  
COMMENT CALC EFFECTIVE VISCOSITY;  
VISCOS(NH);  
COMMENT VORTICITY SUBCYCLE ****************** 1 KERWI;  
FOR J := 2 STEP 1 UNTIL JNH DO  
BEGIN  
END;  
FOR I := 1 STEP 1 UNTIL IH DO  
BEGIN  
COMMENT CALC SOURCE TERM;  
SORCE(SOURCE);  
COMMENT CALC AE = 0, AN = AS;  
CONVF(NH);  
COMMENT CALC RBE = 0, DB = 0, DBS = 0;  
IDENTICAL TO DJ/VP IN SUBST. FORMULA;  
BEGIN  
END;  
COMMENT WHEN J := JNH THE FOLLOWING TERMS ARE CALC TO INTRO, IMPLICIT, FORM;  
BEGIN  
DX2SO := (X2(JNH) - X2(JH));  
END;  
COMMENT BY = -3 * (A[NH, NF] - A[NH, NF])/RSQ/OKP;  
BEGIN  
END;  
GOTO FE;  
GOTO FF;
END!

COMMENT ON PLANE OF SYMMETRY:
DELX2 = X2[2] - X2[1];
FOR I1 = 2 STEP 1 UNTIL IN 'DO'
A[I1, 1, NV1] = (A[I1, 2, NV] - A[I1, NV]) / DELX2 / A[I1, 1, NRO];
END!

COMMENT FINAL PRINTOUT:
PRINTS(1, NV1);
PAPERTHROW;
WRITE(TEXT(1C, 1C) 'REFT DENSITY FOR FLUID'); PRINT(REFT, 2, 2);
WRITE(TEXT(1C, 1C) 'REFT VISCOSITY FOR FLUID'); PRINT(REFT, 2, 2);
WRITE(TEXT(1C, 1C) 'REFT REYNOLDS NUMBER'); PRINT(REFT, 2, 2);
WRITE(TEXT(1C, 1C) 'REFT PRANDTL NUMBER'); PRINT(REFT, 2, 2);
WRITE(TEXT(1C, 1C) 'RELAX PARAMETERS');
FOR K1 = 1 STEP 1 UNTIL IE DO PRINT(RP(K1), 2, 2);
WRITE(TEXT(1C, 1C) 'MAX. NO. ITERATIONS'); PRINT(MAX, 5, 0);
WRITE(TEXT(1C, 1C) 'CONVERG.CRITERIA'); PRINT(CC, 1, 5);
WRITE(TEXT(1C, 1C) 'NO. COLMNSXDIRECT'); PRINT(N1, 3, 0);
WRITE(TEXT(1C, 1C) 'NO. ROWSXDIRECT'); PRINT(J, 3, 0);
WRITE(TEXT(1C, 1C) 'NUSSELTONUMBERS'); JS, 3, 5);
FOR I1 = 1 STEP 1 UNTIL IN 'DO'
BEGIN
PRINT(1, 3, 0); SPACE(2);
PRINT((2+X2[I1]+OK/(A[I1, J, NT]=OKP*XI[11]), 4, 3));
END!
END!
END!
11.3.(vii) STABLE, LAMINAR MOTION.

An investigation was carried out of stable laminar motion through a 1:2 divergence with the boundary conditions specified in 11.3.(v). The downstream section of tube was taken as 50 diameters in length, and the axial grid spacing was 0, 0.1, 0.25, 0.5, 0.75, 1.0, 1.5, 2, 3, 4, 5.5, 7, 8.5, 10, 12, 14, 16, 18, 20, 23, 26, 30, 40, and 50 diameters downstream. Fourteen evenly spaced radial increments were chosen.

A Reynolds number (for the larger tube) of 200 was considered suitable initially, because the experimental results were found to be unstable in this region, exhibiting the 'high' and 'low' values of Nu referred to in part 11.3.(i). Figure 11.19 shows the pattern of streamlines for Re = 200. A standing eddy is clearly indicated, the 'eye' of which occurs at 1.65 diameters. The recirculation zone is approximately 8 diameters in length. Axial velocity profiles are given in figure 11.20, these indicate that the reverse-flow in the outer annulus of fluid occurs at small axial velocity in comparison with the central core.

With a Prandtl number of 100, the temperature distribution was computed, and figure 11.19 shows the pattern of the isotherms. The recirculating flow carries 'hot' fluid to the 'cold', fast moving elements near the inlet causing a point of inflection in the radial temperature profile. The 'hot' outer layers are almost stagnant and tend to insulate the 'cold' core from the tube wall giving rise to low coefficients of heat transfer near the discontinuity.

A graph of Nusselt number versus distance (figure 11.21b) shows that Nu rises rapidly with increasing distance, from 9.6 at the discontinuity to 15.0 at approximately 8.5 diameters downstream. A slow diminution occurs as the distance increases further, until Nu = 10 near the outlet end. Since fairly large axial increments
were selected close to the outlet, the numerical values of Nu in this region may not be very reliable. The value of Nu at large distances was of little interest, however, and the precise magnitude was not a prime objective.

The peak in the axial distribution of Nu almost corresponds to the point of boundary layer reattachment, the latter being defined by zero vorticity at the wall. The point of reattachment is at approximately 9.2 diameters, and Nu occurs at 8.5 diameters.

In figure 11.21b Nu is compared with the corresponding result for entrance-region heat-transfer in a short tube. Initially the local value of Nu is much greater for the short tube, but after 15 diameters the divergence-values converge to within 4% and it is probable that further downstream Nu can be closely estimated from short-tube theory.

Pr was increased for the same value of Re (200) in order to investigate whether a correlation of the kind \( \text{Nu} \sim \text{Pr}^n \) was a valid method of presenting the experimental data at low Reynolds numbers.

Figure 11.24 shows that for \( \text{Pr} = 1,000 \), the axial Nu distribution is qualitatively similar to the previous case (\( \text{Pr} = 100 \)). A peak occurs in the Nu function at the same position as before, (8.5 diameters) but throughout the length of tube Nu is much higher. At the discontinuity Nu = 15.6, rising to a maximum of 24.2, then falling to approximately 19 at the outlet.

In an attempt to form a unique function of distance, the relationship \( \text{Nu} \sim \text{Pr}^n \) was investigated for \( \text{Pr} = 100 \) and 1,000. It was found that the exponent \( n \) would have to be a function of axial position to satisfy both sets of data. At small distances Nu \( \propto \text{Pr}^{0.2} \) whereas near the outlet Nu \( \propto \text{Pr}^{0.3} \). Figure 11.21a indicates that
in plotting $(Nu/Pr^{0.2})$ versus (distance) considerable differences (approximately) 20% arise at large distances.

The effect of increasing $Re$ on the flow pattern, and the subsequent distribution of $Nu$, was determined by setting $Pr = 100$ and $Re = 1,000$. The results obtained in this case were fairly crude estimates because of two practical limitations. First, the high value of $Re$ led to considerable computing time, and convergence of the solution was truncated in the interests of economy. 150 iterations were carried out whereas an estimated 200 to 250 iterations would be required for complete convergence. The ensuing errors were probably quite small, and are indicated by a slight waviness in the $Nu$ - distance curve (figure 11.22). Second, in specifying the outlet boundary conditions a tube length far greater than 50 diameters was desirable. This implies the selection of many more axial increments. The point of reattachment was found to be well in excess of 50 diameters downstream, but this tube length was maintained, despite the artificiality of the system, so that some insight into the heat-transfer process could be gained.

Figure 11.23 shows that when $Re$ is increased to 1,000 the resulting streamlines reveal an elongation of the standing eddy, such that the length of the recirculation zone is of the order 50 diameters. The computed Nusselt numbers given in figure 11.22, when compared with the values for $Re = 200$, indicate that substantial increases in $Re$ cause a stretching of the $Nu$ - distance function without significantly increasing the magnitude. At small axial distances an increase in $Re$ actually reduces $Nu$. The converse is found to be true at large distances.

A direct comparison between the theory and experimental results was unlikely to be meaningful, since stable laminar flow
without a free convectional contribution was unlikely to occur in practice. Two typical sets of test results are shown in figure 11.24, both have \( \text{Re} \approx 200 \), with \( \text{Pr} = 230 \) and 408. \( \text{Nu} \) versus distance is plotted, and it is clear that the experimental results tended to be unstable. Reasonable agreement between theory and experiment is shown however, considering the limitations discussed and the existence of a flow regime associated with 'low' coefficients of heat transfer is established satisfactorily.

11.3.(viii) TURBULENT MOTION IN A DIVERGENCE.

In turbulent flows the kinetic energy associated with the eddying motion is either generated by frictional shear, or diffused from the regions of high intensity to the regions of low intensity. (A good description of the mechanisms is given in Ref: H.22). In most boundary-layer type flows the former mechanism predominates, but in stalled regions of a flow field the transport of turbulence energy by diffusion predominates.

With 'shear' flows the effective turbulent transport properties can be derived from Prandtl's 'mixing length' theory (as in part 11.2.(iv); an example would be the region of flow upstream of a divergence. Downstream of a divergence, at small distances, the field can be treated as 'diffused' flow. The high velocities upstream cause the discontinuity to act as a highly intense source of turbulence, and it can be argued that the effective transport properties downstream differ only slightly from those upstream because the kinetic energy of turbulence is similar. It has been shown (p.175 Ref: K4) that free jets and wakes can be analysed with a constant value for the effective viscosity, as might be predicted from the hypothesis stated.
The properties required are the effective thermal conductivity and viscosity; these can be interrelated using Reynolds analogy (Pr\textsubscript{effective} = 1). In prescribing the effective viscosity a single constant value was postulated which has been used successfully for analysing round turbulent jets (e.g. Ref: K.4) viz:

\[ \text{effective viscosity } \nu'\text{eff} = 0.013 (\nu, V, 0, 0), r_i \]

The equation appears to be a simplified form of Prandtl's mixing length theory in which the 'length' (\( \sim 0.1 r_i \)); such a value is typical of turbulent shear flows in general.

An important conclusion can be derived from the above argument: when analysing turbulent flow through a given divergence it is possible to state an equivalent laminar Reynolds number Re\text{eff} (a particular value) which will exhibit a similar flow pattern to any turbulent flow (arbitrary Re). The system is specified by

\[ \left( \frac{\rho \bar{V}_1 \cdot 2r_0}{\nu'\text{eff}} \right) = \text{Re}_{\text{eff}} \]

which has the value 154(r_i/r_o) from the definition of \( \nu'\text{eff} \).

Initially, this was approximated for a 1:2 divergence by Re\text{eff} \( \approx 80 \), since the true value could only be derived by trial and comparison with experiment.

Along the tube-wall, turbulence must be damped by viscous action. Regions of high and low shear stress exist, so that diffusion and generation of turbulence energy takes place in a complex fashion, and the estimation of local effective properties becomes difficult. This thin 'viscous' layer which persists on the tube wall was ignored for convenience, and an investigation was carried out treating the boundary conditions in an identical way to the laminar analyses discussed previously.
It is reasonable to assume that the true wall-vorticity can be derived from the effective value by the following -

\[ \omega_{\text{wall}} = \omega_{\text{eff, wall}} \left( \frac{J_{\text{eff}}}{J_{\text{f}}} \right) \]

because the region near to the wall can be treated as a constant shear-stress layer. To determine the temperature distribution near the tube wall it is essential to have a knowledge of the transition from the turbulent mainstream conductivity, \( K'_{\text{eff}} \) to the laminar wall value \( K' \). No procedure could be found which enabled such functions to be determined reliably, so it was considered unfeasable to attempt calculating the coefficients of heat transfer.

From the preceding arguments it was postulated that turbulent flow patterns for a 1:2 divergence would resemble laminar flow with \( Re = 80 \). The stream function and vorticity were calculated, and the results are given in figure 11.25.

The standing eddy formed is short in length, the recirculation zone being 2 to 3 diameters long. The 'eye' of the eddy occurs at approximately \( \frac{1}{2} \) diameter downstream. The line of zero rotation (\( \omega = 0 \)) approaches the tube wall at 2.45 diameters, and this corresponds to the point of boundary-layer reattachment.

The laminar results for \( Re = 200 \) indicated that the maximum in the 'Nu versus distance' function occurred at 92% of the distance to the point of reattachment. If the same holds true for the turbulent case then the maximum value of Nu would occur at 2.2 diameters downstream. In practice, it was found that the peak value of Nu corresponded to a distance of 1.8 to 2.0 diameters. The relatively small disparity is probably a consequence of the wrongful selection of \( Re_{\text{eff}} \), which could only be reliably established after studying several different configurations and comparing the theoretical results with experimental work.
A graph of the effective wall vorticity (\(\propto\) shear stress) versus distance is compared with the \(\text{Nu}\) distribution derived by experiment for a particular case (fig. 11.26). In general, \(\text{Nu}\) is seen to be inversely related to the modulus of the shear stress.

In the region of zero wall shear (approximately maximum \(\text{Nu}\)) turbulence energy is diffused from the mainstream to the wall without the generation of turbulence by shear. The local effective transport properties in this region must be a function of the mainstream turbulence level, the true viscous properties, and the distance from the wall. From the definition used for the effective viscosity it would seem that the maximum value of \(\text{Nu}\) is a function of \(Re\) upstream and \(Pr\). The model therefore predicts that \(\text{Nu}\) is independent of the divergence ratio, provided the flow is turbulent throughout. The practical implications of this statement, and empirical justification is given in part 9.7. The argument was supported substantially, but there was a tendency for viscous action to damp the turbulence downstream at low Reynolds numbers, thereby causing a reduction in \(\text{Nu}\).
A programme for solving the equations for laminar flow

Heat transfer with dissipation.

```
BEGIN; COMMENT; FILE;116 DISSIPATION;
REAI D,A,G,G,G,G;
INTEGER N,H,BBD;
REAL! ARRAY Q,R[0;200],F[1;6;0;200],P,I[0;200];
D:=.018;
BBD:=100;
WITH
FOR N:=2 STEP 1 UNTIL 200 DO
BEGIN
Q[N]:=2-2*(N-1)*(N-1)*D*D*D/3-2*(N-1)*D*D*D/3;
END;
F[1,0]:=1;
F[1,1]:=1-D;
F[2,0]:=1;
F[2,1]:=1;
F[3,0]:=1;
F[3,1]:=1;
F[4,0]:=1;
F[4,1]:=1;
M:=1;
P:=0;
IF M=3 OR M=6 THEN
A:=11;
END;
FOR N:=2 STEP 1 UNTIL 200 DO
BEGIN
GGG:=Q[N];
IF M=3 OR M=4 OR M=6 THEN
GGG:=Q[N]+2*(N-1)*D*D*D/3;
END;
M:=M+1;
IF M=5 THEN GOTO PP;
IF M=6 THEN GOTO PP;
IF M=7 THEN GOTO JJ;
F[5,0]:=F[1,200]/F[2,200];
F[5,1]:=F[5,0]-D;
F[6,0]:=F[3,BBD]/F[4,BBD];
F[6,1]:=F[6,0];
IF M=5 THEN GOTO PP;
JJ;
FOR N:=0 STEP 4 UNTIL 200 DO
BEGIN
NEWLINE(11);
PRINT(N*D,2,4,1);
SPACE(5);
PRINT(F[N,N],1,6);
SPACE(5);
PRINT(F[N,N],1,6);
END;
BBD:=BBD+1,25;
IF BBD>160 THEN GOTO EE;
END;
```
LAMINAR FLOW HEAT TRANSFER WITH DISSIPATION. THE FUNCTIONS $g$ AND $\phi$.

\[ f = g + \phi \frac{\eta}{x^\frac{1}{3}} \]

\[ g(0) = 1.2166 \]

\[ \phi(0) = 0.8566 \]

SIMILARITY FUNCTIONS $g(\eta)$ and $\phi(\eta)$.
CALCULATED NUSSELT NUMBERS FOR LAMINAR FLOW, ENTRANCE REGION
HEAT TRANSFER, AT HIGH PRANDTL NUMBERS, WITH VISCOSITY VARIATION.
LAMINAR FLOW HEAT TRANSFER WITH VARIABLE VISCOSITY. THE FUNCTIONS $f_o$ AND $f_1$.

\[ f = f_o + \xi \cdot \sqrt[3]{\beta_f} \]

\[ f_o(0) = 1.2135 \]
\[ f_1(0) = 0.2507 \]

SIMILARITY FUNCTIONS $f_o$ AND $f_1$. SIMILARITY VARIABLE $\eta = y/x^{\frac{1}{3}}$. 
FIGURE 11.6

Turbulent flow heat transfer in the entrance of a tube. The functions \( f_0 \) and \( f_1 \).

\[
f = f_0 + \left( \sigma \alpha R^3 x \right) f_1
\]

The similarity functions \( f_0 \) and \( f_1 \) depend on the similarity variable \( \eta = \frac{y}{z^{1/3}} \).

\[
f_0(0) = 1.9296, \quad f_1(0) = 1.6432
\]
THE EFFECTS OF DISSIPATION ON NUSSELT NUMBER AT THE ENTRANCE OF A TUBE WITH LAMINAR FLOW.

\[ \text{Re} \cdot \text{Pr.} = 10^6, \quad E = \frac{\mu U^2}{q_{1w}} \]

\[ \text{Re} \cdot \text{Pr.} = 10^3. \]

\( E_0, \quad E_1, \quad E_{10} \)

AXIAL DISTANCE FROM START OF HEATING (DIAMETERS).
**Figure 11.8**

The effects of dissipation on Nusselt number at given values of Reynolds number and parameter $E = \frac{\mu_1 L^2}{\kappa_1 W}$.
<table>
<thead>
<tr>
<th>Viscosity Variation Parameter</th>
<th>( \frac{x}{x} \times 10^{-6} )</th>
<th>( 1.002 \times 10^{-5} )</th>
<th>( 1.909 \times 10^{-5} )</th>
<th>( 3.560 \times 10^{-5} )</th>
<th>( 6.570 \times 10^{-5} )</th>
<th>( 1.205 \times 10^{-4} )</th>
<th>( 2.204 \times 10^{-4} )</th>
<th>( 4.024 \times 10^{-4} )</th>
<th>( 7.340 \times 10^{-4} )</th>
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<td>122.0</td>
<td>96.54</td>
<td>77.97</td>
<td>63.40</td>
<td>51.70</td>
<td>42.20</td>
<td>34.45</td>
<td>28.12</td>
<td>22.95</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>99.45</td>
<td>80.92</td>
<td>66.38</td>
<td>54.68</td>
<td>45.15</td>
<td>37.36</td>
<td>30.97</td>
<td>25.72</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
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<td>102.2</td>
<td>83.75</td>
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<td>57.50</td>
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<td>40.08</td>
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<td>1</td>
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</tr>
<tr>
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<td>134.1</td>
<td>109.7</td>
<td>91.42</td>
<td>76.97</td>
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<td>55.45</td>
<td>47.33</td>
<td>40.48</td>
<td>34.66</td>
</tr>
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The values of \( \text{Nu} \), \( \text{Nu}/\text{Nu}_0 \), \( \text{M}_{\text{wall}}(=\mu/\mu_{\text{wall}}) \).
Figure 11.10

The effect of viscosity variation on velocity and temperature distribution in laminar flow at
\[ X = \frac{x/r}{Re, Pr} = 7.34 \times 10^{-4}. \]

Viscosity variation parameter:
\[ \tilde{\gamma} = 0, 10, 20, 50, 500. \]

\[ \frac{T - T_0}{T_{\infty} - T_0} \text{ vs. } r/r_{\infty}. \]

Velocity vs. radius graph, with \( \tilde{\gamma} \) increasing.
FIGURE II.11

THE VARIATION IN NUSSELT NUMBER WITH DISTANCE FROM START OF HEATING, IN VARIABLE VISCOSITY, LAMINAR FLOW.

VISCOOSITY VARIATION PARAMETER \( \gamma \)

\( 0, 20, 50 \)

\( R_e \cdot Pr = 10^6 \)

\( R_e \cdot Pr = 10^5 \)

\( x/2R \) — AXIAL DISTANCE FROM START OF HEATING (DIAMETERS)
THE EFFECT OF VISCOSITY VARIATION ON THE NUSSELT NUMBER FOR LUMINAR FLOW, ENTRANCE REGION HEAT TRANSFER, WITH MODERATE VALUES OF THE RATIO — VISCOSITY AT START OF HEATING/Viscosity AT THE TUBE WALL, $\left( \frac{\mu_0}{\mu_{\text{wall}}} \right)$. 

**Figure 11.12**

VISCOUSITY VARIATION PARAMETER $y$.

- $y_{\text{wall}} = \frac{\mu_0}{\mu_{\text{wall}}}$
- $y_{\text{wall}} = \frac{\mu_0}{\mu_{\text{wall}}}$

Constants of the curves:
- $y_{\text{wall}} = \frac{\mu_0}{\mu_{\text{wall}}}$
- $y_{\text{wall}} = \frac{\mu_0}{\mu_{\text{wall}}}$

Small $x$ theory.
A comparison between the calculated Nusselt numbers (\( \text{Nu} \)) and the constant viscosity value, corrected by the wall viscosity ratio, according to Sieder and Tate (\( \text{Nu}, \text{M}_w^{1/2} \)).

\[ X = \text{dimensionless axial distance}/\text{Re}.\text{Pr}. \]
FIGURE 11.14

The effect of viscosity variation on the Nusselt number for laminar flow, entrance region heat transfer, with very large values of the ratio — viscosity at start of heating/viscosity at the tube wall.
FIGURE 11.15

The dimensionless velocity gradient at tube wall \( \frac{\partial v}{\partial y} \) as a function of Reynolds number \( Re \).
A comparison between theoretical and experimental Nusselt numbers, at small and large axial distances, for turbulent flow in a tube entrance region.
Figure 11.17

The variation in Nusselt number with distance in the turbulent entrance region of a tube. A comparison between theory and experiment.
THE VARIATION IN NUSSELT NUMBER WITH AXIAL DISTANCE IN THE TURBULENT ENTRANCE REGION OF A TUBE. A COMPARISON BETWEEN EXPERIMENT AND SEMI-THEORETICAL INTERPOLATION.
Figure 11.19
Computed streamlines and isotherms. 1:2 divergence. Re = 200, Pr = 100.
Figure 11.20.

Axial velocity profile for 1:2 divergence.

Re = 200.

$\bar{v}_1 = \text{mean axial velocity.}$

Axial distance (diameters):

- 0
- 1.65
- 5.85
- 10.55

Axial velocity $\left( \frac{v}{\bar{v}_1} \right)$ vs. radial distance $\left( \frac{r}{r_1} \right)$.
FIGURE 11.21a

(Nu / Pr^0.2) VERSUS AXIAL DISTANCE IN A DIVERGENCE. THE EFFECTS OF PRANDTL NUMBER FOR Pr = 100 AND 1000.

1:2 DIVERGENCE RATIO.

Re = 200.
Pr = 100.

Re = 200.
Pr = 1000.

FIGURE 11.21b

NUSSELT NUMBER VERSUS DISTANCE. COMPARISON WITH SHORT TUBE THEORY.

Re = 200.
Pr = 100.

1:2 DIVERGENCE.
Figure 11.22
Nusselt Number versus distance for different values of Reynolds number.

1:2 divergence ratio
Prandtl number = 100

Convergence error in numerical solution.

Re = 1,000

Smoothed solution.

Re = 200

Nu.

Axial distance (diameters)
Figure 11.23.

Computed streamlines and isotherms, 1:2 divergence.

Re = 1000   Pr = 100

Stable laminar flow.

Streamfunction $\psi$

Isotherms $\theta = \left( \frac{t - t_0}{\theta_1^2} \right)$

Axial distance $z = 0.10$ (diameters)
FIGURE 11.24

COMPARISON BETWEEN THEORY AND EXPERIMENT FOR NUSSELT NUMBER IN A 1:2 DIVERGENCE. Re = 200 (LAMINAR) PRANDTL NUMBERS 100 AND 1,000.

1:2 DIVERGENCE RATIO.
REYNOLDS NUMBER DOWNSTREAM=200.

REYNOLDS NUMBER DOWNSTREAM=200.

TEST 22.22:
Re = 403/202.
Pr = 408.

TEST 18.2:
Re = 376/89.
Pr = 230.

Pr=100.

Pr=1000.

( Nu )

25

20

15

10

0

10

20

30

40

AXIAL DISTANCE (DIAMETERS).
FIGURE 11.25

CALCULATED STREAMLINES ($\psi$) AND VORTICITY/RADIUS ($\omega/r$) FOR TURBULENT FLOW THROUGH A 1:2 DIVERGENCE.
Figure 11.26

Effective Vorticity at Wall, $\omega_{\text{wall}}$, for Turbulent Flows Through 1:2 Divergence Ratio.

Note: True vorticity at wall given by $\omega_{\text{wall}} = \omega_{\text{eff, wall}} \left( \frac{H_{\text{eff}}}{\mu} \right)$.

Nusselt Number for Turbulent Flow, where $H_{\text{eff}} = 0.013 \cdot V_{(0,0)} \cdot R_i$.

Hence $\omega_{\text{wall}} = \omega_{\text{eff, wall}} \left( \frac{0.013 \cdot R e_{\text{upstream}}}{2} \right)$.

Test 1: $L_2$, $R e = 28122/14122$, $P r = 57$.
12. DISCUSSION OF THE RESULTS AND THEIR APPLICATION.

12.1 THE SHORT STRAIGHT TUBE.

The axial distribution of the local coefficient of heat transfer \( h \) in a short tube can be derived directly by interpolating the data presented in figures 9.16, 9.17, 9.24 and 9.26.

It is necessary to consider the contribution of viscous dissipation, variations in viscosity, and free convection to the value of \( h \).

Viscosity variations are discussed in part 9.5., where \( h \) is shown to be proportional to \( \frac{M_{wall}^{0.14}}{M} \). This parameter is readily determined once the tube and bulk fluid temperatures are known.

If the heat flux is specified, the temperatures must be determined by an iterative procedure.

The theoretical appraisal of viscous dissipation in parts 11.1. (ii), (v) and (vii) showed that \( h \) is reduced in magnitude when the heat flux \( q \) is small. The effect on \( h \) is less than 10% when \( q > 20(\mu u^2/r_w) \) for laminar flow, or \( q > 0.2(\mu u^2/r_w)Re^{0.7}Pr^{-0.3} \) for turbulent flow. Several relationships are proposed in part 11.2. which may be used to correct the value of \( h \) if the above conditions are not satisfied.

Free convection was found to increase the value of \( h \) in laminar flows, and the criterion for determining whether such contributions were significant was as follows -

- Free convection was small if \( \left( \frac{GrPr}{RePr} \right)^{1/4} \left( \frac{x}{2r_w} \right)^{1/2} < 2.5 \)

The corrected value of \( h \) can be determined from figure 9.18.

For some applications a detailed knowledge of the local values of \( h \) is not required. In such instances it is useful to have an 'effective increase in tube length', which is defined as follows -
\[ \Delta L = \int_{0}^{L} \left( \frac{h}{h_\infty} - 1 \right) \, dx \]

where \( h_\infty \) = the value of \( h \) at large axial distances.
\( L \) = tube length.

This concept enables a designer to visualize conditions near the entrance as being equivalent to an extra length of tube, when the flow regime is turbulent. In laminar flow a mean value of \( h \) is sometimes used, as follows:

\[ h_m = \frac{1}{L} \int_{0}^{L} h \, dx \]

The latter result is easily obtained from the theoretical or experimental values of \( h \) discussed above, should the information be required. The value of \( \Delta L \) is worthy of investigation however because of its physical interpretation.

The theoretical results of part 11.2.(vi) permitted the transitional and turbulent values of \( h/h_\infty \) to be given with reasonable accuracy by

\[ \frac{h}{h_\infty} = \frac{33.4}{\gamma (1 - 2.87 \times 10^{-6} \gamma^3)} \]

for \( \gamma < 44.4 \), where \( \gamma = \left( \frac{x}{2r_w} \right) Re^{1.1} \)

The value of \( \Delta L \) becomes 22,100 \( Re^{-1.1} \) diameters after integration, which is numerically 2.4 diameters at \( Re = 4,000 \), and 0.9 diameters at \( Re = 10,000 \) (when \( L \) exceeds the entrance length).

12.2. THE CONVERGENCE.

When a sudden reduction in tube diameter occurs, the local values of \( h \) can be derived from figures 9.27 to 9.38. The magnitude of \( h \) was shown to be dependent on the Prandtl number and diameter ratio in a complicated way, so that it was impossible to define a unique relationship between \( h \) and Pr valid for all axial positions. A tedious but necessary procedure must be undertaken to interpolate the data provided, so that graphs of
(\(\text{Nu}/\text{Pr}^{\frac{3}{2}}\)) versus (distance) can be determined for a particular configuration and type of fluid. Although experiments were not carried out to assess the way in which \(\text{Nu}\) could be influenced by free convection, viscous dissipation, and variations in viscosity it would be reasonable, in the absence of further information, to assume that the methods suggested for application to the short tube configuration will give a fair estimate of such secondary effects. The similarity in the geometry and measured functions of \(\text{Nu}\) substantiate this suggestion.

If local values of \(h\) are not required for a given application it is recommended that for laminar flows \(h_m\) is calculated, and for transitional or turbulent flows \(\Delta L\) is determined. A simple numerical integration of the \((\text{Nu}/\text{Pr}^{\frac{3}{2}})\) functions can be carried out as discussed in part 12.1.

12.3. THE DIVERGENCE.

Consider first the local coefficient of heat transfer in a divergence with \(\text{Re}_1\) (upstream leg) greater than 2,500. From the data presented in figures 9.4-8 to 9.62 it might be found useful to plot graphs of \((h/h_m)\) versus (distance) for a particular diameter-ratio, and for a range of Reynolds numbers \((h_m\) can be assumed to be the value at 40 diameters downstream). This is possible because it has been demonstrated that \(\text{Nu} \times \text{Pr}^{\frac{3}{2}}\) at large axial distances, and in the region of the maximum value of \(h\). Alternatively, if local values of \(h\) are not required the value of \(\Delta L\) can easily be determined as discussed in parts 12.1 and 12.2.

With \(\text{Re}_1 < 2,500\) there is no simple method of estimating local coefficients of heat transfer reliably (for detailed discussion
see part 9.7.). In the first instance, a transition region of $Re_1$ was shown to exist in the approximate range 700 and 2,800 for diameter-ratios up to 1:3.34. In this region the two extremes of turbulent and laminar flow could give rise to values of $h$ differing by a factor of 5 to 7 times. The selection of appropriate data will obviously depend on the design criteria in the transition region. In the second instance, for all values of $Re_1$ less than 2,500 it was demonstrated that the shape and magnitude of the (Nu) versus (distance) functions were primarily dependent on $Re_1$ and Pr, but it was also observed that the functions were sensitive to the heat flux imposed. In view of the preceding arguments it is recommended that part 9.7 is understood before attempting to utilise the data presented in figures 9.48 to 9.62.

12.4. GENERAL CONSIDERATIONS.

The applications for which it is particularly advantageous to have information on local coefficients of heat transfer are those in which the heat-transfer channels, or cooling passages, (in the present discussion circular ducting), comprise short lengths having uniform diameter. A typical example is illustrated below, for which it is difficult to imagine how a reliable rate of heat transfer per unit temperature difference could be estimated without the data presented herein. The configuration might represent a cooling passage in a piece of machinery, and the designer could be interested in the minimum number of such passages required to hold down the temperature of a component, there being a limit on the available space. Many similar applications can be envisaged where large variations in the value of $h$ occur continuously with distance.
Other configurations will certainly arise where the engineer must use some discretion in applying the results of these tests. Typical cases might incorporate severe variations in the axial distribution of heat-flux, or the geometry of the system might not correspond closely with those included in this work, for example:

Unfortunately, no general guidance can be given for dealing with problems of this kind, each case must be considered independently. In the above diagram, the convergence nearest the outlet end might alter the flow pattern within the centre section substantially, although it is probable that the axial distribution of Nu in this region would be significantly different from the divergence results stated during laminar flow only.
13. CONCLUSIONS AND FUTURE WORK.

Measurements have been obtained of the local coefficient of heat transfer in short lengths of horizontal tube, and in the region of a discontinuity in tube diameter. Laminar, transitional and turbulent flows were investigated utilising several viscous fluids.

The effects of free convection, temperature dependent viscosity, and viscous dissipation on the heat transfer process were assessed with the proviso that such contributions were small in magnitude. In the latter case the assessment was purely theoretical, since dissipation was shown to play an insignificant role in the experiments carried out. Both theoretical and empirical analyses were instigated for the cases of free convection and variable viscosity.

In the experiments with the short tube it was first shown theoretically that Nu was proportional to Pr$\frac{3}{2}$, then the group $(NuPr^{-\frac{1}{2}})$ was shown by experiment to be a unique function of axial position for particular Re. It is implied in the foregoing that no secondary effects (e.g. free convection) were present, and the flow was fully developed at the entrance of the tube. When the flow was undeveloped at the entrance, the parameter $(\delta u/Pr^{\frac{3}{2}})$ was found to be a unique function, as described, only for the transitional and turbulent flow regimes. With laminar flow, $(NuPr^{-\frac{3}{2}})$ was found to be a weak function of Pr as well as Re and distance. The maximum difference between Nu-developed and Nu-undeveloped in these experiments was approximately 40% with Pr $\geq 90$, and 20% with Pr $\leq 500$. In general, the 'developed' correlations could therefore be used to give a reasonable estimate of 'undeveloped' coefficients of heat transfer when a viscous medium was utilised.
The axial distribution of $\text{Nu}$ close to a sudden convergence in diameter was determined experimentally for four different Prandtl numbers. The parameter $(\text{NuPr}^{-\frac{3}{2}})$ was shown to be a function of the axial position and $\text{Re}$ when turbulence was present in the section of tube downstream. With laminar flow downstream, the values of $(\text{NuPr}^{-\frac{3}{2}})$ close to the discontinuity were shown to increase as $\text{Pr}$ was reduced, for a particular Reynolds number. With particular values of $\text{Re}$ and $\text{Pr}$ downstream, the Nusselt numbers measured close to the convergence were found to increase with the ratio $(\text{upstream diameter})/(\text{downstream diameter})$. The magnitudes of $\text{Nu}$ were found to be greater than the corresponding values obtained with a short tube having undeveloped flow at the entrance, provided the turbulent regime prevailed. With laminar flows the measured values of $\text{Nu}$ were generally less than those obtained in the short tube experiments.

The separated region of flow, which occurred just downstream of a sudden divergence in diameter, gave rise to an axial distribution of $\text{Nu}$ with a well defined peak-value near to the discontinuity. When the Reynolds number for the section of tube upstream was greater than 2,500 the flow regime close to the step could be described as turbulent. The axial distribution of $\text{Nu}$ for such flows was expressible in the graphical form $(\text{NuPr}^{-\frac{3}{2}})$ as a function of axial distance, with $\text{Re}$ constant. The position of maximum $\text{Nu}$ was found to be invariant with $\text{Re}$ for a particular diameter ratio. Some theoretical substantiation of this was derived.

For lower Reynolds numbers (than 2,500 upstream) a complicated flow pattern was evident which was investigated using flow visualization experiments and numerical analysis. Laminar
flow was present when Re was of the order 100 and coefficients
of heat transfer were measured which were relatively much lower
than turbulent values. An intermediate flow regime was shown
to exist in which either turbulence or lamination could occur,
it was clear that there was no smooth transition from one regime
to the other.

At the lower Reynolds numbers just described, the
axial distribution of Nu was found to be dependent on the heat
flux. Generally, the peak in the Nu - distance function moved
upstream as the flux was increased.

A theoretical analysis indicated that the position of
maximum Nu was just upstream of the point of boundary layer
reattachment. It was shown also that a relationship of the kind
Nu \propto Pr^n was inadmissible as a means of correlating the experimental
data at low Reynolds numbers because the exponent n would have to
be a function of the axial position.

Certain aspects of these investigations could be extended
still further to give an 'in depth' appreciation of the heat
transfer mechanisms, and to provide information on a greater
range of practical conditions than are covered herein. For
example, the use of more highly viscous oils arises in practice,
and extremely small Reynolds numbers (creeping flows) are likely
to be encountered. In this thesis it has been indicated that a
geometrical discontinuity can be used to prevent the occurrence
of large coefficients of heat transfer at the entrance of a tube
having low Reynolds number flow. Further, it became apparent
that a geometrical discontinuity has a significant effect on
Nu at large axial distances. The rate of heat transfer could
easily be overestimated in these circumstances if one is led by
intuition in the absence of recorded measurements.

Other types of fluid could be investigated to advantage,
in particular the non-newtonian fluids. Comparatively, little
experimentation has been carried out in this area of heat-transfer,
and information on the effects of a sudden change in diameter
would prove a valuable contribution. Initially, pseudoplastic,
time-independent fluids (i.e. with a non-linear stress-strain
relationship) should be considered since the results would have
wide application in the process industries. A suitable dilute
polymer solution might be considered initially.

Other topics which should be pursued are as follows -
the investigation of a sudden divergence when the diameter ratio
is large (not necessarily with viscous fluids), the theoretical
analysis of laminar and turbulent heat transfer in a divergence
or convergence, and heat transfer in two-phase flow through a
discontinuous tube. The scope of the research could be increased
by considering other practical geometrical configurations; changes
in the flow direction such as occur in a helical tube would be a
suitable subject.
APPENDIX (A)

SPECIFICATION AND DESCRIPTION OF APPARATUS.

(1) Pump and Motor

Drysdale and Co. Ltd. Type Unilac U14/25
Supply 415 v, a.c. 3ph. 50 c/s.
Motor 25 h.p.
Centrifugal Pump 14 h.p. 100 lbf/in².

(2) Cooler

Sorek Radiators Ltd. Type 509304.
Cast iron shell, alum-brass tubes.
Specification with oil-viscosity 72 cP.

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<td>Temperature</td>
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<td>out</td>
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<td>2</td>
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<td>-</td>
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<td>Heat Transferred = 135,000 Btu/hr.</td>
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(3) Filters

Birfield Filtration Ltd.
One type 620 SP, with 80 mesh monel gauze strainer
One type FC1, with 5 micron filter cartridge.

(4) Flowmeters

Meterflow Ltd.
Frequency meter, type M707, 0 - 2,000 c/s.
Turbine flowmeter, type M2/1500/3150, 0 - 150 gal/min.
Turbine flowmeter, type M2/0625/115, 0 - 15 gal/min.
Meters calibrated on water, Aeroshell, and Tellus 27.
(5) **Thermocouple selector switch**

The Croydon Precision Instrument Co.

3 - 100 way selector switches, type 3F2.

(6) **Calibrated shunt**

The Cambridge Instrument Co. Ltd.

N.P.L. calibrated shunt.

Resistance 50 $\mu\text{Ohm}^{+0.025}_{-0}$ at 20°C with 200-2000A.

(7) **Potentiometer**

H. Tinsley & Co. Ltd.

Vernier potentiometer type 4025A.

(8) **Voltage controller**

Midland Transformer Co. Ltd.

On load power regulator type RDCM 50/250

Input 415 v 0 - 50A

Output 0 - 415 v 50A

(9) **Rectifier and smoothing unit**

Hackbridge and Hewittic Electrical Co. Ltd.

Rectifier transformer type 106029A

Primary 415 v, 103A

Secondary 26 v, 1640A

Metal rectifier type 111.2

Output 30 v, 2000A.
Heat Transfer in Slug Flow Using the Lavelle Procedure as Modified by Sellar.

The problem is to determine the axial distribution of $Nu$, in a tube passing fluid at uniform velocity, when a constant heat flux is applied downstream of a point $x = 0$. When utilising Sellar's procedure (Ref: C.13) a solution must first be obtained for the case of constant tube temperature, then this becomes a particular solution which can be modified to suit other boundary conditions.

The energy equation is written
\[
\frac{d\Theta}{dx} = 2 \frac{d^2\Theta}{dy^2}
\]
where $t_w$ = tube wall temperature,
$t_\infty$ = the inlet temperature,
\[
\Theta = \frac{t - t_\infty}{t_w - t_\infty}
\]
\[
X = \frac{x}{r_w \operatorname{RePr}} \quad , \quad Y = \frac{y}{r_w}
\]

Now choosing the similarity variable $\gamma = \frac{Y}{X^3}$, so that $\Theta(x, y) = \Theta(\gamma)$, the equation becomes,
\[
\Theta'' + \frac{2}{4} \Theta' = 0
\]
or
\[
\frac{d}{d\gamma} \left( \gamma^{\frac{3}{2}} \Theta' \right) = 0.
\]
Integrating with $\Theta(\infty) = 0$, $\Theta(0) = 1$, gives
\[
\Theta = \left( \int_0^{\infty} e^{-\frac{7}{2} \gamma} d\gamma \right) / \left( \int_0^{\infty} e^{-\frac{7}{2} \gamma} d\gamma \right).
\]

The Nusselt number at constant temperature is found from
\[
Nu_{\text{temp}} = 2 \frac{d\Theta}{dy} \bigg|_{Y=0} = 2 \frac{d\gamma}{dy} \Theta'(0)
\]

hence
\[
Nu_{\text{temp}} = 2 X^{-\frac{3}{2}} \Theta'(0).
\]
\( \theta'(0) \) is as follows

\[
\theta'(0) = \left[ \int_0^{\infty} e^{-\gamma^2} d\gamma \right]^{-1} = \frac{1}{4.1.\Gamma(\frac{1}{2})}
\]

so that \( N_u_{\text{temp}} = 0.798 X^{-\frac{1}{2}} \)

To obtain a similar result for uniform heat flux, \( N_u_{\text{heat}} \), the following integral is utilized.

\[
q = \int_0^X h' (x, \xi) \, dt_w
\]

Where \( h' \) is coefficient of heat transfer for a step in temperature, with \( \xi \) unheated length, or \( \left( \frac{2h'}{K} \right) = 0.798 (X-\xi)^{-\frac{1}{2}} \).

Rewriting,

\[
1 = \frac{0.798}{2} \int_0^X (X-\xi)^{-\frac{1}{2}} \frac{d\theta_w}{d\xi} \, d\xi, \quad \text{where} \quad \theta_w = \frac{(t_w-t_{\infty})}{q_0 r_w} K
\]

Suppose \( N_u_{\text{heat}} = A X^{-\frac{3}{2}} \), and constant \( A \) is to be determined. The wall temperature is given by \( \theta_w = 2/N_u_{\text{heat}} \), so that

\[
\frac{d\theta_w}{d\xi} = (A \xi^{\frac{3}{2}})^{-1}
\]

Putting this into the integral

\[
\frac{2A}{0.798} = \int_0^X (X-\xi)^{-\frac{1}{2}} \xi^{-\frac{3}{2}} \, d\xi
\]

\[
= \frac{\Gamma(\frac{1}{2}) \cdot \Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2} + \rac{3}{2})}
\]

Therefore \( A = 1.252 \)

or \( N_u_{\text{heat}} = 1.252 X^{-\frac{1}{2}} \).
BIBLIOGRAPHY.

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